PERFORMANCE EVALUATION FOR SCATTERING MIMO CHANNEL MODELS

D. Gesbert

Iospan (formerly Gigabit) Wireless Inc.,
3099 North First Street,
San Jose, CA 95134
gesbert@iospanwireless.com

H. Bölcskei, D. A. Gore, A. J. Paulraj

Information Systems Laboratory
Department of Electrical Engineering
Stanford University, Stanford, CA 94305

Abstract – Using a multiple-input multiple-output (MIMO) outdoor wireless fading channel model recently introduced by the authors, we study the impact of physical propagation parameters in a full scattering scenario on outage capacity and ergodic capacity. In particular, introducing the concept of scatterer-to-scatterer (STS) rank, we show that a MIMO channel with correlated inputs (or outputs) but with full STS rank yields better performance from an outage capacity point-of-view than a channel with i.i.d. inputs and outputs and low STS rank.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless systems have been shown theoretically to have significantly higher capacity than more traditional single-input multiple-output (SIMO) systems [1, 5, 2]. In addition to enhanced diversity advantages, MIMO links can offer multiplexing gains by opening parallel spatial data pipes within the same bandwidth provided rich enough scattering is present [7]. Recently the authors have introduced a general statistical model for MIMO wireless outdoor channels [4, 3]. The model presented by the authors improves upon previously reported models [1, 9, 6, 7, 2] in that it allows to investigate the behavior of channel capacity as a function of parameters such as the local scattering radii at the transmitter and the receiver, the distance between the transmitter and the receiver, and the antenna beamwidths and spacing. The model suggests that spatial fading correlation alone is not sufficient to describe the overall rank of the channel. It allows, for example, to describe as one particular case, MIMO channels with uncorrelated spatial fading at the transmitter and the receiver but reduced channel rank (and hence low capacity) termed pin-hole [4] or key-hole channels [11].

Contributions. In this paper, we introduce the concept of scatterer-to-scatterer (STS) rank, which along with the rank of the input correlation matrix ("input rank") and the rank of the output correlation matrix ("output rank"), govern the MIMO channel capacity. We will find that the STS rank has a significant impact both on ergodic (average) capacity and on outage capacity. The latter is due to the fact that the STS rank determines the fading distribution of the MIMO channel, which can range from Rayleigh to double Rayleigh. In particular, the new model suggests that a MIMO channel with uncorrelated fading at both the transmitter and the receiver but low STS rank will perform worse in terms of outage capacity than a channel with fully correlated spatial fading at either the transmitter or the receiver and full STS rank. We provide an intuitive explanation for this phenomenon and describe realistic physical situations for which this effect occurs. Finally, we validate our claims through Monte-Carlo simulations.

2. CAPACITY OF MIMO CHANNELS

Throughout the paper, we assume that N transmit and M receive antennas are employed. We restrict our discussion to the frequency-flat fading case and assume that the transmitter has no channel knowledge whereas the receiver has perfect channel knowledge. Two situations are considered, the *ergodic* and the *nonergodic* case. In both cases we assume that the channel remains fixed within one symbol interval and then changes in an independent fashion to a new realization. Outage capacity tends to describe the diversity advantage of the channel, while ergodic capacity is more representative for the average throughput achievable on the channel.

Ergodic Capacity. The ergodic (mean) capacity in bits/sec/Hz of a random MIMO channel under an

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average transmitter power constraint is given by[1] [1]

$$C_{erg} = \mathcal{E}_H \left\{ \log_2 \left[\det \left(\mathbf{I}_M + \frac{\rho}{N} \mathbf{H}\mathbf{H}^* \right) \right] \right\}, \quad (1)$$

where $\mathbf{H}$ is the $M \times N$ random channel matrix, $\mathcal{E}_H$ stands for expectation over all channel realizations, $\mathbf{I}_M$ denotes the identity matrix of size M, and ρ is the average signal-to-noise ratio (SNR) at each receiver branch. Assuming that coding is performed over many independent fading intervals, C_{erg} can be interpreted as the Shannon capacity of the random MIMO channel [5].

