Space-Time Signal Design for Fading Relay Channels

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***Abstract*— Cooperative diversity is a transmission technique where multiple users pool their resources to form a virtual antenna array that realizes spatial diversity gain in a distributed fashion. In this paper, we examine space-time signal design for a simple amplify-and-forward relay channel. We show that the code design criteria for the relay case consist of the traditional rank and determinant criteria as well as appropriate power control rules. While proper signal design and power control can indeed achieve full spatial diversity gain, the potential benefit of relay-assisted communication over direct communication depends strongly on the channel conditions. In particular, we present a switching criterion based on which the source terminal may opt to forego relay-assisted communication and communicate with the destination terminal directly. The criterion is based on the cut-off rate of the effective channel (physical channel in conjunction with finite constellation and maximum-likelihood (ML) decoding).**

I. Introduction

Transmission over wireless channels suffers from random fluctuations in signal level known as fading and from co-channel interference. Diversity is a powerful technique to mitigate fading and improve robustness to interference. In classical diversity techniques, the data signal is conveyed to the receiver over multiple (ideally) independently fading signal paths (in time/frequency/space). Appropriate combining at the receiver realizes diversity gain, thereby improving link reliability.

Spatial diversity techniques are particularly attractive since they do not incur an expenditure of transmission time or bandwidth, thereby improving spectral efficiency. Signal design for multi-antenna systems (i.e., space-time coding) aimed at extracting spatial diversity has been studied extensively in the literature [1], [2], [3], [4].

A new way of realizing spatial diversity gain (in a distributed fashion) has recently been introduced in [5], [6], [7], [8] under the name of *user cooperation diversity* or *cooperative diversity*. In [5] it has been shown that uplink capacity may be increased via user cooperation diversity. A variety of cooperation protocols have been studied and analyzed in [6], [9], [10], [11] for channels with a single relay (two users). In [7] it is shown that for channels with multiple relays, cooperative diversity with appropriate code construction offers full spatial diversity gain. We note that signal design for achieving cooperative diversity gain is a specific form of network coding, which is being increasingly studied as a means to improve the performance of wireless networks [12], [13], [14]. Finally, we refer to [15] for results on non-fading relay channels and to [16] and [17] for recent results on scaling laws in large (relay) networks.

Contributions. In this paper, we examine space-time signal design for the simple single relay channel (see Fig. 1) where the relay terminal simply amplifies and forwards the received signal. Our detailed contributions are as follows:

- For an amplify-and-forward relay channel, we derive an *upper-bound* on the *pairwise error probability* associated with a (distributed) space-time code. Based on this result, we demonstrate that optimal space-time code design in the relay case consists of satisfying the classical rank and determinant criteria as well as conditions on power allocation between nodes.
- We demonstrate that the use of relay-assisted communication is not always beneficial when compared to direct transmission. We then provide a (channel dependent) *switching criterion* based on which the source terminal may opt to forego relay-assisted communication and allocate all transmit power for direct communication with the destination terminal. Our criterion requires numerical evaluation and is based on the *cut-off rate* of the effective channel (physical channel in conjunction with finite constellation and maximum-likelihood (ML) decoding).

Notation. $\mathcal{E}$ denotes the expectation operator, $\mathbf{I}_M$ is the $M \times M$ identity matrix and $\mathbf{0}$ denotes the all zeros matrix of appropriate size. The superscripts ^{T,*,H} stand for transpose, elementwise conjugation, and conjugate transpose, respectively. A circularly symmetric complex Gaussian random variable is a random variable $z = (x + jy) \sim \mathcal{CN}(0, \sigma^2)$, where x and y are i.i.d. $\mathcal{N}(0, \sigma^2/2)$. Finally, $\|\mathbf{a}\|$ stands for the Euclidean norm of the vector $\mathbf{a}$.

Organization of the paper. The rest of this paper is organized as follows. Sec. II describes the channel and signal model. Sec. III analyzes the space-time codeword construction criteria for the relay channel under consideration. Sec. IV discusses the switching criterion based on which the source terminal may decide to forego relay-assisted communication. We present simulation results in Sec. V and conclude in Sec. VI.

