How to Achieve the Optimal DMT of Selective Fading MIMO Channels?

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Abstract—In this paper, we extend the non-vanishing determinant (NVD) criterion to the selective fading channel case. A new family of split NVD parallel codes, that satisfies this design criterion, is proposed to achieve the optimal diversity multiplexing tradeoff (DMT) derived by Coronel and Bölcskei, IEEE ISIT, 2007. This result is of significant interest not only in its own right, but also because it settles a long-standing debate in the literature related to the optimal DMT of selective fading channels.

I. INTRODUCTION AND MOTIVATIONS

The diversity multiplexing tradeoff (DMT) proposed by Zheng and Tse in [1] is a powerful approach to characterizing the dual benefits in terms of diversity and spatial multiplexing in the high SNR regime. In order to achieve the optimal diversity multiplexing tradeoff for the flat fading MIMO channel, Belfiore et al. introduced the non-vanishing determinant criterion in [2]. Later, Elia et al. [3] proved that this criterion is a sufficient condition to achieve the optimal DMT using a full rate code.

While most of the above results address the case of flat fading channels, the general channel model of time-frequency selective channels has been considered by Coronel et al. in [4], [5] where the optimal DMT is derived. Moreover, a DMT optimal coding scheme based on a joint precoder and parallel codes construction, is proposed. As the block fading channel is a special case of the time-frequency selective channel, it is expected that the DMT expression in [5] matches with the corresponding result in [1]. This is, however, not the case and has given rise to lots of debate in the literature e.g. [6]. A rigorous interpretation of this incoherence in results remains an open problem. The present paper settles the issue and shows that the DMT derived in [5] is, indeed, achievable.

Contributions: We consider a particular class of the general channel model considered in [4], [5] where the channel is selective either in time or in frequency. For this class of channels, we propose a systematic way to achieve the optimal DMT by extending the non-vanishing determinant criterion to the selective channel case. A new code construction based on split NVD parallel codes is then proposed to satisfy the NVD parallel criterion. Moreover, for the block fading channel, we provide an extension of the geometrical interpretation to show the achievability of the optimal DMT. This result is of significant interest not only in its own right, but also as it shows that the optimal DMT in [5] is achievable for all the classes of fading channels including the block fading channel.

Outline of the paper: The rest of the paper is organized as follows. In Section II, we define the selective fading channel model. We derive in Section III the code design criterion required to achieve the optimal DMT for this class of channels. We propose, in Section IV, a new family of split NVD parallel codes to satisfy this code design criterion. Then, we provide in Section V a geometrical interpretation of the achievable DMT for the particular case of block fading channels. Finally, Section VI concludes the paper.

Notation: The notation used in this paper is as follows. Boldface lower case letters v denote vectors, boldface capital letters M denote matrices. M† denotes conjugate transposition. ∥H∥F = Tr(HH†) is the Frobenius norm of a matrix. Tr{A} refers to the trace of matrix A. I_N stands for the N × N identity matrix. diag{A_n}_{n=0}^{N-1} denotes the block diagonal matrix containing A_n on its diagonal. The non zero eigenvalues of A ordered in ascending order are denoted by λ_i(A). CN represents the complex Gaussian random variable. \mathbb{E}_X is the mathematical expectation w.r.t. to the random variable X. Equality in distribution between two random variables X and Y is represented by X \sim Y. Exponential equality is denoted by f(x) = x^\beta, i.e. \lim_{x \to \infty} \frac{\log f(x)}{\log x} = b, and ≥_1, ≤_1 denote the exponential inequality. |A| denotes the cardinality of a set A. Finally, A \otimes B denotes the Kronecker product of the matrices A and B.