Outage Capacity. The outage capacity tends to be a measure of the diversity advantage of MIMO channels. The channel capacity at a given outage probability q is denoted as $C_{out,q}$. To be precise, the channel capacity is less than $C_{out,q}$ with probability q, i.e.,

$$\text{Prob} \left\{ \log_2 \left[\det \left(\mathbf{I}_M + \frac{\rho}{N} \mathbf{H}\mathbf{H}^* \right) \right] \leq C_{out,q} \right\} = q. \quad (2)$$

3. A STOCHASTIC MIMO MODEL FOR DISTRIBUTED SCATTERING

We consider non-line-of-sight (NLOS) multipath frequency-flat fading channels, where fading is induced by the presence of scatterers at both ends of the radio link. For the sake of simplicity, we consider the effect of near-field scatterers only. We ignore remote scatterers assuming that path loss will tend to limit their contribution. Transmit and receive antenna spacing is denoted as d_t and d_r, respectively.

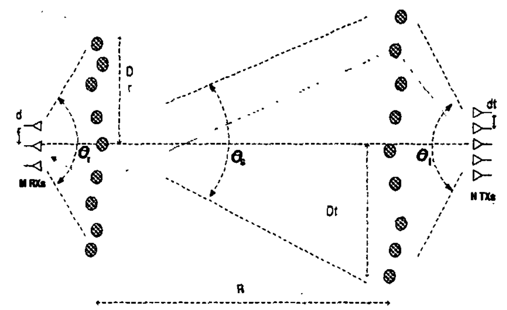

Figure 1: *Propagation scenario for a fading MIMO channel with local scattering at both ends.*

On both sides of the link, the propagation path between the two arrays is obstructed by a set of significant local scatterers (such as buildings and large objects) refered to as transmit or receive scatterers. The distance between the transmit scatterers and the receive scatterers is denoted as R. All scatterers are modeled as omni-directional ideal reflectors. The extent of the scatterers from the horizontal axis is D_t and D_r, respectively. When omni-directional antennas are used D_t and D_r correspond to the transmit and receive *scattering radius*, respectively. On the receive side, the signal reflected by the scatterers onto the antennas impinges on the array with an angular spread denoted by θ_r, where θ_r is a function of the position of the array with respect to the scatterers. Similarly on the transmit side we define an angular spread θ_t. The scatterers are assumed to be located sufficiently far from the antennas for the plane-wave assumption to hold. We furthermore assume that $D_t, D_r \ll R$ (local scattering condition).

We assume S scatterers on both sides of the link, where S is an arbitrary, large enough number for random fading to occur (typically $S > 10$ is sufficient). The exact locations of the scatterers is irrelevant here. Every transmit scatterer captures the radio signal and re-radiates it in the form of a plane wave towards the receive scatterers. The receive scatterers are viewed as an array of S *virtual antennas* with average spacing $2D_r/S$, and as such experience an angle spread defined by $\tan(\theta_S/2) = D_t/R$.

3.1. Stochastic channel model

Following the assumptions and notations above, a model for the MIMO channel is as follows (see [4] for more details):

$$\mathbf{H} = \frac{1}{\sqrt{S}} \mathbf{R}^{1/2}_{\theta_r,d_r} \mathbf{G}_r \mathbf{R}^{1/2}_{\theta_S,2D_r/S} \mathbf{G}_t \mathbf{R}^{1/2}_{\theta_t,d_t}, \quad (3)$$

where $\mathbf{R}_{\theta_r,d_r}$ is the receive antenna fading correlation matrix of size $M \times M$, $\mathbf{R}_{\theta_t,d_t}$ is the transmit antenna fading correlation matrix of size $N \times N$, $\mathbf{R}_{\theta_S,2D_r/S}$ is the receive scatterers' fading correlation matrix of size $S \times S$, and $\mathbf{G}_t$ and $\mathbf{G}_r$ are $S \times N$ and $M \times S$ matrices with i.i.d. complex Gaussian entries with zero mean and unit variance. The matrix $\mathbf{R}^{1/2}_{\theta_S,2D_r/S}$ describes the correlation between the fading signals captured by the receive scatterers when they are seen as virtual antenna elements with average spacing $2D_r/S$. The factor $\frac{1}{\sqrt{S}}$ normalizes the channel energy regardless of how many scatterers are considered.

