BLIND EQUALIZATION IN OFDM-BASED MULTI-ANTENNA SYSTEMS

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Abstract—Wireless systems employing multiple antennas at the transmitter and the receiver can achieve extraordinary bit rates. Orthogonal frequency division multiplexing (OFDM) significantly reduces receiver complexity in multi-antenna broadband systems. In this paper, we introduce an algorithm for blind channel identification and equalization in OFDM-based multi-antenna systems. Our approach employs nonredundant antenna precoding, uses second-order cyclostationary statistics, and yields unique estimates (up to a phase rotation for each symbol stream). Furthermore, it does not require knowledge of the channel order, imposes no restrictions on the channel zeros, and exhibits low sensitivity to stationary noise. We present simulation results demonstrating the channel estimator and the corresponding equalizer performance.

1. INTRODUCTION AND OUTLINE

Deploying *multiple antennas* at both the transmitter and the receiver of a wireless system has recently been shown to yield extraordinary bit rates [1]-[4]. The corresponding technology, known as *spatial multiplexing* [1] or *BLAST* [2, 5], allows an impressive increase in data rate in a wireless radio link without additional power or bandwidth consumption. *Orthogonal frequency division multiplexing* (OFDM) [6]-[9] significantly reduces receiver complexity in wireless broadband systems and has recently been proposed for use in wireless broadband multi-antenna systems [4, 10]. In practice, in order to get the promised increase in data rate, accurate channel state information is required in the receiver. This information can be obtained by sending training data and estimating the channel [4]. The training overhead required, unfortunately, is more significant in estimating multiple-input multiple-output (MIMO) channels. To avoid this problem, we propose an algorithm for blind channel identification and equalization in OFDM-based MIMO systems.

Blind identification and equalization of MIMO channels has been a very active area of research during the past few years. We refer the interested reader to [11] which contains an excellent overview of the subject and an extensive reference list until 1996. More recent references are for example [12]-[15]. To the best of our knowledge previous work on blind MIMO channel estimation was for single-carrier systems only. The use of cyclostationary statistics to accomplish blind MIMO channel estimation has first been proposed for frequency-flat fading channels in [16, 17]. More recently, the use of *conjugate cyclostationary statistics* in combination with *constant-modulus* antenna precoding has been suggested in [18] to accomplish blind MIMO channel estimation in the single-carrier case.

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Contributions. In this paper, using a nonredundant *nonconstant-modulus* precoding scheme, we introduce an algorithm for the blind identification and equalization of OFDM-based MIMO systems. Our method uses second-order *cyclostationary statistics* and identifies the matrix channel on a subchannel by subchannel basis, i.e., each scalar subchannel is identified individually. The proposed algorithm does *not require knowledge of the channel order*, it does *not impose restrictions on channel zeros*, and *it exhibits low sensitivity* to stationary noise. Our equalization method recovers the transmitted symbol streams up to a phase rotation (which will in general be different for different symbol streams). This remaining ambiguity can be resolved using higher-order statistics or short training sequences.

Relation to previous work. Our approach extends an idea first proposed by the authors for the single-carrier case in [19]. Besides applying to OFDM, our algorithm differs from that suggested in [18] in that we use nonconstant-modulus precoding instead of constant-modulus precoding and cyclostationarity instead of conjugate cyclostationarity. Furthermore, our approach works for arbitrary symbol constellations and arbitrary stationary noise, the phase ambiguity is resolved up to a diagonal matrix of phase terms, and most importantly the performance of our estimator does not degrade significantly if the number of sources (number of transmit antennas) increases.

Organization of the paper. The rest of this paper is organized as follows. In Section 2, we describe broadband OFDM-based spatial multiplexing systems and we state our assumptions and introduce some notation. In Section 3, we introduce the novel algorithm by considering the case of two transmit and two receive antennas. Section 4 contains simulation results demonstrating the estimator and the corresponding equalizer performance. Finally, Section 5 provides our conclusions.

2. OFDM-BASED SPATIAL MULTIPLEXING

Spatial multiplexing [1] requires multiple antennas at the transmitter and the receiver and works as follows. In the transmitter the (possibly coded) information symbol stream is split up into independent substreams, which are OFDM-modulated and then sent simultaneously and within the same frequency band from the M_T transmit antennas. In the receiver, the individual data streams are OFDM-demodulated, separated, demultiplexed, and decoded. The use of OFDM turns the frequency-selective MIMO fading channel into a set of parallel Gaussian MIMO channels each of which is frequency-flat fading. This makes equalization very simple. In fact only a constant matrix has to be inverted for each OFDM tone [4, 10].