II. Channel and Signal Model

General setup. Consider the relay channel shown in Fig. 1. Data is to be transmitted from the source terminal S to the destination terminal D with the assistance of the relay terminal R. All terminals are equipped with single antenna transmitters and receivers. Throughout the paper, we assume that a terminal cannot transmit and receive simultaneously.

The relay terminal amplifies and forwards the received signal to the destination. The[1] S→R link is orthogonalized (via time, frequency, or code division) to the direct S→D link. The R→D link is assumed to be co-channel to the S→D link, i.e., the transmitted signals collide at the destination terminal. This setup could correspond to a scenario where the source employs a TDMA-based scheme and devotes the first half of the time-slot to communication with the relay and the second half to communication with the destination terminal. The destination terminal chooses not to receive the signal transmitted during the first time slot as it may be engaged in data transmission to

[1] A→B signifies communication from terminal A to terminal B.

another terminal. Hence, data is buffered in the relay terminal for subsequent retransmission. Note that this protocol differs from the protocol described in [8] where there is no collision at the destination terminal between the S→D and R→D links. More fundamentally, protocols can be designed to vary the degree of collision (broadcasting or receive) in the network, resulting in varying complexity-performance tradeoffs [18]. Finally, we assume that the source terminal allocates equal transmit energy for direct and relay-assisted communication.

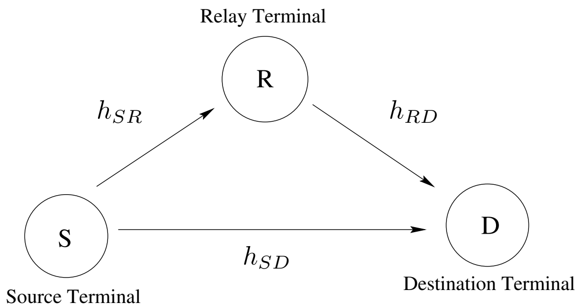

Fig. 1. Schematic of fading relay channel.

Channel model. Throughout the paper we assume frequency-flat block fading with block length $T \geq 2$, no channel knowledge in the transmitter, perfect channel state information in the receivers and perfect synchronization between the nodes. The channel gains h_{SD} and h_{SR} are assumed to consist of a fixed (possibly line-of-sight) and a variable component according to

$$h_{SY} = \sqrt{\frac{K_{SY}}{1+K_{SY}}} e^{j\phi_{SY}} + \sqrt{\frac{1}{1+K_{SY}}}\, \widetilde{h}_{SY}, \ \ Y = R, D,$$

where ϕ_{SY} is a deterministic phase factor, K_{SY} denotes the Ricean K-factor and $\widetilde{h}_{SY} \sim \mathcal{CN}(0, 1)$ is the variable component. Note that $\mathcal{E}\{|h_{SY}|^2\} = 1$. Unless specified otherwise, we assume that the channel between the relay terminal and the destination terminal is AWGN with $h_{RD} = 1$. Physically, this assumption could correspond to a scenario where the destination and relay terminals are static and have line-of-sight connection (e.g. two base-stations), while the source is moving. We emphasize that this assumption is conceptual and simplifies the performance analysis significantly. The general case is rather difficult to deal with analytically. In the last paragraph of Sec. III, we shall discus a special case where the assumption of h_{RD} being a fading link leads to analytically tractable results.

Signal model. The input-output relation for the S→R link is given by

$$y_R = \sqrt{E_{SR}}\, h_{SR} s_1 + n_R,$$

where y_R is the signal received at the relay terminal, E_{SR} is the average energy available at the transmitter over a symbol period (having accounted for path loss and shadowing in the S→R link), s_1 is a complex-valued data symbol drawn from a scalar constellation with unit average energy, and $n_R \sim \mathcal{CN}(0, N_o)$ is additive white noise. The relay terminal normalizes the received signal by a factor of $\sqrt{\mathcal{E}\{|y_R|^2\}}$ (so that the average energy is unity) and retransmits the normalized signal to the destination terminal. The signal received at the destination terminal is given by

