CUT-OFF RATE BASED TRANSMIT OPTIMIZATION FOR SPATIAL MULTIPLEXING ON GENERAL MIMO CHANNELS

Rohit U. Nabar, Helmut Bölcskei *

Communication Technology Laboratory
ETH Zurich
Email: {nabar, boelcskei}@nari.ee.ethz.ch

Arogyaswami J. Paulraj

Information Systems Laboratory
Stanford University
Email: apaulraj@stanford.edu

ABSTRACT

The use of spatial multiplexing (SM) in multiple-input multiple-output (MIMO) wireless systems promises a linear (in the minimum of the number of transmit and receive antennas) increase in data rate. In practice, the performance of SM depends critically on a variety of channel conditions including antenna height and spacing, polarization of antennas, and richness of scattering. Transmit correlation has been shown to be detrimental to the performance of SM, since it leads to the existence of preferred spatial directions. In addition, the presence of an ill-conditioned fixed (possibly line-of-sight) component in the channel can severely degrade performance. In this paper, we present a simple transmit optimization strategy to partially mitigate the impact of unfavorable channel statistics on the performance of SM. The proposed strategy takes the scalar symbol constellation and the channel statistics into account and relies on simple phase-shifting of the multiplexed symbol streams at the transmitter. The phase shifts are chosen such that the cut-off rate of the effective channel (physical channel in combination with finite constellation and ML decoding) is maximized. We find SNR gains of up to 4 dB over the case when no transmit optimization is employed.

1. INTRODUCTION

The use of spatial multiplexing (SM) in systems with multiple antennas at the transmitter and the receiver can yield a dramatic increase in spectral efficiency [1, 2, 3]. In practice, the achievable multiplexing gain depends on a variety of channel conditions including antenna height and spacing, polarization of the antennas, and richness of scattering. With rich (omni-directional and isotropic) scattering and all antenna elements identically polarized, the elements of the matrix channel can be modeled as i.i.d. zero-mean circularly symmetric complex gaussian random variables (i.i.d. Rayleigh fading model). However, measurements [4] have shown that the MIMO channel can deviate significantly from this idealistic model and exhibit correlated fading, Ricean fading as well as gain imbalances between channel elements (if polarization diversity is employed). In [5] it has been shown that the presence of transmit correlation due to lack of scattering and/or insufficient transmit antenna spacing can have a detrimental impact on multi-antenna signaling techniques. Furthermore, the presence of an ill-conditioned fixed component (Ricean fading) in the channel can severely degrade the performance of SM [6].

Contributions. In this paper, we introduce a simple transmit optimization strategy which improves the performance of SM over unfavorable channels. The proposed technique is based on relative phase adjustments [7, 8] of the multiplexed symbol streams at the transmitter and exploits knowledge of the channel statistics. In practice, this assumption requires that the channel statistics vary slowly which is certainly true for the fixed wireless case. The optimum phase adjustments are obtained by maximizing the cut-off rate [9, 10] of the effective MIMO channel (physical MIMO channel in conjunction with a finite scalar symbol constellation and ML decoding). We assume a general MIMO channel model with correlated Rayleigh and/or Ricean fading. Simulation results show that SNR gains of up to 4 dB can be obtained. The idea of relative phase adjustment between the multiplexed symbol streams was first proposed by the authors in [7] for channels with two transmit antennas to reduce the average scalar symbol error rate in the presence of Rayleigh fading. The analysis in this paper extends the idea to MIMO channels with greater than two transmit antennas and to the Ricean case with the objective of increasing the cut-off rate of the effective channel.

Organization of the paper. The organization of this paper is as follows. Section 2 introduces the general MIMO channel and signal model. In Section 3, we present our transmit optimization technique. Section 4 provides simulation results, and Section 5 contains our conclusions.

2. GENERAL MIMO CHANNEL MODEL

Consider a MIMO channel with M_T transmit antennas and M_R receive antennas. Assuming flat fading over the frequency band of interest, the input-output relation for the channel is given by[1]

$$\mathbf{y} = \sqrt{E_s}\,\mathbf{H}\mathbf{x} + \mathbf{n},$$

where

*This work was supported in part by ETH grant TH-6 02-2.

[1] The superscript ^{H} stands for conjugate transpose. $\mathcal{E}$ is the expectation operator. $\mathbf{I}_m$ is the $m \times m$ identity matrix and $\mathbf{0}$ stands for an all zeros matrix of appropriate size.