Let us next introduce some notation and some basic results needed later in the paper. Throughout the paper, we will assume that the OFDM system employs N tones and a cyclic prefix (CP) of length L, i.e., the OFDM symbol length is given by $M = N + L$. Furthermore, M_T and M_R denote the number of transmit and receive antennas, respectively. The transmitted signal corresponding to the i-th ($i = 0, 1, ..., M_T - 1$) antenna is given by

$$s_i[n] = \sum_{l=-\infty}^{\infty} g[n - lM] \sum_{k=0}^{N-1} c_{k,l}^{(i)} e^{j\frac{2\pi}{N}k(n-lM)},$$

where $g[n] = \text{rect}_{[0,M-1]}[n]$ with

$$\text{rect}_{[N_1,N_2]}[n] = \begin{cases} 1, & n = N_1, N_1 + 1, ..., N_2 \\ 0, & \text{else.} \end{cases}$$

Furthermore $c_{k,l}^{(i)}$ denotes the complex data symbol transmitted on the k-th tone in the l-th OFDM symbol from the i-th antenna. The signal received at the m-th antenna can now be written as

$$r_m[n] = \sum_{i=0}^{M_T-1} \left[\sum_{l=-\infty}^{\infty} h_{m,i}[l]\, s_i[n - l] \right] + \rho_m[n], \quad (1)$$

where $h_{m,i}[n]$ denotes the impulse response of the scalar subchannel between the i-th transmit and the m-th receive antenna and $\rho_m[n]$ ($m = 0, 1, ..., M_R - 1$) is stationary additive (potentially colored) noise observed at the m-th receive antenna. Using the following notation

$$\begin{aligned} \mathbf{r}[n] &= [r_0[n]\ r_1[n]\ ...\ r_{M_R-1}[n]]^T \\ \mathbf{s}[n] &= [s_0[n]\ s_1[n]\ ...\ s_{M_T-1}[n]]^T \\ \boldsymbol{\rho}[n] &= [\rho_0[n]\ \rho_1[n]\ ...\ \rho_{M_R-1}[n]]^T \end{aligned}$$

we can rewrite the input-output relation (1) in vector-matrix form as

$$\mathbf{r}[n] = \sum_{l=-\infty}^{\infty} \mathbf{H}_l \mathbf{s}[n - l] + \boldsymbol{\rho}[n],$$

where $[\mathbf{H}_l]_{m,i} = h_{m,i}[l]$ ($m = 0, 1, ..., M_R - 1, i = 0, 1, ..., M_T - 1$). In the following, we assume that the delay spread of the $M_R \times M_T$ matrix channel is L taps, i.e.,

$$\mathbf{H}(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_l\, e^{-j2\pi l\theta}.$$

Denoting the reconstructed data symbol corresponding to $c_{k,l}^{(i)}$ as $\hat{c}_{k,l}^{(i)}$ and organizing the data symbols into frequency vectors according to $\hat{\mathbf{c}}_{k,l} = [\hat{c}_{k,l}^{(0)}\ \hat{c}_{k,l}^{(1)}\ ...\ \hat{c}_{k,l}^{(M_T-1)}]^T$ and $\mathbf{c}_{k,l} = [c_{k,l}^{(0)}\ c_{k,l}^{(1)}\ ...\ c_{k,l}^{(M_T-1)}]^T$ ($k = 0, 1, ..., N - 1,\ l \in \mathbb{Z}$), it can be shown that

$$\hat{\mathbf{c}}_{k,l} = \mathbf{H}(e^{j\frac{2\pi}{N}k})\mathbf{c}_{k,l} + \boldsymbol{\rho}_{k,l}, \quad k = 0, 1, ..., N-1,\ l \in \mathbb{Z}, \quad (2)$$

where $\boldsymbol{\rho}_{k,l}$ is stationary additive Gaussian noise. Thanks to (2) the convolutive mixtures observed at the receive antennas can be separated and equalized by computing

$$\mathbf{y}_{k,l} = \left[\mathbf{H}(e^{j\frac{2\pi}{N}k})\right]^{-1} \hat{\mathbf{c}}_{k,l} = \mathbf{c}_{k,l} + \left[\mathbf{H}(e^{j\frac{2\pi}{N}k})\right]^{-1} \boldsymbol{\rho}_{k,l} \quad (3)$$

for $k = 0, 1, ..., N - 1$. We note that alternatively an MMSE equalizer or an ML receiver can be employed on a tone by tone basis. In this paper, we will be concerned with the blind estimation of the matrices $\mathbf{H}(e^{j\frac{2\pi}{N}k})$ ($k = 0, 1, ..., N - 1$). Note, however, that a direct estimation of the $\mathbf{H}(e^{j\frac{2\pi}{N}k})$ would be very inefficient, since it requires the estimation of NM_RM_T parameters, which can be a significant number in practical systems. Rather, we will provide an algorithm for blindly estimating the channel impulse response matrix taps $\mathbf{H}_l$ from which we can compute estimates of the $\mathbf{H}(e^{j\frac{2\pi}{N}k}) = \sum_{l=0}^{L-1} \mathbf{H}_l\, e^{-j2\pi\frac{k}{N}l}$ using the FFT.

3. THE BLIND CHANNEL IDENTIFICATION ALGORITHM

In this section, we shall introduce the novel channel identification algorithm and we demonstrate the basic idea using a simple example with two transmit and two receive antennas. The generalization to an arbitrary number of antennas is discussed in [20].

3.1. Preparation

The basic idea of our algorithm is to perform *nonredundant nonconstant-modulus precoding* in the transmitter such that the cyclostationary statistics allow a separate identification of the individual scalar subchannels $h_{m,i}[n]$ ($m = 0, 1, ..., M_R - 1, i = 0, 1, ..., M_T - 1$). This is achieved by providing each transmit antenna with a different signature in the cyclostationary domain with the signatures chosen such that for a given cycle all but one transmit antennas are nulled out. It is therefore possible to identify the matrix channel on a column by column basis up to a constant diagonal matrix of phase rotations. After equalization according to (3) the individual symbol streams will be decoded up to a phase rotation, which will in general be different for different symbol streams. This remaining ambiguity can be resolved by using higher-order statistics or by sending short training sequences. Alternatively, if differential detection is employed, the phase rotation can be ignored.