II. CHANNEL AND SIGNAL MODEL

The input-output relation for the class of channels considered in this paper is given by

\[ Y_{n}^{[n_r \times T]} = \sqrt{\frac{\text{SNR}}{n_t}} H_{n}^{[n_r \times n_t]} X_{n}^{[n_t \times T]} + Z_{n}^{[n_r \times T]}, \]

where \( n = 0, 1, \ldots, N - 1 \) represents the sub-channel \( n \), the sub-channel \( H_{n}^{[n_r \times n_t]} \) is a \( n_r \times n_t \) MIMO channel that remains constant during all the duration of the transmission \( T \), \( X_{n} \) represents the transmitted signal, and \( Z_{n} \) denotes the additive i.i.d. \( \mathcal{CN}(0, I) \) noise. The channels \( H_{n} \) are correlated across the sub-channels \( n = 0 \ldots N - 1 \) according to

\[ H = [H_0 \ldots H_{N-1}] = H_{w} (R_{\text{det}}^{1/2} \otimes I_{n_t}), \]
where \( \mathbf{R}_{\text{fi}} \) is the \( N \times N \) correlation matrix between the scalar subchannels with rank equal to \( \rho \leq N \), \( \mathbf{H}_w \) is an \( n_r \times N n_t \) matrix with i.i.d. \( \mathcal{CN}(0,1) \) entries. The transmitted signal satisfies the following power constraint,

\[
\sum_{i=0}^{N-1} \mathbb{E}[\| \mathbf{X}_i \|_F^2] \leq T N. \tag{3}
\]

Throughout this paper, we set \( m = \min(n_t, n_r) \) and \( M = \max(n_t, n_r) \).

The input-output relation considered in (1) models the case when the channel is selective either in time or in frequency. For the frequency selective channel, \( n \) stands for the frequency and the channel is constant across time. In this case, \( N \) represents the total number of subcarriers and \( \mathbf{R}_{\text{fi}} \) is a circulant matrix. For the time selective case (or the block fading channel), the channel remains constant during a block \( n \) of \( T \) time slots and changes in a statistically independent manner across blocks. For this case, \( N \) represents the total number of blocks and \( \mathbf{R}_{\text{fi}} = \mathbf{I}_N \).

III. TOWARDS THE OPTIMAL DMT OF SELECTIVE FADING CHANNEL

In this section, we extend the NVD space time code design criterion to the case of the MIMO selective fading channels. We start by briefly reviewing some basic preliminaries on the DMT in Subsection III-A and on the outage bound of the selective fading channel in Subsection III-B. Then, we derive, in Subsection III-C, a sufficient condition on the code required to achieve the optimal DMT for this class of channels.

A. Diversity multiplexing tradeoff (DMT)

Let \( \mathcal{X}_p(\text{SNR}) \) be a family of coding schemes operating at a given SNR, and let \( R(\text{SNR}) \) denote the rate transmitted per sub-channel, such that,

\[
R(\text{SNR}) = r \log \text{SNR},
\]

where \( r \) is the multiplexing gain per sub-channel.

The diversity multiplexing tradeoff (DMT) of the coding scheme \( \mathcal{X}_p(\text{SNR}) \) is defined as the SNR exponent of the error probability \( P_{e, \mathcal{X}_p}(r, \text{SNR}) \) using maximum-likelihood decoding such that

\[
d(r) = -\lim_{\text{SNR} \to \infty} \frac{\log P_{e, \mathcal{X}_p}(r, \text{SNR})}{\log \text{SNR}}.
\]

B. Outage bound on the DMT of selective fading channel

The discussion about the outage formulation will be revisited in the last Section V. We just recall here the Jensen outage bound derived by Coronel et al. in [5]. We refer the interested reader to [4] and [5] for more details.

Theorem 1 (Outage bound on the DMT): For a selective fading channel, the outage probability is lower-bounded by,

\[
P_{\text{out}}(r) \geq P_J(r) \equiv \text{SNR}^{-d_J(r)}
\]

where,

\[
d_J(r) = (\rho M - r)(m - r). \tag{4}
\]

C. Optimal design criterion

When the channel is selective either in time or in frequency, the optimal code design criterion required to achieve the optimal DMT is summarized in the following theorem.