$$y_D = \sqrt{E_{SD}}\, h_{SD} s_2 + \sqrt{E_{RD}}\, \frac{y_R}{\sqrt{\mathcal{E}\{|y_R|^2\}}} + n_D, \qquad (1)$$

where E_{SD} is the average energy available over a symbol period at the source terminal (having accounted for path loss and shadowing in the S→D link) for communication on the S→D link, s_2 is a complex-valued data symbol drawn from a scalar constellation with unit average energy, E_{RD} is the average energy available at the relay terminal over a symbol period (having accounted for path loss and shadowing in the R→D link) and $n_D \sim \mathcal{CN}(0, N_o)$ is additive white noise. Using $\mathcal{E}\{|y_R|^2\} = E_{SR} + N_o$, we can rewrite (1) as

$$y_D = \sqrt{E_{SD}} h_{SD} s_2 + \sqrt{\frac{E_{SR}E_{RD}}{E_{SR}+N_o}}\, h_{SR} s_1 + \widetilde{n}, \qquad (2)$$

where $\widetilde{n} \sim \mathcal{CN}(0, N_o')$ with $N_o' = N_o\left(1 + \frac{E_{RD}}{E_{SR}+N_o}\right)$. Finally, we assume that the receiver normalizes y_D by a factor $\omega = \sqrt{1 + \frac{E_{RD}}{E_{SR}+N_o}}$. This normalization does not alter the signal-to-noise ratio but simplifies the ensuing presentation. Note that although equal energy is allocated by the source terminal for direct and relay-assisted communication, E_{SD} is not necessarily equal to E_{SR} due to path-loss and shadowing differences between the S→D and S→R links. In matrix notation the effective input-output relation may now be written as

$$y_D/\omega = \mathbf{h}^T \mathbf{\Lambda} \mathbf{s} + n,$$

where $\mathbf{h} = [h_{SR}\ h_{SD}]^T$, $\mathbf{s} = [s_1\ s_2]^T$, $n \sim \mathcal{CN}(0, N_o)$ is noise and

$$\mathbf{\Lambda} = \text{diag}\left\{ \frac{1}{\omega}\sqrt{\frac{E_{SR}E_{RD}}{E_{SR}+N_o}},\ \frac{\sqrt{E_{SD}}}{\omega} \right\}.$$

The effective input-output relation resembles that of a point-to-point MISO (multiple-input single-output) channel. For notational convenience we introduce the 2×2 matrices $\overline{\mathbf{K}}$ and $\widetilde{\mathbf{K}}$ defined as

$$\overline{\mathbf{K}} = \text{diag}\left\{ \sqrt{\frac{K_{SR}}{1+K_{SR}}},\ \sqrt{\frac{K_{SD}}{1+K_{SD}}} \right\},$$

$$\widetilde{\mathbf{K}} = \text{diag}\left\{ \sqrt{\frac{1}{1+K_{SR}}},\ \sqrt{\frac{1}{1+K_{SD}}} \right\}.$$

Finally, we decompose $\mathbf{h}$ as $\mathbf{h} = \overline{\mathbf{h}} + \widetilde{\mathbf{h}}$, where

$$\overline{\mathbf{h}} = \overline{\mathbf{K}}\ [e^{j\phi_{SR}} \quad e^{j\phi_{SD}}]^T$$

is the fixed component of the channel and

$$\widetilde{\mathbf{h}} = \widetilde{\mathbf{K}}\ [\widetilde{h}_{SR} \quad \widetilde{h}_{SD}]^T$$

denotes the variable component.

III. Code Design Criteria

In this section, we derive the code design criteria for the amplify-and-forward relay channel introduced in Sec. II. In particular, we show that the design criteria consist of the traditional rank and determinant criteria for co-located antennas [2] and additional criteria on the power allocation between terminals. Based on these results, we then prove that amplify-and-forward based relay-assisted communication can indeed achieve second-order diversity.