Nonredundant precoding. Our approach is based on nonredundant nonconstant-modulus precoding which consists of multiplying the individual data streams by P-periodic precoding sequences prior to transmission. The precoding sequences have to be different for different transmit antennas. More specifically, the precoded transmit signal corresponding to the i-th transmit antenna is given by ($i = 0, 1, ..., M_T - 1$)

$$s_i[n] = \sum_{l=-\infty}^{\infty} a_l^{(i)} g[n - lM] \sum_{k=0}^{N-1} c_{k,l}^{(i)}\, e^{j\frac{2\pi}{N}k(n-lM)}, \quad (4)$$

where $a_l^{(i)} = a_{l+P}^{(i)}$ is the i-th P-periodic precoding sequence. In order to keep the average transmit SNR constant the precoding sequences have to be normalized such that $\sum_{l=0}^{P-1} |a_l^{(i)}|^2 = P$ ($i = 0, 1, ..., M_T - 1$). From (4) it follows that all the (complex) data symbols in the l-th OFDM symbol transmitted from the i-th antenna are multiplied by $a_l^{(i)}$ before the IFFT is applied. We note that this form of precoding has previously been suggested by Serpedin and Giannakis in [21] to introduce cyclostationarity in the transmit signal thereby making blind channel identification in single-antenna symbol-rate sampled single-carrier systems possible. The more general idea of transmitter-induced cyclostationarity has been suggested previously in [22, 23].

Cyclostationary statistics. Now, defining the $M_R \times M_R$ correlation matrix of the vector random process $\mathbf{r}[n]$

as[1] $\mathbf{c}_r[n,\tau] = \mathcal{E}\{\mathbf{r}[n]\mathbf{r}^H[n-\tau]\}$ and assuming that the data sequences $c_{k,l}^{(i)}$ are statistically independent of the noise and satisfy $\mathcal{E}\{c_{k,l}^{(i)} c_{k',l'}^{(i')^*}\} = \sigma_i^2 \delta[k-k']\delta[l-l']\delta[i-i']$, it is shown in [20] that

$$\mathbf{c}_r[n,\tau] = \sum_{l=-\infty}^{\infty} \mathbf{H}_l \sum_{r=-\infty}^{\infty} \mathbf{c}_s[n-l,r]\mathbf{H}_{r-\tau+l}^H + \mathbf{c}_\rho[\tau], \quad (5)$$

where $\mathbf{c}_\rho[\tau] = \mathcal{E}\{\boldsymbol{\rho}[n]\,\boldsymbol{\rho}^H[n-\tau]\}$ and $\mathbf{c}_s[n,\tau] = \mathcal{E}\{\mathbf{s}[n]\,\mathbf{s}^H[n-\tau]\} = \mathrm{diag}\{c_s^{(i)}[n,\tau]\}_{i=0}^{M_T-1}$ (recall that the different transmit signals were assumed to be uncorrelated). The scalar correlation functions $c_s^{(i)}[n,\tau]$ are given by $c_s^{(i)}[n,\tau] =$

$$\begin{cases} \sigma_i^2 \sum_{l=-\infty}^{\infty} |a_l^{(i)}|^2 \,\mathrm{rect}_{[0,M-1]}[n-lM], & \tau = 0 \\ \sigma_i^2 \sum_{l=-\infty}^{\infty} |a_l^{(i)}|^2 \,\mathrm{rect}_{[N,M-1]}[n-lM], & \tau = N \\ \sigma_i^2 \sum_{l=-\infty}^{\infty} |a_l^{(i)}|^2 \,\mathrm{rect}_{[0,L-1]}[n-lM], & \tau = -N \\ 0, & \text{else.} \end{cases}$$

Now, using the P-periodicity of the precoding sequences $a_l^{(i)}$ it can be verified that $c_s^{(i)}[n,\tau] = c_s^{(i)}[n+PM,\tau]$ for $i = 0,1,\ldots,M_T-1$. Consequently, we have

$$\mathbf{c}_s[n,\tau] = \mathbf{c}_s[n+PM,\tau] \quad (6)$$

which shows that $\mathbf{s}[n]$ is a cyclostationary vector random process with period PM. By cyclostationary vector random process with period PM we mean that each of the entries in the vector $\mathbf{s}[n]$ is a scalar cyclostationary random process with cyclostationarity period PM. Using (6), it follows from (5) that $\mathbf{c}_r[n,\tau] = \mathbf{c}_r[n+PM,\tau]$ and hence $\mathbf{r}[n]$ is a cyclostationary vector random process with period PM as well. Note that cyclostationarity is introduced in the transmitter thanks to the CP and the nonredundant precoding operation.