Theorem 2 (Sufficient condition for DMT achievability): A coding scheme \( \mathbf{X} \in \mathcal{X}_p(\text{SNR}) \) achieves the optimal DMT \((\rho M - r)(m - r)\), if for any two different codewords \( \mathbf{X}, \hat{\mathbf{X}} \in \mathcal{X}_p(\text{SNR}) \), the eigenvalues of the block diagonal matrix \( \mathbf{D} \mathbf{D}^\dagger \), where \( \mathbf{D} = \text{diag}(\{ \mathbf{X}_n - \hat{\mathbf{X}}_n \})_{n=0}^{N-1} \) satisfy

\[
\min_{\mathbf{X}, \hat{\mathbf{X}} \in \mathcal{X}_p(\text{SNR})} \prod_{i=1}^{m} \lambda_i(\mathbf{D} \mathbf{D}^\dagger) \geq \frac{1}{2^{\rho(\text{SNR})}+\alpha(\text{SNR})}. \tag{5}
\]

Proof: The proof of this theorem will not be detailed here due to the lack of space. The same steps as the proof of [Theorem 1 in [4]] should be followed. The main difference here is that the NVD parallel criterion in (5) guarantees to have a full rank matrix without requiring any additional precoder.

IV. SPLIT NVD PARALLEL CODES FOR SELECTIVE FADING CHANNEL

In this section, we propose a new family of split NVD parallel codes to achieve the optimal DMT of \((\rho M - r)(m - r)\). Before studying the optimality of these split NVD parallel codes, we briefly review the structure of the NVD parallel codes in Subsections IV-A and IV-B. An equivalent system model for the selective channel is defined in Subsection IV-C. Finally, the code construction and the optimality of the split NVD parallel code is addressed in Subsection IV-D.

A. NVD parallel scheme

Let \( \mathbf{X} = \text{diag}(\{ \mathbf{X}_n \})_{n=0}^{N-1} \in \mathcal{X}_p(\text{SNR}) \) be the block diagonal matrix containing the transmitted codeword \( \mathbf{X} \), in (1), and constructed such that \( \mathbf{X} = \theta \mathbf{X} \), where \( \theta \) is a scaling factor that depends on the structure of the code, and chosen to ensure the power constraint in (3). The block diagonal matrix \( \mathbf{X} = \text{diag}(\{ \mathbf{X}_n \})_{n=0}^{N-1} \) is an NVD parallel code denoted by \( \mathcal{C}(\text{SNR}) \), and defined as follows:

Definition 1 (NVD parallel scheme): Let \( \mathcal{A}(\text{SNR}) \) be an alphabet\(^2\) that is scalably dense, such that

\[
\forall s \in \mathcal{A}(\text{SNR}) \quad \Rightarrow \quad |s|^2 \leq |\mathcal{A}(\text{SNR})|.
\]

Then, \( \mathcal{C}(\text{SNR}) \) is called NVD parallel code if,

1) Each entry of \( \mathbf{X} \) is a linear combination of symbols carved from \( \mathcal{A}(\text{SNR}) \).
2) The total number of transmitted symbols carved from \( \mathcal{A}(\text{SNR}) \) is equal to \( T N n_t \).
3) For any pair of different codewords \( \mathbf{X} \) and \( \hat{\mathbf{X}} \in \mathcal{C}(\text{SNR}) \), the NVD property is satisfied

\[
\det((\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^\dagger) \geq \kappa > 0, \tag{6}
\]

\(^2\)We assume here without restriction that the signal constellation is a QAM constellation, \( i.e., \mathcal{A}(\text{SNR}) = \mathcal{A}_{\text{QAM}}(\text{SNR}) \). This can be also extended to the case of HEX constellations.
with $\kappa$ is a constant independent of SNR.