- $\mathbf{y}$ is the $M_R \times 1$ receive signal vector
- E_s is the average energy available per transmit antenna over a symbol period
- $\mathbf{x}$ is the $M_T \times 1$ transmitted codevector whose elements are drawn independently from a scalar constellation (such as QAM) with unit average energy and alphabet size A
- $\mathbf{n}$ is zero-mean spatially white complex gaussian noise with $\mathcal{E}\{\mathbf{n}\mathbf{n}^H\} = N_o \mathbf{I}_{M_R}$.

The MIMO channel transfer matrix $\mathbf{H}$ is expressed as the sum of a fixed component and a fading component according to

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\,\overline{\mathbf{H}} + \sqrt{\frac{1}{1+K}}\,\widetilde{\mathbf{H}},$$

where $K > 0$ is the Ricean K-factor of the channel (defined as the ratio of the power in the fixed component to the power in the fading component of the channel). The elements of $\overline{\mathbf{H}}$ are complex constants that satisfy $|[\overline{\mathbf{H}}]_{m,n}|^2 = 1$ $(m = 1, 2, ..., M_R, n = 1, 2, ..., M_T)$. Furthermore, the elements of $\widetilde{\mathbf{H}}$ are modeled as (possibly correlated) circularly symmetric complex gaussian random variables with unit variance. Correlation between the elements of $\mathbf{H}$ may be captured through the following model [11, 5]

$$\widetilde{\mathbf{H}} = \mathbf{R}^{1/2}\mathbf{H}_w\mathbf{S}^{1/2},$$

where the elements of $\mathbf{H}_w$ are uncorrelated circularly symmetric complex gaussian random variables with unit variance and $\mathbf{R}$ and $\mathbf{S}$ are Hermitian positive semi-definite matrices that reflect the receive and transmit correlation, respectively.

3. TRANSMIT OPTIMIZATION

The transmit optimization strategy consists of premultiplying the codevector to be transmitted, $\mathbf{x}$, by an $M_T \times M_T$ diagonal precoding matrix according to

$$\mathbf{x}' = \mathbf{D}\mathbf{x},$$

where

$$\mathbf{D} = \mathrm{diag}\{e^{j\theta_m}\}_{m=1}^{M_T}.$$

This translates to a phase-shift applied to each of the M_T scalar sub-streams at the transmitter. In the following we assume that the channel $\mathbf{H}$ and the precoding matrix $\mathbf{D}$ are perfectly known to the receiver. The performance criterion that we seek to optimize is the cut-off rate $R_o(\mathbf{D})$ of the effective channel (i.e., physical channel in conjunction with finite scalar constellation and ML decoding) given by (in bps/Hz)

$$R_o(\mathbf{D}) = \log_2\left(\frac{1}{A^{M_T}} + \frac{\sum_{\mathbf{f}}\sum_{\mathbf{g}\neq\mathbf{f}}\beta(\mathbf{f}\to\mathbf{g},\mathbf{D})}{A^{2M_T}}\right)^{-1}, \tag{1}$$

where $\beta(\mathbf{f} \to \mathbf{g}, \mathbf{D})$ is the Chernoff upper-bound on the average (over the channel $\mathbf{H}$) probability that the receiver decodes transmitted codevector $\mathbf{f}$ as codevector $\mathbf{g}$ (assuming ML detection) for a given precoding matrix $\mathbf{D}$. Furthermore, we assume that the same scalar constellation is employed on all transmit antennas and that the symbols transmitted from the individual antennas are independent. The cut-off rate satisfies

$$\overline{P}_e \leq 2^{-n(R_o(\mathbf{D})-R)}, \tag{2}$$

where R is the number of information bits transmitted per vector symbol and $\overline{P}_e$ is the probability of codeword error averaged over the ensemble of coded systems with block length n (under standard assumptions on the choice of codebooks as detailed in [12]). Clearly, if $R < R_o(\mathbf{D})$ then $\overline{P}_e \to 0$ as $n \to \infty$. Thus for a given $\mathbf{D}$, $R_o(\mathbf{D})$ lower-bounds the capacity of the effective channel.