Since $\mathbf{c}_r[n,\tau]$ is PM-periodic in n, we can expand it into a Fourier series with respect to n with the Fourier series coefficient matrices given by

$$\mathbf{C}_r[k,\tau] = \frac{1}{PM}\sum_{n=0}^{PM-1} \mathbf{c}_r[n,\tau]\, e^{-j\frac{2\pi}{PM}kn}, \; k = 0,1,\ldots,PM-1.$$

Next applying a z-transform with respect to τ, we obtain the cyclic power spectral matrices

$$\mathbf{S}_r[k,z] = \sum_{\tau=-\infty}^{\infty} \mathbf{C}_r[k,\tau]\, z^{-\tau}$$

which are shown in [20] to be given by

$$\mathbf{S}_r[k,z] = \mathbf{H}\left(ze^{j\frac{2\pi}{PM}k}\right)\mathbf{S}_s[k,z]\,\tilde{\mathbf{H}}(z) + \mathbf{S}_\rho(z)\delta[k], \quad (7)$$

where $\tilde{\mathbf{H}}(z) = \mathbf{H}^H\left(\frac{1}{z^*}\right)$, $\mathbf{S}_\rho(z) = \sum_{\tau=-\infty}^{\infty} \mathbf{c}_\rho[\tau]\, z^{-\tau}$ and

$$\begin{aligned}\mathbf{S}_s[k,z] &= \mathrm{diag}\{S_s^{(i)}[k,z]\}_{i=0}^{M_T-1} = \sum_{\tau=-\infty}^{\infty} \mathbf{C}_s[k,\tau]\, z^{-\tau} \\ &= \sum_{\tau=-\infty}^{\infty} \frac{1}{PM}\sum_{n=0}^{PM-1} \mathbf{c}_s[n,\tau]\, e^{-j\frac{2\pi}{PM}kn}\, z^{-\tau}.\end{aligned}$$

[1] $\mathcal{E}$ stands for the expectation operator and the superscript ^{H} denotes conjugate transposition.

The Fourier series coefficient matrices $\mathbf{C}_s[k,\tau]$ are given by

$$\begin{aligned}\mathbf{C}_s[k,\tau] &= \mathrm{diag}\{C_s^{(i)}[k,\tau]\}_{i=0}^{M_T-1} \\ &= \frac{N}{PM}\delta_N[\tau] A^{(g,g)}\left[\tau, \frac{k}{PM}\right)\mathrm{diag}\{\sigma_i^2 \Phi_P^{(i)}[k]\}_{i=0}^{M_T-1}, \quad (8)\end{aligned}$$

where $\delta_N[\tau] = \sum_{l=-\infty}^{\infty} \delta[\tau - lN]$,

$$\Phi_P^{(i)}[k] = \sum_{r=0}^{P-1} |a_r^{(i)}|^2\, e^{-j\frac{2\pi}{P}rk}, \quad (9)$$

and $A^{(g,g)}\left[\tau, \frac{k}{PM}\right) =$

$$\frac{1}{N}\, e^{-j2\pi\frac{k}{PM}\frac{M+\tau-1}{2}} \begin{cases} \frac{\sin\left(\frac{\pi k}{PM}(M-\tau)\right)}{\sin\left(\pi\frac{k}{PM}\right)}, & 0 \le \tau \le M-1 \\ \frac{\sin\left(\frac{\pi k}{PM}(M+\tau)\right)}{\sin\left(\pi\frac{k}{PM}\right)}, & -M+1 \le \tau < 0. \end{cases} \quad (10)$$

3.2. The basic identification algorithm

We shall next explain our algorithm using a simple example with 2 transmit and 2 receive antennas. Considering the 2 by 2 case simplifies the presentation and conveys the basic ideas underlying the algorithm.

We use the normalized 4-periodic precoding sequences $a^{(0)} = [0.9513 \;\; 1.0464 \;\; 0.9513 \;\; 1.0464]$ and $a^{(1)} = [1.0464 \;\; 0.9513 \;\; 0.9513 \;\; 1.0464]$. From (9) it follows that $\Phi_P^{(0)} = [4.0000 \;\; 0.0000 \;\; -0.3801 \;\; 0.0000]$ and $\Phi_P^{(1)} = [4.0000 \;\; 0.1900{+}j0.1900 \;\; 0.0000 \;\; 0.1900{-}j0.1900]$. In the following, in order to keep the discussion more general, we shall stick to the notation P for the period of the precoding sequences instead of specializing to $P = 4$. From (7) we obtain

$$\begin{aligned}[\mathbf{S}_r[k,z]]_{0,0} &= H_{0,0}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(0)}[k,z]\tilde{H}_{0,0}(z) \\ &+ H_{0,1}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(1)}[k,z]\tilde{H}_{0,1}(z) \quad (11)\end{aligned}$$

$$\begin{aligned}[\mathbf{S}_r[k,z]]_{1,1} &= H_{1,0}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(0)}[k,z]\tilde{H}_{1,0}(z) \\ &+ H_{1,1}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(1)}[k,z]\tilde{H}_{1,1}(z), \quad (12)\end{aligned}$$