General design criteria. Assume that the source terminal constructs a $2 \times T$ space-time codeword $\mathbf{C}$. Elements of the first and second rows of $\mathbf{C}$ are transmitted serially over the direct and relay-assisted channels, respectively. The signals received at the destination terminal may be stacked to form a $T \times 1$ vector $\mathbf{y}$ which satisfies

$$\mathbf{y}^T = \mathbf{h}^T \mathbf{\Lambda} \mathbf{C} + \mathbf{n}^T,$$

where $\mathbf{n}$ is a $T \times 1$ zero-mean circularly symmetric temporally white complex Gaussian noise vector with $\mathcal{E}\{\mathbf{n}\mathbf{n}^H\} = N_o \mathbf{I}_T$. We furthermore assume perfect knowledge of $\mathbf{h}$ and $\mathbf{\Lambda}$ at the destination terminal, which can be maintained through training and tracking (at the relay and destination terminals) and suitable communication protocols. The destination terminal constructs an ML estimate of the transmitted space-time codeword according to

$$\widehat{\mathbf{C}} = \arg\min_{\mathbf{C}} \|\mathbf{y}^T - \mathbf{h}^T \mathbf{\Lambda} \mathbf{C}\|^2, \quad (3)$$

where the minimization is performed over all possible codeword matrices $\mathbf{C}$. From (3) it follows immediately that the decoding complexity in the relay case is inherited from the underlying space-time code.

Now, the probability that a transmitted codeword $\mathbf{C}$ is mistaken for another codeword $\mathbf{E}$ for a given channel realization $\mathbf{h}$ is obtained as the pairwise error probability (PEP)

$$P(\mathbf{C} \to \mathbf{E}|\mathbf{h}) = Q\left(\sqrt{\frac{\|\mathbf{h}^T \mathbf{\Lambda}(\mathbf{C}-\mathbf{E})\|^2}{2N_o}}\right).$$

Setting $\mathbf{x}^T = \mathbf{h}^T \mathbf{\Lambda}(\mathbf{C}-\mathbf{E})$ and applying the Chernoff bound $Q(x) \leq e^{-\frac{x^2}{2}}$, we obtain

$$P(\mathbf{C} \to \mathbf{E}|\mathbf{h}) \leq e^{-\frac{\|\mathbf{x}\|^2}{4N_o}}. \quad (4)$$

The average (over h_{SD} and h_{SR}) of the right-hand-side (RHS) of (4) is completely characterized by the mean vector

$$\overline{\mathbf{x}} = (\mathbf{C}-\mathbf{E})^T \mathbf{\Lambda} \overline{\mathbf{h}}$$

and the covariance matrix

$$\mathbf{C}_{\mathbf{x}} = \mathcal{E}\{(\mathbf{x}-\overline{\mathbf{x}})(\mathbf{x}-\overline{\mathbf{x}})^H\} = (\mathbf{C}-\mathbf{E})^T \mathbf{\Lambda} \widetilde{\mathbf{K}}^2 \mathbf{\Lambda} (\mathbf{C}-\mathbf{E})^*.$$

Using the eigendecomposition[2] $\mathbf{C}_{\mathbf{x}} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$ where $\mathbf{\Sigma} = \text{diag}\{\sigma_j\}_{j=1}^2$, $P(\mathbf{C} \to \mathbf{E}) = \mathcal{E}_{\mathbf{h}}\{P(\mathbf{C} \to \mathbf{E}|\mathbf{h})\}$ may be upper-bounded as [19]

$$P(\mathbf{C} \to \mathbf{E}) \leq e^{-\frac{\|\overline{\mathbf{x}}\|^2}{4N_o}} \prod_{j=1}^{2} \frac{e^{\frac{(|b_j|/(4N_o))^2}{1+\sigma_j/(4N_o)}}}{1+\frac{\sigma_j}{4N_o}}, \quad (5)$$

where b_j $(j = 1, 2)$ is the j-th element of $\mathbf{b} = \mathbf{\Sigma}^{1/2}\mathbf{U}^H \overline{\mathbf{x}}$.