For the general MIMO channel, defining $\mathbf{e} = \mathbf{f} - \mathbf{g}$,

$$\begin{aligned}\overline{\mathbf{y}} &= \sqrt{\frac{K}{1+K}}\,\overline{\mathbf{H}}\mathbf{D}\mathbf{e}\\ \widetilde{\mathbf{y}} &= \sqrt{\frac{1}{1+K}}\,\widetilde{\mathbf{H}}\mathbf{D}\mathbf{e},\end{aligned}$$

and $\mathbf{C}_{\widetilde{\mathbf{y}}} = \mathcal{E}\{\widetilde{\mathbf{y}}\widetilde{\mathbf{y}}\} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ where $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_i\}_{i=1}^{M_R}$ we can show[2] [13, 6]

$$\beta(\mathbf{f}\to\mathbf{g},\mathbf{D}) = e^{-\frac{E_s}{4N_o}\|\overline{\mathbf{y}}\|^2}\prod_{i=1}^{r(\mathbf{C}_{\widetilde{\mathbf{y}}})}\frac{e^{\frac{\left(\frac{E_s}{4N_o}|d_i|\lambda_i\right)^2}{1+\frac{E_s}{4N_o}\lambda_i}}}{1+\frac{E_s}{4N_o}\lambda_i}, \tag{3}$$

where d_i $(i = 1, 2, \cdots, M_R)$ is the i-th element of

$$\mathbf{d} = \mathrm{diag}\left\{\frac{1}{\sqrt{\lambda_1}}, \frac{1}{\sqrt{\lambda_2}}, ..., \frac{1}{\sqrt{\lambda_{r(\mathbf{C}_{\widetilde{\mathbf{y}}})}}}, 0, ..., 0\right\}\mathbf{U}^H\overline{\mathbf{y}}.$$

Straightforward manipulations reveal[3]

$$\mathbf{C}_{\widetilde{\mathbf{y}}} = \left(\frac{\|\mathbf{S}^{1/2}\mathbf{D}\mathbf{e}\|^2}{1+K}\right)\mathbf{R}, \tag{4}$$

and hence $r(\mathbf{C}_{\widetilde{\mathbf{y}}}) = r(\mathbf{R})$ (assuming $\mathbf{D}\mathbf{e}$ does not lie perfectly in the nullspace of $\mathbf{S}$) with

$$\lambda_i = \left(\frac{\|\mathbf{S}^{1/2}\mathbf{D}\mathbf{e}\|^2}{1+K}\right)\gamma_i, \tag{5}$$

where $\gamma_i (i = 1, 2, \cdots, M_R)$ is the i-th eigenvalue of $\mathbf{R}$.

Now, if knowledge of the channel statistics (K, $\overline{\mathbf{H}}$, $\mathbf{R}$, and $\mathbf{S}$) is available to the transmitter, the optimization problem can be posed as

[2] $r(\mathbf{A})$ stands for the rank of the matrix $\mathbf{A}$.

[3] $\|\mathbf{a}\| = \sqrt{\mathbf{a}^H\mathbf{a}}$ is the Euclidean norm of the vector $\mathbf{a}$.

$$\boldsymbol{\theta}^{opt} = \arg\max_{\boldsymbol{\theta}} R_o(\boldsymbol{\theta}),$$

since $\boldsymbol{\theta} = [\theta_1\ \theta_2\ ...\ \theta_{M_T}]^T$ completely parameterizes $\mathbf{D}$. In the simulations we resort to numerical search to compute $\boldsymbol{\theta}^{opt}$. The complexity of this procedure can be reduced by coarser quantization of the phases at the expense of performance – i.e., the resulting solution may not be globally optimal. Further, it is straightforward to show that it is only the relative phase difference between the multiplexed symbol streams that affects performance. Therefore, without loss of generality we set $\theta_1 = 0$.

Note that if we do not impose a constraint on the structure of $\mathbf{D}$ (recall that in our case $\mathbf{D}$ is a diagonal matrix with unit modulus entries) we can expect improved performance. However, a more general $\mathbf{D}$ is likely to alter the peak power constraints of the power amplifiers at the transmitter which is an expensive proposition. The diagonal structure of $\mathbf{D}$ with simple phase-shifting leaves the peak power constraints of the transmit amplifiers unaltered thereby minimizing cost.

Physical Intuition. The physical intuition behind our optimization scheme is as described in [7]. Codevectors that lie along input singular directions with low gain are likely to be mistaken for other codevectors, thereby resulting in a reduction in cut-off rate. The precoding matrix $\mathbf{D}$ seeks to steer codevectors in the direction of the input singular vectors with high gain thereby making them more distinguishable at the receiver and increasing the cut-off rate. Further, different codevectors need to be aligned differently depending on their geometry. Hence $\mathbf{D}^{opt}$ seeks to balance the phase-shift settings through the in-built averaging over all codevector differences (cf. Eq. (1)).