where $\tilde{H}(z) = H^*\left(\frac{1}{z^*}\right)$ and $k \neq 0$. The basic idea of our channel identification algorithm now is to find cycles $k_1 \neq 0$ and $k_2 \neq 0$ such that $S_s^{(0)}[k_1,z] = 0, S_s^{(1)}[k_1,z] \neq 0, S_s^{(1)}[k_2,z] = 0$, and $S_s^{(0)}[k_2,z] \neq 0$. Since $S_s^{(i)}[k,z] = \sum_{\tau=-\infty}^{\infty} C_s^{(i)}[k,\tau]\, z^{-\tau}$ it follows that $S_s^{(i)}[k,z] = 0 \;\forall z \in \mathbb{C}$ if and only if $C_s^{(i)}[k,\tau] = 0 \;\forall \tau \in \mathbb{Z}$. From (8) we can see that the $C_s^{(i)}[k,\tau]$ differ only in the $\Phi_P^{(i)}[k]$. Consequently picking k_1 and k_2 such that $\Phi_P^{(0)}[k_1] = 0, \Phi_P^{(1)}[k_1] \neq 0, \Phi_P^{(1)}[k_2] = 0$, and $\Phi_P^{(0)}[k_2] \neq 0$ solves the problem. Clearly, setting $k_1 = 1$ and $k_2 = 2$ is a possible choice. In the following, in order to keep the discussion more general, we shall stick to the general notation k_1 and k_2 for the cycles. Specializing (11) and (12) and using $\Phi_P^{(i)}[-k] = \Phi_P^{(i)^*}[k]$ we obtain

$$[\mathbf{S}_r[\pm k_1, z]]_{0,0} = H_{0,1}\left(ze^{\pm j\frac{2\pi}{PM}k_1}\right) S_s^{(1)}[\pm k_1, z]\tilde{H}_{0,1}(z) \quad (13)$$

$$[\mathbf{S}_r[\pm k_1, z)]_{1,1} = H_{1,1}\left(ze^{\pm j\frac{2\pi}{PM}k_1}\right) S_s^{(1)}[\pm k_1, z)\tilde{H}_{1,1}(z) \tag{14}$$

$$[\mathbf{S}_r[\pm k_2, z)]_{0,0} = H_{0,0}\left(ze^{\pm j\frac{2\pi}{PM}k_2}\right) S_s^{(0)}[\pm k_2, z)\tilde{H}_{0,0}(z) \tag{15}$$

$$[\mathbf{S}_r[\pm k_2, z)]_{1,1} = H_{1,0}\left(ze^{\pm j\frac{2\pi}{PM}k_2}\right) S_s^{(0)}[\pm k_2, z)\tilde{H}_{1,0}(z). \tag{16}$$

Now we can use a modified version of an algorithm first proposed by Tong et. al. in [24] and later extended by Serpedin and Giannakis [21] to identify the scalar subchannels $H_{0,1}(z)$ and $H_{1,1}(z)$ from $[\mathbf{S}_r[\pm k_1, z)]_{0,0}$ and $[\mathbf{S}_r[\pm k_1, z)]_{1,1}$, respectively, and the subchannels $H_{0,0}(z)$ and $H_{1,0}(z)$ from $[\mathbf{S}_r[\pm k_2, z)]_{0,0}$ and $[\mathbf{S}_r[\pm k_2, z)]_{1,1}$, respectively. By proper design of the periodic precoding sequences we have broken up the 2×2 matrix channel identification into the identification of 4 scalar subchannels. Thus, the basic idea of our algorithm is to perform nonredundant antenna precoding such that only one of the two transmit antennas appears as "active" at a time in the cyclostationary statistics of the received signal vector.

The algorithm we are using in the following to identify the individual scalar subchannels has been used previously in CP OFDM systems [25] and in pulse shaping OFDM/OQAM systems [26]. We shall therefore keep the presentation of the identification algorithm short. Furthermore, we restrict the discussion to the identification of $H_{0,0}(z)$. The remaining subchannel filters can be identified using the same procedure. Starting from (15) we get

$$[\mathbf{S}_r[k_2, z)]_{0,0} S_s^{(0)}[-k_2, z)\, H_{0,0}(ze^{-j\frac{2\pi}{PM}k_2}) - [\mathbf{S}_r[-k_2, z)]_{0,0} S_s^{(0)}[k_2, z)\, H_{0,0}(ze^{j\frac{2\pi}{PM}k_2}) = 0$$

which can be rewritten as

$$\underbrace{[\mathbf{T}_{r,s}^{(k_2,-k_2)}\,\mathbf{D}^{-k_2} - \mathbf{T}_{r,s}^{(-k_2,k_2)}\,\mathbf{D}^{k_2}]}_{\mathbf{G}_{r,s}^{(k_2,-k_2)}}\mathbf{h}_{0,0} = \mathbf{0} \tag{17}$$

with the $(4M + 3L_{h_{0,0}} - 6) \times L_{h_{0,0}}$ Toeplitz matrices $\mathbf{T}_{r,s}^{(k,l)}$ with first row

$$[a_{r,s}^{(k,l)}[-2M - L_{h_{0,0}} + 3]\ 0\ \dots\ 0]$$

and first column

$$[a_{r,s}^{(k,l)}[-2M - L_{h_{0,0}} + 3]\ \dots\ a_{r,s}^{(k,l)}[2M + L_{h_{0,0}} - 3]\ 0\ \dots\ 0],$$

and the $L_{h_{0,0}} \times L_{h_{0,0}}$ diagonal matrix $\mathbf{D} = \mathrm{diag}\{e^{-j\frac{2\pi}{PM}l}\}_{l=0}^{L_{h_{0,0}}-1}$. Here, $a_{r,s}^{(k,l)}[n] =$