As shown in [7], the scaling factor $\theta$ that ensures the power constraint is,

$$\theta^2 \leq |A(\text{SNR})|^{-1}. \quad (7)$$

Using the NVD parallel criterion in (6) and the value of $\theta^2$ in (7), the eigenvalues of the block diagonal matrix $D = X - \hat{X} = \tau(\Xi - \hat{\Xi})$ for any different codewords $X, \hat{X}$, are such that,

$$\prod_{i=1}^{Nn_t} \lambda_i((DD^T)^f) = \left| \det(\Xi - \hat{\Xi}) \right|^2 \geq \frac{1}{|A(\text{SNR})|^{Nn_t}}.$$

Due to the power constraint in (3), these eigenvalues necessarily satisfy $\lambda_i((DD^T)^f) \leq 1$. Then, the NVD parallel criterion is equivalent to,

$$\min_{X, \hat{X} \in \chi_p(\text{SNR})} \prod_{i=1}^{m} \lambda_i((DD^T)^f) \geq \frac{1}{|A(\text{SNR})|^{Nn_t}}. \quad (8)$$

B. Cyclic division algebra (CDA) code structure

We recall here the most relevant concepts of the construction of the codeword matrix $\Xi = \text{diag}(\Xi_i)_{i=0}^{N-1}$ based on cyclic division algebra. We refer the reader to [8], [7] for more details on the NVD parallel code construction. In the following, let $\mathbb{F}$ denote the Galois extension of degree $N$ over $\mathbb{Q}(i)$ and with generator $\tau$, and $\mathbb{K}$ be a cyclic extension of degree $n_t$ over $\mathbb{F}$ having $\sigma$ as generator. The code $\Xi$ is constructed by setting $\Xi_i = \tau_i(\hat{\Xi})$ where $\hat{\Xi} = \Xi_0$ belongs to the cyclic division algebra $\mathcal{C} = (\mathbb{K}/\mathbb{F}, \sigma, \gamma)$, and $\gamma \in \mathbb{F}$ chosen such that $\gamma, \gamma^2, \ldots, \gamma^{n_t-1}$ are not norms of an element of $\mathbb{K}$. The matrix $\hat{\Xi}$ is defined such that

$$\hat{\Xi} = \begin{pmatrix} x_0 & x_1 & \cdots & x_{n_t-1} \\ \gamma \sigma(x_{n_t-1}) & \sigma(x_0) & \cdots & \sigma(x_{n_t-2}) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma \sigma^{n_t-2}(x_1) & \gamma \sigma^{n_t-1}(x_2) & \cdots & \sigma^{n_t-1}(x_0) \end{pmatrix},$$

where, $x_i = \sum_{j=1}^{Nn_t} s_{i,j} \omega_j$, $s_{i,j} \in A(\text{SNR})$ and $\omega_j \in \mathbb{K}$.

C. Equivalent model scheme

The equivalent model representing the selective fading channel defined in (1) has the following form,

$$Y_e[Nn_r \times NT] = \sqrt{\frac{\text{SNR}}{n_t}} H X_e[Nn_t \times NT] + Z_e[Nn_r \times NT], \quad (9)$$

where $H = \text{diag}(H_{i=0}^{N-1}) \in \mathbb{C}^{Nn_r \times Nn_t}$ is the channel block diagonal matrix and $X_e = [X_{e,0}^{T} \ldots X_{e,N-1}^{T}]^T$ is the transmitted coding scheme such that $E[X_eX_e^H] = I_{NT}$. The model in (9) operates at the same multiplexing gain as generator. The optimal DMT of the selective fading channel $d_s(r)$ is related to the equivalent scheme by $d_s(r) = d^*(\frac{r}{N})$, where $d^*(r)$ is the optimal DMT of the equivalent model.