Additional Comments. Note from Eq. (4) the asymmetry in the impact of transmit and receive correlation on the performance of SM. This asymmetry has been noted before in [5], and has important implications on the transmit optimization scheme at hand.

For example, consider pure Rayleigh fading ($K = 0$) with no transmit correlation ($\mathbf{S} = \mathbf{I}_{M_T}$). Since $\mathbf{D}^H\mathbf{D} = \mathbf{I}_{M_T}$, we get

$$\mathbf{C}_{\tilde{\mathbf{y}}} = \|\mathbf{e}\|^2\mathbf{R}, \tag{6}$$

making the optimization technique redundant irrespectively of the receive correlation. Such a scenario is likely to occur in the uplink (communication from the subscriber to the base) at large ranges since scattering is typically rich around the subscriber. An extension of this observation is that the transmit optimization technique will not provide any gains in an i.i.d. Rayleigh fading MIMO channel ($\mathbf{R} = \mathbf{I}_{M_R}$ and $\mathbf{S} = \mathbf{I}_{M_T}$). This is intuitively clear as the i.i.d. channel is spatially white with no preferred directions.

The gains of the transmit optimization technique proposed in this paper are available in Rayleigh fading environments with transmit correlation. Such propagation characteristics are likely in the downlink (communication from the base to the subscriber) at large ranges since scattering is typically poor at the base-station as the base-station antennas are elevated with narrow vertical beamwidth and restricted horizontal beamwidth. Further, at high SNR $\left(\frac{E_s}{N_o} \gg 1\right)$ when $K = 0$, combining Eqs. (1) and (3) and assuming that $\mathbf{De}$ does not lie perfectly in the null-space of $\mathbf{S}$, we get

$$R_o(\mathbf{D}) = \log_2\left(\frac{1}{A^{M_T}} + \frac{\sum_{\mathbf{f}}\sum_{\mathbf{g}\neq\mathbf{f}} \|\mathbf{S}^{1/2}\mathbf{De}\|^{-2r(\mathbf{R})}}{A^{2M_T}\prod_{i=1}^{r(\mathbf{R})}\left(\frac{E_s\gamma_i}{4N_o}\right)}\right)^{-1},$$

which indicates that in the presence of Rayleigh fading at high SNR knowledge of the transmit correlation and rank of the receive correlation matrix at the transmitter is sufficient to optimize the cut-off rate. Additionally, $\boldsymbol{\theta}^{opt}$ is independent of the actual receive correlation (but for $r(\mathbf{R})$) and the SNR (in practice 10-15dB).

The above comments pertain to the case of pure Rayleigh fading. With increasing K, the geometry of the transmitted vector constellation relative to the fixed component of the channel $\overline{\mathbf{H}}$ plays an important role. Simulations in the following section will demonstrate the performance gains through phase-shifting at the transmitter when the channel contains a fixed component.

4. SIMULATION RESULTS

We consider a channel with $M_T = M_R = 3$ employing SM with BPSK modulation. The SNR is defined as $10\log_{10}\left(\frac{M_T E_s}{N_o}\right)$ (in dB). Further, we assume that $\mathbf{R} = \mathbf{I}_{M_R}$ and that the fixed component of the channel and the transmit correlation matrix $\mathbf{S}$ are given by

$$\overline{\mathbf{H}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 1 & t_{1,2} & t_{1,3} \\ t_{1,2}^* & 1 & t_{2,3} \\ t_{1,3}^* & t_{2,3}^* & 1 \end{bmatrix},$$

where $t_{i,j}$ is the coefficient of correlation between the i-th and j-th transmit antennas. Note that the above fixed component of the channel results when the antennas at the transmitter and receiver are closely spaced and appear at broadside to each other. In the following, we set $t_{1,2} = 0.95$, $t_{1,3} = 0.8$, and $t_{2,3} = 0.9$. Figs. 1 and 2 show the cut-off rate for the channel with and without the optimal phase-shift settings for $K = 0$ and $K = 10$. One can see a 4 dB gain in SNR for the case of Rayleigh fading, restoring almost all of the loss due to spatial fading correlation, and an even higher gain for the case of Ricean fading. The gain is expected to decrease when higher order codebooks are employed due to the increased density of the input-signal space which increases the probability that a favorable phase-setting for a particular signal vector is detrimental to another signal vector. The double-humped nature of the cut-off rate curve in the case of Ricean fading with no relative phase adjustment can be explained as follows. Since $K = 10$, the effects of the fading component of the channel remain buried beneath the noise floor up to an SNR of about 10 dB. Beyond this SNR the fading in the channel results in a more conducive channel geometry resulting in two distinct behavior regions.