$$[\mathbf{C}_r[k, n]]_{0,0}\, C_s^{(0)}[l, 0] + [\mathbf{C}_r[k, n-N]]_{0,0}\, C_s^{(0)}[l, N] + [\mathbf{C}_r[k, n+N]]_{0,0}\, C_s^{(0)}[l, -N] \tag{18}$$

and $\mathbf{h}_{0,0} = \left[h_{0,0}[0]\ \ h_{0,0}[1]\ \dots\ h_{0,0}[L_{h_{0,0}} - 1]\right]^T$. Finally, $\mathbf{0}$ denotes the $(4M + 3L_{h_{0,0}} - 6) \times 1$ all zero vector. Using Theorem 1 in [21] it can be shown that the channel $\mathbf{h}_{0,0}$ is uniquely identifiable within a phase ambiguity (irrespectively of channel zeros) if and only if there is no $l \in [1, L_{h_{0,0}} - 1]$ such that $e^{j\frac{4\pi}{PM}k_2 l} = 1$. This implies that identifiability is guaranteed for $L_{h_{0,0}} \leq \frac{PM}{2k_2}$. Since OFDM symbols are very long in practice (i.e. M is typically chosen to be at least 4 times the channel length), this condition will easily be met.

The channel estimate $\widehat{\mathbf{h}}_{0,0}$ can now be found by first estimating the cyclostationary statistics $\mathbf{C}_r[k, \tau]$ from a finite data record $\{\mathbf{r}[n]\}_{n=0}^{L-1}$ of length L according to [17, 27]

$$\widehat{\mathbf{C}}_r[k, \tau] = \frac{1}{L}\sum_{n=0}^{L-1}\mathbf{r}[n]\mathbf{r}^H[n-\tau]e^{-j\frac{2\pi}{PM}kn} \tag{19}$$

and then solving the following optimization problem:

$$\widehat{\mathbf{h}}_{0,0} = \underset{\mathbf{h}_{0,0} \neq 0}{\arg\min} \left\| \hat{\mathbf{G}}_{r,s}^{(k_2,-k_2)}\mathbf{h}_{0,0} \right\|^2, \tag{20}$$

where $\hat{\mathbf{G}}_{r,s}^{(k_2,-k_2)}$ is an estimate of $\mathbf{G}_{r,s}^{(k_2,-k_2)}$ obtained by replacing $[\mathbf{C}_r[k, \tau]]_{0,0}$ in (18) by the estimates $[\widehat{\mathbf{C}}_r[k, \tau]]_{0,0}$. The solution of (20) is found as the eigenvector of $\hat{\mathbf{G}}_{r,s}^{(k_2,-k_2)^H}\hat{\mathbf{G}}_{r,s}^{(k_2,-k_2)}$ associated to the smallest eigenvalue. The remaining subchannel filters $H_{0,1}(z), H_{1,1}(z)$, and $H_{1,0}(z)$ can be identified by performing the same procedure as above for (13),(14) and (16), respectively. We note that periodic precoding increases the cyclostationarity period by a factor of P and hence longer data records are required to arrive at good estimates of $\mathbf{C}_r[k, \tau]$. Therefore, in practice one should aim at precoding sequences with small P.

3.3. Resolving the complex scale ambiguity

Each of the scalar subchannels has now been identified up to a complex constant, which in general will be different for different subchannels. We shall next describe a two-step procedure for resolving this remaining ambiguity up to a diagonal matrix of phase rotations. Assume that the scalar subchannel filters have been identified up to a complex constant denoted as $r_{m,i}e^{j\phi_{m,i}}$ for the filter $H_{m,i}(z)$ or equivalently the true filter $H_{m,i}(z)$ is given by $H_{m,i}(z) = r_{m,i}e^{j\phi_{m,i}}H_{m,i}^{(e)}(z)$, where $H_{m,i}^{(e)}(z)$ is the estimate of the filter $H_{m,i}(z)$ obtained using the subspace-based algorithm described above. Let us first show how the $r_{m,i}$ can be estimated. Again concentrating on the filter $H_{0,0}(z)$ we have

$$[\mathbf{S}_r[k_2, z)]_{0,0} = r_{0,0}^2 H_{0,0}^{(e)}\left(ze^{j\frac{2\pi}{PM}k_2}\right) S_s^{(0)}[k_2, z)\tilde{H}_{0,0}^{(e)}(z).$$