D. Split NVD parallel codes and optimality

The NVD parallel codes as put straightforwardly by Lu in [7] and Yang et al. in [8] are sub-optimal, as the DMT achieved by these codes is only $\rho(n_t - r)(n_r - r) < (\rho M - r)(m - r)$. The main idea of the new split code construction is to design a coding scheme for the equivalent model in subsection IV-C that guarantees to transmit a rate of $R(\text{SNR})$ over each sub-channel and to satisfy the NVD parallel criterion in Theorem 2. The two possible ways of splitting the data over the parallel channels are depicted in Figure 1.

For the first transmission scheme studied in [7] and depicted in Figure 1(a), the total rate is transmitted during only $T$ slots over each sub-channel. Each block $\tau_i(\hat{\Xi})$ contains $TNn_t$ symbols carved from a signal constellation $A_d(\text{SNR})$. In order to maintain a rate of $R(\text{SNR})$ over each sub-channel, the size of the constellation $|A_d(\text{SNR})|$ should be chosen such that,

$$R(\text{SNR}) = r \log \text{SNR} = \frac{1}{NT} \log |A_d(\text{SNR})|^{n_tTN}.$$ 

i.e., $|A_d(\text{SNR})| = \text{SNR}^{\frac{r}{NT}}$. It can easily be verified that for this choice of signal constellation size, the NVD parallel criterion in (8) is,

$$\min_{X, \hat{X} \in \chi_p(\text{SNR})} \prod_{i=1}^{m} \lambda_i((DD^T)^f) \geq \frac{1}{2^{N \log |A_d(\text{SNR})|^{r/NT}}} \frac{1}{2^{N \log \text{SNR}^{\frac{r}{NT}}}}.$$ 

Obviously, the sufficient condition in Theorem 2 is not satisfied in this case. The achievable DMT by this transmission
scheme is only $\rho(n_t - r)(n_r - r)$ as shown in [7], and it is therefore sub-optimal.

In the second transmission scheme shown in Figure 1(b), the total rate is split equally among all the $NT$ slots. Each block $\Xi_i$ transmits $TNn_t$ symbols carved from a signal constellation $A_i(SNR)$. The same $TNn_t$ symbols are transmitted over blocks $\Xi_i \ldots \Xi_{N-1}$, but encoded differently. However, different symbols are transmitted over two different blocks $\Xi_i$ and $\Xi_j$. In order to maintain the rate of $R(SNR)$ over each sub-channel, the signal constellation $A_i(SNR)$ should be chosen such that,

$$R(SNR) = r \log SNR = \frac{1}{T} \log |A_i(SNR)|^T.$$  

The size of the signal constellation for the split NVD parallel scheme is therefore reduced compared to the block diagonal case, and

$$|A_i(SNR)| = SNR^{\frac{TN}{r}} = |A_d(SNR)|^T.$$  

Due to the block diagonal channel matrix structure in (9), it can be deduced that the split NVD parallel code is equivalent to a concatenation of $N$ independent parallel NVD codes, where the symbols of each NVD parallel code are carved from a constellation $A_i(SNR)$ with size $SNR^{\frac{TN}{r}}$. The system is in error if at least one of the NVD parallel codes is in error. For each NVD parallel code with symbols carved from $A_i(SNR)$, it can be easily verified by replacing the cardinality of $A_i(SNR)$ in (8) that the NVD parallel criterion in Theorem 2 is satisfied, i.e.,

$$\min_{X \in X_p(SNR)} \prod_{i=1}^{m} \lambda_i(DD) \geq 1 \frac{1}{2^R(SNR)+\alpha(SNR)}.$$  

The split NVD parallel codes\(^3\) in Figure 1(b) achieve therefore the optimal DMT of $(\rho M - r)(m - r)$.

The comparison of both schemes is depicted in Figure 2 for the case of $2 \times 2$ MIMO block fading channel with $N = 2$ blocks. For a rate per channel use equal to 4 bpcu (resp. 8 bpcu), we need to use a BPSK (resp. QPSK) constellation for the scheme with split code and QPSK (resp. 16QAM) constellation for the scheme with NVD parallel code. As it can be shown, the gain of the split codes compared to the NVD parallel case is significant when the spectral efficiency of the code increases.