3.2. Correlation matrix

The general form of the fading correlation matrices used above is as follows. Consider a uniform linear array of K omni-directional receive antennas with spacing d and L (odd) point sources radiating narrowband signals towards the array. We assume the plane-wave

[1]The superscripts * and ^{T} stand for Hermitian transpose and transpose, respectively.

directions of arrival (DOAs) of these signals span an angular spread of θ radians (see Fig. 2).

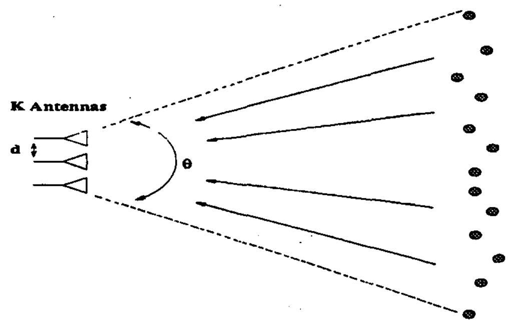

Figure 2: *Propagation scenario for SIMO fading correlation. Each point source transmits a plane-wave signal to a linear array.*

The resulting $K \times K$ fading correlation matrix $\mathbf{R}_{\theta,d}$ is governed by the angle spread, the antenna spacing and the wavelength. For uniformly distributed DOAs for instance, we find (e.g.[8])

$$[\mathbf{R}_{\theta,d}]_{m,k} = \frac{1}{L}\sum_{i=-\frac{L-1}{2}}^{i=\frac{L-1}{2}} e^{-2\pi j(k-m)\frac{d}{\lambda}\cos(\frac{\pi}{2}+\theta_i)}, \quad (4)$$

where θ_i denotes the direction of arrival of the i-th point source. Depending on the angle spread and antenna spacing, $\mathbf{R}_{\theta,d}$ ranges from the identity matrix (uncorrelated spatial fading) to the all ones matrix (full correlation). This correlation model can readily be applied to an array of transmit antennas with corresponding antenna spacing and signal departure angle spread.

3.3. Scatterer-to-scatterer (STS) rank

We introduce the STS rank, here simply defined as the rank of the correlation matrix $\mathbf{R}_{\theta_S,2D_r/S}$. Note that this matrix is completely independent of transmit and receive antenna geometry. Instead, the STS rank is a function of the angular spread experienced by the scatterer to scatterer signals on both transmitting and receiving ends. This angular spread is governed by the transmit and receive scattering radii and the distance between transmitter and receiver.

The STS rank drops when the product $\theta_S D_r$ becomes small compared to the wavelength. For small θ_S we have $\theta_S \approx 2D_t/R$. Hence, the STS rank drops if the product $D_t D_r/R$ becomes small compared to the wavelength.

Note that the STS rank, along with the input rank and the ouptut rank, govern the global rank of the MIMO channel. It follows that antenna correlation causes rank loss but the converse is not true.

3.4. Fading distribution

The distribution of the individual elements in $\mathbf{H}$ defined in (3) is in general not Rayleigh. Indeed, the model contains the product of two independent Rayleigh fading matrices. In the interesting particular case of uncorrelated transmit and receive fading, the model reduces to

$$\mathbf{H} = \frac{1}{\sqrt{S}}\mathbf{G}_r\mathbf{R}^{1/2}_{\theta_S,2D_r/S}\mathbf{G}_t. \quad (5)$$

Consequently, the STS rank causes the distribution of $\mathbf{H}$ to range continuously from "double Rayleigh" to Rayleigh, as shown below.

3.5. Low STS rank

A double Rayleigh distribution is obtained in the case where $\mathbf{R}_{\theta_S,2D_r/S}$ is the all ones matrix, which corresponds to the case where the scatterers' signals are fully correlated. In this case, we obtain

$$\mathbf{H} = \mathbf{g}_{rx}\mathbf{g}_{tx}^{*}, \quad (6)$$

where $\mathbf{g}_{rx}$ and $\mathbf{g}_{tx}$ are independent receive and transmit fading vectors with i.i.d. complex-valued Gaussian components, i.e., $\mathbf{g}_{rx} \sim \mathcal{CN}(\mathbf{0},\mathbf{I}_M)$ and $\mathbf{g}_{tx} \sim \mathcal{CN}(\mathbf{0},\mathbf{I}_N)$. The existence of this peculiar channel has been noted in previous contributions under the name of pin-hole [4] or key-hole channels [11]. The distribution for the uncorrelated pin-hole channel has been given in [12]. The model in (3) contains this particular case as a special case.