Now, we can obtain an estimate of $r_{0,0}$ as

$$\widehat{r}_{0,0} = \frac{1}{|\mathcal{I}|}\sum_{n \in \mathcal{I}}\sqrt{\frac{\left[\widehat{\mathbf{C}}_r[k_2, n]\right]_{0,0}}{r_h^{(e)}[n]}},$$

where $r_h^{(e)}[n] =$

$$\sum_u C_s^{(0)}[k_2, n-u]\sum_l h_{0,0}^{(e)}[u+l]e^{-j\frac{2\pi}{PM}k_2(u+l)}h_{0,0}^{(e)^*}[l]$$

and $\mathcal{I} := \{n | r_h^{(e)}[n] \neq 0\}$. The remaining subchannel gains $r_{0,1}, r_{1,1}$ and $r_{1,0}$ can be estimated by performing the same procedure as above for (13), (14), and (16), respectively. Let us next show how the phase ambiguity due to the $e^{j\phi_{m,i}}$ can be reduced by considering the cross-terms of $\mathbf{S}_r[k, z)$. Assuming that the channel gain estimates $\hat{r}_{m,i}$ are perfect we can set $H_{m,i}(z) = e^{j\phi_{m,i}}H_{m,i}^{(e)}(z)$, where now $H_{m,i}^{(e)}(z)$ denotes the estimate of $H_{m,i}(z)$ including the channel gain $r_{m,i}$. It follows from (7) that

$$[\mathbf{S}_r[k, z)]_{1,0} = H_{1,0}^{(e)}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(0)}[k, z)\tilde{H}_{0,0}^{(e)}(z)\, e^{j(\phi_{1,0} - \phi_{0,0})}$$

$$+H_{1,1}^{(e)}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(1)}[k,z]\tilde{H}_{0,1}^{(e)}(z)\, e^{j(\phi_{1,1}-\phi_{0,1})}$$

$$[\mathbf{S}_r[k,z)]_{0,1} = H_{0,0}^{(e)}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(0)}[k,z)\tilde{H}_{1,0}^{(e)}(z)\, e^{j(\phi_{0,0}-\phi_{1,0})}$$

$$+H_{0,1}^{(e)}\left(ze^{j\frac{2\pi}{PM}k}\right) S_s^{(1)}[k,z)\tilde{H}_{1,1}^{(e)}(z)\, e^{j(\phi_{0,1}-\phi_{1,1})}.$$

In the following we shall focus on the estimation of $\phi_{1,0} - \phi_{0,0}$. Since $S_s^{(1)}[k_2,z) = 0$ we get $[\mathbf{S}_r[k_2,z)]_{1,0} =$

$$H_{1,0}^{(e)}\left(ze^{j\frac{2\pi}{PM}k_2}\right) S_s^{(0)}[k_2,z)\tilde{H}_{0,0}^{(e)}(z)\, e^{j(\phi_{1,0}-\phi_{0,0})}, \qquad (21)$$

from which we can derive an estimate of the phase difference $\phi_e = \phi_{1,0} - \phi_{0,0}$ as

$$\widehat{\phi_e} = \frac{1}{|\mathcal{I}|}\sum_{n\in\mathcal{I}} \arg\left\{\frac{[\widehat{\mathbf{C}}_r[k_2,n]]_{1,0}}{r_h^{(e)}[n]}\right\},$$

where $r_h^{(e)}[n] =$

$$\sum_u C_s^{(0)}[k_2, n-u] \sum_l h_{1,0}^{(e)}[u+l] e^{-j\frac{2\pi}{PM}k_2(u+l)} h_{0,0}^{(e)^*}[l]$$

and $\mathcal{I} := \{n | r_h^{(e)}[n] \neq 0\}$. The phase difference $\phi_{0,1} - \phi_{1,1}$ can be estimated similarly from $[\widehat{\mathbf{C}}_r[k_1,n]]_{1,0}$ and $[\widehat{\mathbf{C}}_r[k_1,n]]_{0,1}$. Now, since we have estimated the channel gains $r_{m,i}\,(m,i = 0,1)$ and the phase differences $\phi_{1,0} - \phi_{0,0}$ and $\phi_{0,1} - \phi_{1,1}$, we have identified the 2×2 channel transfer matrix $\mathbf{H}(z)$ up to a diagonal matrix of phase rotations. The only ambiguities left are the $\phi_{i,i}\,(i = 0,1)$ or equivalently any matrix

$$\mathbf{H}'(z) = \mathbf{H}(z)\,\mathrm{diag}\{e^{j\phi_{i,i}}\}_{i=0,1}$$

with arbitrary $\phi_{i,i}\,(i = 0,1)$ is a valid solution of our algorithm. Now, since

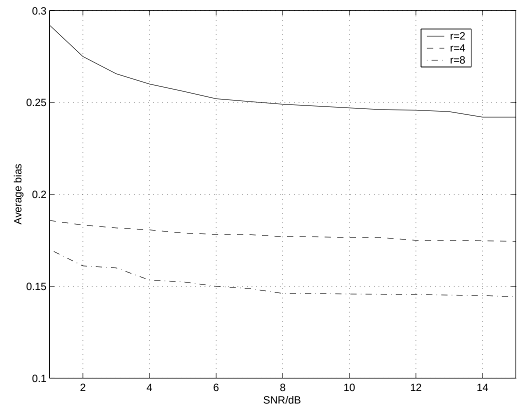
$$\mathbf{H}'(e^{j\frac{2\pi}{N}k}) = \mathbf{H}(e^{j\frac{2\pi}{N}k})\,\mathrm{diag}\{e^{j\phi_{i,i}}\}_{i=0,1}$$

we can see that equalization according to (3) for example recovers the individual data streams up to a phase rotation.