V. APPLICATION TO THE BLOCK FADING CHANNEL

The block fading channel is a particular case of the selective fading channel model considered in (1) with covariance matrix $R_{\Xi} = I_N$. The optimal DMT expression is therefore $d^*(r) = (NM - r)(m - r)$, which is the DMT expression of the general channel model considered in [4], [5] applied to this particular channel setting. Obviously, this result does not match with the corresponding result in [1], i.e., $d_i(r) = N(M - r)(m - r) \leq d^*(r), \forall r$. This incoherence in results has given rise to lots of debate in literature e.g. [6]. The authors of [6] base their arguments on a non-accurate outage probability derivation ($P_{out}(r) \doteq SNR^{-d_i(r)}$) to claim that the DMT of the block fading channel cannot exceed $d_i(r) \leq d^*(r)$. As we will show in the following, deriving the analytical outage probability is not a straightforward generalization of the flat fading channel and should be carefully performed. The outage derivation we provide here is based on the geometrical argument previously used for the flat fading channel in [1] and for the selective fading case in [4].

1) Geometrical interpretation: For the block fading channel, the outage probability is,

$$P_{out}(r) = P \{ \log \det \left( I + \frac{SNR}{nt} \mathcal{H}_i \right) < Nr \log SNR \},$$

where $\mathcal{H} = \text{diag}(\mathcal{H}_j)_{j=0}^{N-1}$ is the block diagonal channel matrix.

In order to generalize the geometrical interpretation in [4] to the block fading channel, we start first by finding an equivalent expression of the outage probability. For this, we consider $h_{ij}$ the $N \times 1$ Gaussian vector $\sim CN(0, I_N)$ containing the $N$ independent channel realisations between transmit antenna $j$ and receive antenna $i$. It is well-known that the Gaussian vector $h_{ij}$ is identically distributed as $\mathcal{F} h_{\omega,ij}$ for any unitary matrix $\mathcal{F}$, i.e., $h_{ij} \sim \mathcal{F} h_{\omega,ij}$.

In the following, we specify our result to the case where $\mathcal{F}$ is a $N \times N$ Fast Fourier Transform (FFT) matrix. This means that each channel realisation is identically distributed as,

$$h_{ij}^n \sim \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} h_{ij,\omega}^n e^{-j2\pi ln/N}, \quad n = 0 \ldots N - 1.$$  

The block diagonal matrix $\mathcal{H}$ is therefore identically dis-
distributed as $D_H$, i.e., $\mathcal{H} \sim D_H$, where,

$$D_H = \frac{1}{\sqrt{N}} \begin{bmatrix}
\sum_{l=0}^{N-1} H_{w,l}^0 & \cdots & \sum_{l=0}^{N-1} H_{w,l}^{N-1}
\end{bmatrix}, \quad (10)
$$

with $\omega_l = e^{-j 2\pi l / N}$ and $H_{w,t} = (h_{ij,\omega})_{1 \leq i \leq n_r, 1 \leq j \leq n_t}$.

Consequently, the mutual information is identically distributed as,

$$I(x, y | H) \sim \log \det \left( I + \frac{\text{SNR}}{Nn_t} D_H D_H^\dagger \right) \sim I(x, y | H).$$

By using an FFT precoder and an FFT equalizer as in an OFDM system to transmit over the channel $D_H$ in (10), the matrix $D_H D_H^\dagger$ can be made unitarily equivalent to $C_H C_H^\dagger$, where

$$C_H = \begin{bmatrix}
H_{w,0} & H_{w,1} & \cdots & H_{w, N-1}
H_{w,N-1} & H_{w,0} & \cdots & H_{w,N-2}
\vdots & \vdots & \ddots & \vdots
H_{w,1} & H_{w,2} & \cdots & H_{w,0}
\end{bmatrix}. \quad (11)
$$