The case of pure Rayleigh fading. We now consider the case of pure Rayleigh fading, i.e., $K_{SD} = K_{SR} = 0$ which implies $\overline{\mathbf{K}} = \mathbf{0}$ and $\widetilde{\mathbf{K}} = \mathbf{I}_2$. Then,

$$P(\mathbf{C} \to \mathbf{E}) \leq \prod_{j=1}^{2} \frac{1}{1+\frac{\sigma_j}{4N_o}},$$

with σ_j $(j = 1, 2)$ denoting the eigenvalues of the 2×2 matrix $\mathbf{\Lambda}(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^H \mathbf{\Lambda}$. Applying Ostrowski's theorem [20], σ_j $(j = 1, 2)$ may be lower-bounded as

$$\sigma_j \geq \min\left\{\frac{E_{SR}E_{RD}}{\omega^2(E_{SR}+N_o)}, \frac{E_{SD}}{\omega^2}\right\}\gamma_j,$$

[2]Note that $\mathbf{C}_{\mathbf{x}}$ has at most two non-zero eigenvalues. $\mathbf{U}$ is a $T \times 2$ matrix satisfying $\mathbf{U}^H\mathbf{U} = \mathbf{I}_2$.

where γ_j $(j = 1, 2)$ denotes the eigenvalues of $(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^H$. Consequently, the average PEP can be upper-bounded as

$$P(\mathbf{C} \to \mathbf{E}) \leq \prod_{j=1}^{2} \frac{1}{1+\min\left\{\frac{E_{SR}E_{RD}}{\omega^2(E_{SR}+N_o)N_o}, \frac{E_{SD}}{\omega^2 N_o}\right\}\frac{\gamma_j}{4}}. \quad (6)$$

We set $\beta = \min\left\{\frac{E_{SR}E_{RD}}{\omega^2(E_{SR}+N_o)N_o}, \frac{E_{SD}}{\omega^2 N_o}\right\}$ which may be interpreted as an effective SNR. With this substitution we can write (6) as

$$P(\mathbf{C} \to \mathbf{E}) \leq \prod_{j=1}^{2} \frac{1}{1+\frac{\beta}{4}\gamma_j}. \quad (7)$$

For $\gamma_j > 0$ $(j = 1, 2)$ and $\beta \gg 1$, we get

$$P(\mathbf{C} \to \mathbf{E}) \leq \left(\prod_{j=1}^{2} \gamma_j\right)^{-1} (\beta/4)^{-2}, \quad (8)$$

which implies that relay-assisted communication with a properly designed space-time code achieves second-order diversity as a function of the effective SNR β. We emphasize, however, that the error-rate performance being determined by β implies that simply increasing the power on one of the links will not necessarily improve the error rate according to a second-order diversity behavior.

In order to further illustrate the necessity for power control, consider the case where a space-time code achieving full spatial diversity gain in the case of co-located antennas is used, E_{SR} and E_{RD} are kept constant, and E_{SD} is increased. For $E_{SD} > \frac{E_{SR}E_{RD}}{E_{SR}+N_o}$ we have $\beta = \frac{E_{SR}E_{RD}}{\omega^2(E_{SR}+N_o)N_o}$ which using (8) shows that the error probability does not improve according to a second-order diversity behavior. Instead, careful balancing between E_{SR}, E_{RD}, and E_{SD} is necessary which can be achieved through power control.

Hence, optimum signal design for the relay case consists of constructing a space-time code satisfying the classical rank and determinant criteria for the case of co-located antennas [2] as well as ensuring proper power control.

Comments on orthogonal designs. Orthogonal space-time block codes [3], [4] are popular due to their low decoding complexity. The Alamouti scheme is a simple orthogonal space-time block code where the codewords $\mathbf{C}$ are of the form

$$\mathbf{C} = \begin{bmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{bmatrix}.$$

Note that the Alamouti scheme can be applied to our setup without altering the simplified decoding procedure specified in [3]. For Alamouti transmission, $(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^H = \gamma \mathbf{I}_2$, where $\gamma = |c_1 - e_1|^2 + |c_2 - e_2|^2$. Therefore, the eigenvalues of $\mathbf{\Lambda}(\mathbf{C}-\mathbf{E})(\mathbf{C}-\mathbf{E})^H \mathbf{\Lambda}$ take a particularly simple form

$$\sigma_1 = \frac{E_{SD}}{\omega^2}\gamma, \;\; \sigma_2 = \frac{E_{SR}E_{RD}}{\omega^2(E_{SR}+N_o)}\gamma.$$