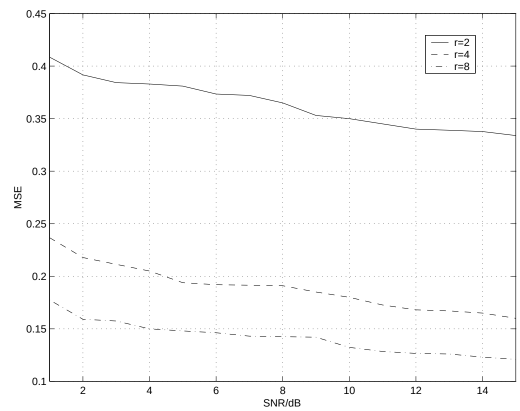
The extension of this algorithm to an arbitrary number of antennas and the systematic design of precoding sequences is discussed in [20].

4. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the performance of the proposed matrix channel estimation algorithm and the corresponding multi-channel equalizer. In all simulation examples the estimator performance was measured in terms of the average bias and the mean square error (MSE) both averaged over all subchannels. We simulated a system with 2 transmit and 2 receive antennas, $N = 12$ subcarriers and cyclic prefix of length $L = 4$ (i.e., $M = N + L = 16$) and used the length-4 precoding sequences $\mathbf{a}^{(0)} = [\alpha\ \beta\ \alpha\ \beta]^T$, $\mathbf{a}^{(1)} = [\beta\ \alpha\ \alpha\ \beta]^T$ with varying $r = |\beta/\alpha|$. The data symbols were i.i.d. 4-PSK symbols and the channel code employed was a convolutional code of rate 1/2 and constraint length 5. The signal-to-noise-ratio (SNR) for the i-th data stream was defined as $\mathrm{SNR}_i = 10\,\log_{10}\left(\frac{\sigma_i^2}{\sigma_\rho^2}\right)$, where σ_ρ^2 is the variance of the white noise process $\rho[n]$. Since in all simulations the SNR was chosen to be the same for all data streams in the following we drop the index i in SNR_i in order to simplify the notation. All results were obtained by averaging over 500 independent Monte Carlo trials, where each realization consisted of 8256 data symbols (i.e. 516 OFDM symbols). The length of the matrix channel we simulated was 3. In all simulation examples the precoding sequences were normalized.



(a)



(b)

Fig. 1. Estimator performance: (a) Average bias and (b) MSE as a function of SNR in dB for $r = |\beta/\alpha| = 2, 4,$ and 8.

Simulation Example 1. In the first simulation example we computed the average bias and the MSE of the channel estimator as a function of SNR in dB. Fig. 1 shows the results for $r = |\beta/\alpha| = 2, 4,$ and 8. We can see that the performance of the estimator generally improves with increasing SNR and that the best estimator performance is obtained for $r = 8$. Making r larger improves the estimator performance due to better separation of the individual transmit signals' cyclostationary statistics (cf. (11) and (12)). On the other hand in terms of overall system performance it is desirable to keep the degree of amplitude variation in the precoding coefficients as small as possible in order to have the transmit SNR as constant as possible over time. The second simulation example investigates this tradeoff in terms of performance of the multi-channel equalizer. Finally, we can observe that going from $r = 2$ to $r = 4$ improves the estimator performance significantly, while a further increase to $r = 8$ yields less improvement.

Simulation Example 2. For the channel estimates obtained in Simulation Example 1, we investigate the performance of the corresponding multi-channel equalizer. For each SNR value we took the average of the channel estimates over all Monte Carlo runs and computed the corresponding $\widehat{\mathbf{H}}(e^{j\frac{2\pi}{N}k}) = \sum_{l=0}^{2} \widehat{\mathbf{H}}_l\, e^{-j\frac{2\pi}{N}kl}$. Fig. 2 shows the BER as a function of the SNR for $r = |\beta/\alpha| = 2, 4$, and 8. We can see that $r = 4$ consistently yields the best performance in terms of BER.

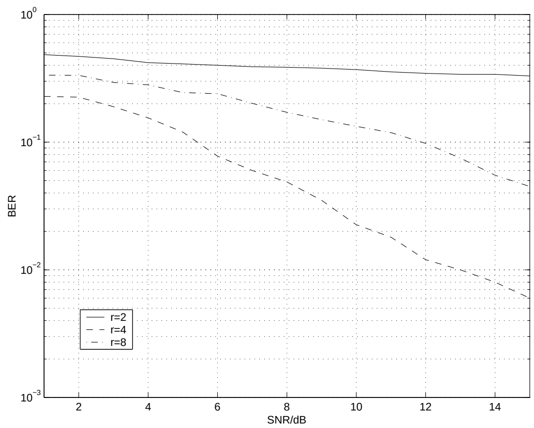


Fig. 2. Multi-channel equalizer performance. Bit error rate as a function of SNR in dB for $r = |\beta/\alpha| = 2, 4$ and 8.

5. CONCLUSION

We introduced an algorithm for blind channel identification and equalization in OFDM-based spatial multiplexing systems. Our approach uses second-order cyclostationary statistics and employs a form of nonredundant antenna precoding. The basic idea of our method is to provide each transmit antenna with a different signature in the cyclostationary domain to null out the influence of all but one transmit antenna at a time. This makes a scalar subchannel by subchannel identification of the matrix channel possible. Our algorithm does not require knowledge of the channel order, imposes no restrictions on channel zeros, and exhibits low sensitivity to stationary noise. We provided simulation examples demonstrating the channel estimator performance and the corresponding multi-channel equalizer performance.

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