Thus, the corresponding mutual information $I_D(\text{SNR})$ can be written as,

$$I_D(\text{SNR}) = \log \det \left( I + \frac{\text{SNR}}{Nn_t} C_H C_H^\dagger \right) \sim I(x, y | H).$$

It follows therefore that the outage probability is such that,

$$P_{out}(r) = \mathbb{P} \left\{ \log \det \left( I + \frac{\text{SNR}}{Nn_t} C_H C_H^\dagger \right) < N' \log \text{SNR} \right\}.$$

Following the geometrical interpretation of the flat fading channel in [1], the typical outage event occurs when the channel matrix $C_H$ is close to the manifold of all matrices with rank $Nr$ denoted by $\mathcal{R}_{N_r}$, such that,

$$\mathcal{R}_{N_r} = \{ C_H : \text{rank}\{C_H\} = Nr \}.$$

By following the same reasoning as in [1], this requires that the $d(r)$ components of $C_H$ orthogonal to $\mathcal{R}_{N_r}$ to be collapsed, i.e., be on the order of $\text{SNR}^{-1}$. The probability of this event is $P_{out}(r) \leq \text{SNR}^{-d(r)}$. The number of these components is given by

$$d(r) = N M m - \dim(\mathcal{R}_{N_r}),$$

where $\dim(\mathcal{R}_{N_r})$ is the sufficient minimal number of parameters required to specify matrix $C_H$ with rank $Nr$.

2) Dimensionality of $\mathcal{R}_{N_r}$: We first note that due to the structure of $C_H$ in (11), the number of parameters required to characterize a matrix $C_H$ in $\mathcal{R}_{N_r}$ is equal to the number of parameters required to specify an $m \times N M$ matrix $(m = \min(n_t, n_r)$ and $M = \max(n_t, n_r))$ with rank $r$ that contains the $n_t$ first columns if $n_t \leq n_r$, and the $n_r$ first rows if $n_r \leq n_t$, Characterising a matrix $C_H$ with rank $Nr$ reduces therefore to the problem of characterizing a matrix of dimension $m \times N M$ with rank $r$ that requires only $N M r + (m - r) r$, i.e.

$$\dim(\mathcal{R}_{N_r}) = N M r + (m - r) r,$$

where $N M r$ is the number of independent parameters needed to identify $r$ independent vectors and $(m - r) r$ parameters are needed to identify the linear dependent vectors as a function of the $r$ independent vectors. It can be easily verified here that the $N M r$ free i.i.d. Gaussian parameters that identify the $r$ linear independent vectors generate a block circulant matrix with rank $Nr$ with a probability equal to one.

It should be finally emphasized that the minimal number of parameters needed to describe the subspace $\mathcal{R}_{N_r}$ is therefore different than the number of parameters needed to characterize $N$ independent subspaces $\mathcal{R}_r$ separately, i.e., $\dim(\mathcal{R}_{N_r}) \neq N \dim(\mathcal{R}_r) = N(Mr + (m - r)r) > N M r + (m - r) r$, $\forall r$.

It can be deduced that the optimal DMT for the class of block fading channel is,

$$d_{out}(r) = N M m - \dim(\mathcal{R}_{N_r}) = (N M - r)(m - r),$$

and not,

$$d_{out}(r) \neq N M r - N \dim(\mathcal{R}_r) = N(M - r)(m - r).$$

VI. CONCLUSION

In this paper, we propose a new family of split NVD parallel codes to achieve the optimal DMT in [5] for the case of time or frequency selective channels. We show that the optimal DMT expression in [5] is, indeed, achievable for all the classes of selective fading channels, including the block fading channel. This result is also supported by the geometrical interpretation of this optimal achievable DMT for the block fading channel.

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