3.6. Full STS rank

In the other extreme situation, where the signals captured by the scatterers are completely independent, we have $\mathbf{R}_{\theta_S,2D_r/S} = \mathbf{I}_S$ and hence

$$\mathbf{H} = \frac{1}{\sqrt{S}}\mathbf{G}_r\mathbf{G}_t, \quad (7)$$

which by the central limit theorem implies that for a sufficiently large S the $[\mathbf{H}]_{i,j}$ are complex Gaussian. We thus find the i.i.d. model used e.g. in [1] as a special case of our model.

3.7. Correlated inputs and full STS rank

We still assume that $\mathbf{R}_{\theta_S,2D_r/S} = \mathbf{I}_S$. If the geometry of the transmit antennas is such that the transmit fading correlation is high (e.g. transmit antennas too closely spaced), $\mathbf{R}_{\theta_t,d_t}$ becomes the $N \times N$ all ones matrix and

$$\mathbf{H} = \mathbf{g}_{rx}(1, \ldots, 1), \quad (8)$$

where it was assumed that receive fading is uncorrelated, and $\mathbf{g}_{rx}$ is the uncorrelated receive fading vector. A converse case with fully correlated outputs and uncorrelated inputs can be defined in a similar way.

3.8. Diversity and STS rank

Unlike general intuition, the input rank and output rank alone do not suffice to characterize the diversity order offered by the MIMO channel. It is well known that an uncorrelated i.i.d. $M \times N$ **Gaussian** channel offers MN-th order diversity. In contrast, the case of fully correlated inputs but fully uncorrelated outputs (with full STS rank) (8) offers exactly M-th order diversity. In comparison, the low STS rank case offers *strictly less* than $\min(M, N)$-th order diversity and will therefore always result in worse outage probability even in the case of uncorrelated fading at both the input and the output (see subsequent section). The reason for this is, that in (6) an outage can be caused independently by the transmit *or* the receive antennas.

4. MONTE CARLO SIMULATIONS

We study the impact of correlation matrix rank loss on ergodic and outage capacity for the 3 by 3 case. In the following, we denote $\mathbf{R}_t = \mathbf{R}_{\theta_t,d_t}$, $\mathbf{R}_r = \mathbf{R}_{\theta_r,d_r}$, and $\mathbf{R}_s = \mathbf{R}_{\theta_S,2D_r/S}$. We simulate both theoretical (correlation is either 1 or 0) and real (correlation parameterized by propagation parameters) channels.

Fig. 3 shows the behavior of ergodic capacity as a function of rank for ideal correlations[2]. Each point on every curve is the average capacity of 10,000 realizations of the channel with the channel model as per (3). Fig. 4 shows the outage capacity generated from 10,000 realizations of the channel for six different conditions as tabulated in Tab. 1. Fig. 5 depicts a comparison of the outage capacities between channels with ideal correlation matrices and real correlation matrices according to (4) derived from propagation parameters. The propagation parameters (in meters) are as per Tab. 2. R_r (resp. R_t) indicate the distance from the receive (resp. transmit) antenna array to the local receive (resp. transmit) scatterers, used to compute angle spread.

Note how the ergodic capacity is driven by the overall channel rank, which drops whenever any of the STS rank or input/output rank is low. The pinhole channel offers roughly the same ergodic capacity as a fully correlated channel (all correlation matrices have rank 1).