Consequently, the upper bound on $P(\mathbf{C} \to \mathbf{E})$ can be evaluated directly without applying Ostrowski's theorem to yield

$$P(\mathbf{C}\rightarrow\mathbf{E}) \leq \frac{1}{1+\frac{E_{SD}/(4N_o)}{\omega^2}\gamma} \times \frac{1}{1+\frac{E_{SR}E_{RD}/(4(E_{SR}+N_o)N_o)}{\omega^2}\gamma} \leq \left(\frac{1}{1+\frac{\beta}{4}\gamma}\right)^2. \quad (9)$$

For β large

$$P(\mathbf{C}\rightarrow\mathbf{E}) \leq \gamma^{-2}\left(\beta/4\right)^{-2},$$

which conforms with (8) and shows that second-order diversity in the effective SNR β can indeed be achieved. Furthermore (9) illustrates the need for appropriate power control. For, example it is clear that simply increasing E_{SD}/N_o in (9) will decrease the PEP according to a first-order rather than a second-order diversity behavior.

PEP for fading R→D link. So far, we have considered the case where the R→D link is static. In certain cases the average PEP for a fading R→D link ($h_{RD} \sim \mathcal{CN}(0,1)$) becomes analytically tractable. For example, continuing with the Alamouti scheme, the PEP for a given realization of h_{RD} (averaged over the S→R and S→D channels) can be upper-bounded as

$$P(\mathbf{C}\rightarrow\mathbf{E}|h_{RD}) \leq \frac{1}{1+\frac{E_{SD}/(4N_o)}{1+\frac{E_{RD}|h_{RD}|^2}{E_{SR}+N_o}}\gamma} \times \frac{1}{1+\frac{E_{SR}E_{RD}|h_{RD}|^2/(4(E_{SR}+N_o)N_o)}{1+\frac{E_{RD}|h_{RD}|^2}{E_{SR}+N_o}}\gamma}. \quad (10)$$

Now, assuming $E_{SR}/N_o \gg 1$ and $E_{SR} \gg E_{RD}$ we have $\beta \approx \min\{\frac{E_{SD}}{N_o}, \frac{E_{RD}}{N_o}\}$. For $\beta \gg 1$ the average PEP, $P(\mathbf{C}\rightarrow\mathbf{E}) = \mathcal{E}_{h_{RD}}\{P(\mathbf{C}\rightarrow\mathbf{E}|h_{RD})\}$, can be upper-bounded as

$$P(\mathbf{C}\rightarrow\mathbf{E}) \leq \left(\frac{\gamma\beta}{4}\right)^{-2}\Gamma\left(0,\frac{1}{\gamma\beta/4}\right)\exp\left(\frac{1}{\gamma\beta/4}\right), \quad (11)$$

where $\Gamma(a,x)$ is the *incomplete gamma function* defined as $\Gamma(a,x)=\int_x^\infty t^{a-1}e^{-t}dt$. It can now be shown that asymptotically in β the RHS of (11) decays proportional to β^{-2} with a coding gain loss (compared to the case where the R→D link is static) [18]. These observations are verified in Simulation Example 1.

IV. To Relay or not to Relay

Based on the results of the previous section one may be tempted to conclude that the relay link should always be used in order to improve the performance. In this section, we shall show that there are situations where using the direct link only is superior to relay-assisted communication. More specifically, depending on the channel conditions, it may be beneficial to allocate all transmit energy for direct S→D communication rather than splitting energy evenly between the S→D and S→R links. The channel conditions under which this may occur are varied. We base our decision to relay (or not to relay) on the cut-off rate [21] of the effective channel (physical channel in conjunction with finite scalar constellation and ML decoding), which lower-bounds the capacity of the physical channel. Simple decision rules on whether the relay should be used or not are difficult to obtain. We will therefore have to rely on numerical evaluation of our decision guidelines.

Let us start by examining the cut-off rate for direct S→D communication. In this case the input-output relation is given by

$$y_D = \sqrt{E_{SD}}\sqrt{2}h_{SD}s + n,$$

where the factor $\sqrt{2}$ reflects the additional energy allocated for direct communication, s is a complex-valued data symbol with unit average energy and $n \sim \mathcal{CN}(0,N_o)$ is white noise. The cut-off rate R_o^{Direct} of this channel (using the standard assumptions on codeword construction detailed in [21]) is given by

$$R_o^{Direct} = \log_2 A - \log_2\left(1+A^{-1}\sum_c\sum_{e,c\neq e}\zeta(c\rightarrow e)\right),$$

where A denotes the alphabet size and

$$\zeta(c\rightarrow e) = \frac{(1+K_{SD})e^{-\frac{\frac{2E_{SD}|c-e|^2}{4N_o}K_{SD}}{1+\frac{2E_{SD}|c-e|^2}{4N_o}+K_{SD}}}}{1+K_{SD}+\frac{2E_{SD}|c-e|^2}{4N_o}}$$

is the average (over the channel) Chernoff upper-bound on the probability of mistaking transmitted constellation point c for constellation point e. The cut-off rate for relay-assisted communication with a specific space-time transmission technique is defined similarly. For the sake of simplicity, considering Alamouti transmission, we obtain

$$R_o^{Relay} = \log_2 A - \frac{1}{2}\log_2\left(1+A^{-2}\sum_{\mathbf{C}}\sum_{\mathbf{E},\mathbf{C}\neq\mathbf{E}}\zeta(\mathbf{C}\rightarrow\mathbf{E})\right),$$

where $\zeta(\mathbf{C}\rightarrow\mathbf{E})$ is the RHS of (5) and the total available transmit energy is split evenly between the S→D and S→R links. The decision whether or not to relay is now based on the ratio

$$\eta = R_o^{Relay}/R_o^{Direct}, \quad (12)$$

with $\eta \leq 1$ meaning that the source terminal should forego relay-assisted communication and $\eta > 1$ suggesting otherwise.

V. Simulation Examples

In this section, we provide simulation results illustrating and quantifying our analytical results. We consider the transmission scenario described in Sec. II, assume uncoded 4-QAM modulation and Alamouti transmission for relay-assisted communication, and set $\phi_{SD} = \pi/3$ and $\phi_{SR} = -\pi/3$.

Simulation Example 1. In the first simulation example, we demonstrate that relay-assisted communication can indeed achieve second-order diversity. Fig. 2 depicts the simulated symbol error rate (Monte Carlo simulation) with and without the assistance of the relay as a function of E_{SD}/N_o for $K_{SD} = K_{SR} = 0$. We assume $E_{SD} = E_{RD}$, i.e., the S→D and R→D links are balanced (this can be achieved through power control) and for now that the R→D link is static. The uncoded symbol error rate for relay-assisted communication is shown for two different values of E_{SR}/N_o. Note that for $E_{SR}/N_o = 30$dB, $\omega \approx 1$ over the plotted range of E_{SD}/N_o. Furthermore, for $\omega \approx 1$ and $E_{SD} = E_{RD}$, $\beta = \frac{E_{SD}}{N_o}\min\left\{1, \frac{E_{SR}/N_o}{1+E_{SR}/N_o}\right\}$ which for E_{SR}/N_o large yields $\beta \approx E_{SD}/N_o$. Hence for E_{SR}/N_o large relay-assisted communication indeed achieves second-order diversity in E_{SD}/N_o (reflected by the slope of symbol error probability as a function of E_{SD}/N_o). However, performance degrades dramatically as the S→R link deteriorates to $E_{SR}/N_o = 5$dB. We observe an error flooring effect, due to the amplified noise received at the destination terminal through the R→D link which offsets any increase in received signal energy. In fact for E_{SR}/N_o sufficiently small and E_{RD}/N_o large the effective SNR is given by $\beta = \frac{E_{SR}}{N_o}$, which shows that β is governed by E_{SR}/N_o. For the sake of comparison we plot on the same graph the symbol error rate for the Alamouti scheme for the case where the R→D is Rayleigh fading and $E_{SR}/N_o = 30$dB. The simulation result conforms with the discussion and observations made subsequent to (11) showing that the error rate decays according to a second-order diversity behavior with a coding gain loss compared to the case where the R→D channel is static.