[2]Full rank implies a correlation matrix of identity and a rank one correlation matrix is the all ones matrix

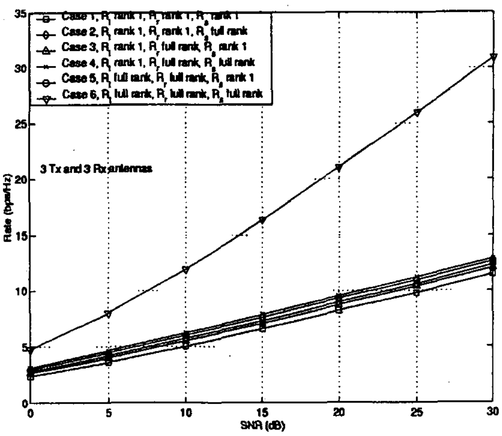

Figure 3: *Ergodic capacity behavior.*

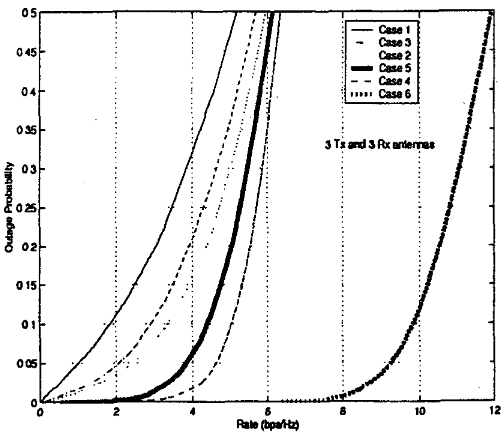

Figure 4: *Outage Capacity Behavior.*

Outage capacity is very sensitive to the particular correlation matrix that loses rank. From an outage capacity stand-point a loss of STS rank alone (case 5) is worse than a loss of rank in either of the other correlation matrices (case 4) because the fading probability increases. Case 2 is the Rayleigh fading case with no diversity. Even though case 3 has a full rank $\mathbf{R}_r$ (which case 2 does not have), the overall channel offers no diversity; moreover the rank loss of $\mathbf{R}_s$ causes the fading distribution to be worse (in terms of outage properties) than Rayleigh. Finally case 1, where all correlation matrices have rank 1 is the worst of all, because it has double Rayleigh fading and offers no spatial diversity.

Fig. 5 shows a good fit between the outage capacity curves computed from ideal correlation matrices and those derived from the real propagation parameters from Tab. 2.

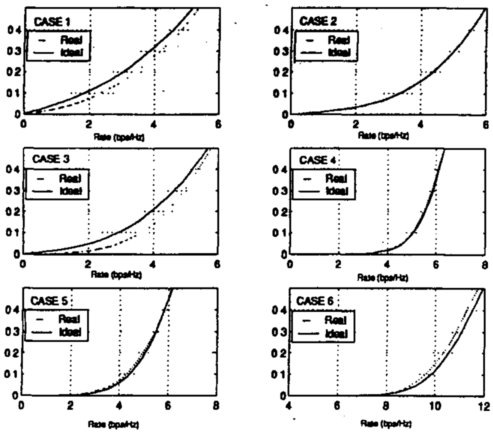

Figure 5: *Comparison of outage capacity behavior between real and ideal correlation matrices.*

Case	$\mathbf{R}_t$	$\mathbf{R}_r$	$\mathbf{R}_s$
1	rank 1	rank 1	rank1
2	rank 1	rank 1	full rank
3	rank 1	full rank	rank 1
4	rank 1	full rank	full rank
5	full rank	full rank	rank 1
6	full rank	full rank	full rank

Table 1: *Cases depicted in Figs. 3 and 4.*

5. CONCLUSION

For a recently introduced model for wireless MIMO outdoor channels, we studied the impact of rank loss of certain correlation matrices on ergodic and outage capacities. We found that while a rank loss due to spatial fading correlation at the transmitter or the receiver or due to large distance between the transmitter and the receiver has more or less the same impact on ergodic capacity, the impact on outage capacity can vary depending on where the rank loss occurs. The STS rank is shown to have a particular impact.

Case	D_t	D_r	R_t	R_r	R	$d_t(\lambda)$	$d_r(\lambda)$
1	20	20	100	100	30000	0.1	0.1
2	20	20	100	100	1000	0.1	0.1
3	20	50	100	50	40000	0.1	1
4	20	50	100	50	1000	0.1	1
5	15	15	20	20	40000	0.5	0.5
6	20	20	50	50	1000	1	1

Table 2: *Real propagation parameters for Fig. 5.*

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