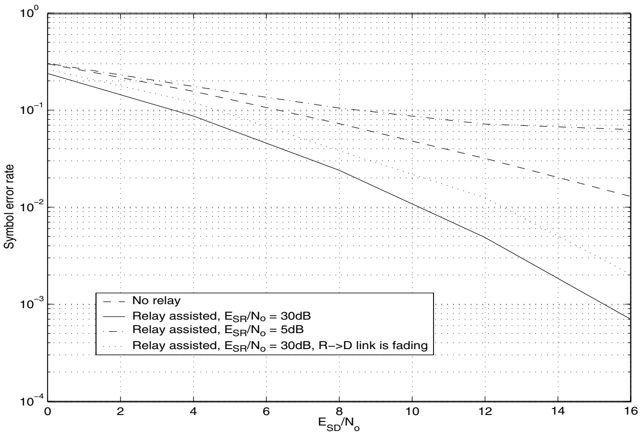

Fig. 2. Symbol error rate as a function of E_{SD}/N_o. Second-order diversity is achieved under suitable channel conditions.

Simulation Example 2. This simulation example serves to illustrate the fact that relay-assisted communication is not always superior to direct transmission. Assuming that $h_{RD} = 1$ is static, $K_{SR} = 0$ and $E_{SR}/N_o = E_{RD}/N_o = 15$dB, Fig. 3 plots the parameter η defined in (12) as a function of K_{SD} (on a linear scale) for different values of E_{SD}/N_o. It is clear that η decreases with increasing K_{SD} on account of the improved quality of the direct channel. Further, as expected, with increasing E_{SD}/N_o, η decreases for a given K_{SD}. Specifically, for $E_{SD}/N_o = 5$ dB and $K_{SD} > 15$, direct communication is preferred over relay-assisted communication. This threshold will in general decrease with increasing E_{SD}/N_o as observed in Fig. 3.

VI. CONCLUSION

We examined space-time signal design for a simple amplify-and-forward based fading relay channel. We showed that the error performance of relay-assisted communication is governed by an effective SNR which is an intricate function of path loss and the power assignment among the nodes. We demonstrated that the code design criteria for the amplify-and-forward relay channel consist of the traditional rank and determinant criteria for the case of co-located antennas [2] as well as appropriate power control rules. We presented a switching criterion based on which the source terminal may opt to forego relay-assisted communication and communicate with the destination terminal directly. Our criterion is based on the cut-off rate of the effective channel (physical channel in conjunction with finite constellation and maximum-likelihood (ML) decoding) and requires numerical evaluation.

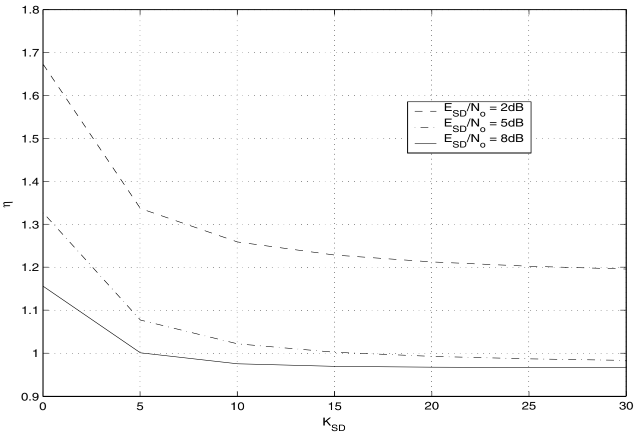

Fig. 3. The quantity η defined in (12) as a function of K_{SD} for the relay channel. The benefit of relay-assisted communication decreases with increasing K_{SD}.

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