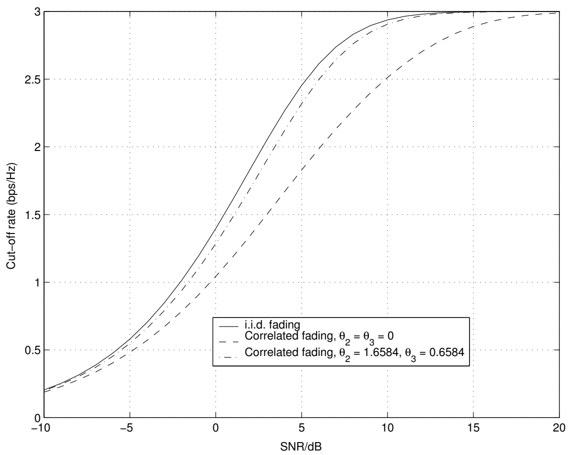

Fig. 1. Impact of relative-phase adjustment on cut-off rate performance in a correlated Rayleigh fading environment.

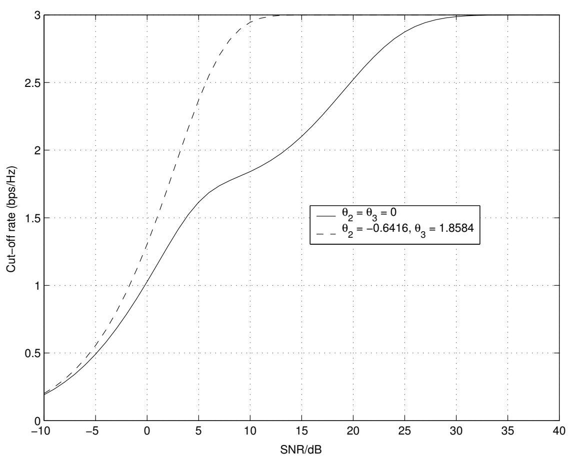

Fig. 2. Relative phase adjustment can result in significant gains in cut-off rate performance in the presence of Ricean fading.

5. CONCLUSION

We introduced a novel simple transmit optimization technique for spatial multiplexing in general MIMO channels. The proposed strategy relies on relative phase adjustments of the transmitted symbol streams with the phase shift values obtained by maximizing the cut-off rate of the effective MIMO channel (physical channel in conjunction with finite scalar constellation and ML decoding). The scheme is inexpensive in terms of hardware complexity and simulation examples demonstrate gains of up to 4 dB depending on the channel conditions.

6. REFERENCES

[1] A. J. Paulraj and T. Kailath, "Increasing capacity in wireless broadcast systems using distributed transmission/directional reception," *U. S. Patent*, no. 5,345,599, 1994.

[2] G. J. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Comm.*, vol. 6, no. 3, pp. 311–335, March 1998.

[3] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Tel.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[4] D. Baum, D. Gore, R. U. Nabar, S. Panchanathan, K. Hari, V. Erceg, and A. J. Paulraj, "Measurement and characterization of broadband MIMO fixed wireless channels at 2.5 GHz," in *Proc. IEEE ICPWC*, Hyderabad, India, Dec. 2000, pp. 203–206.

[5] H. Bölcskei and A. J. Paulraj, "Performance analysis of space-time codes in correlated Rayleigh fading environments," in *Proc. Asilomar Conf. on Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2000, vol. 1, pp. 687–693.

[6] R. U. Nabar, H. Bölcskei, V. Erceg, D. Gesbert, and A. J. Paulraj, "Performance of multi-antenna signaling techniques in the presence of polarization diversity," *IEEE Trans. Sig. Proc.*, vol. 50, no. 10, pp. 2553–2562, Oct. 2002.

[7] R. U. Nabar, H. Bölcskei, and A. J. Paulraj, "Transmit optimization for spatial multiplexing in the presence of spatial fading correlation," in *Proc. IEEE GLOBECOM*, San Antonio, TX, Nov. 2001, vol. 1, pp. 131–135.

[8] Z. Hong, K. Liu, R. Heath, and A. Sayeed, "Spatial multiplexing in correlated fading via the virtual channel representation," *IEEE J. Sel. Areas Comm.*, accepted for publication.

[9] D. Rainish, "Diversity transform for fading channels," *IEEE Trans. Comm.*, vol. 44, no. 12, Dec. 1996.

[10] A. O. Hero and T. L. Marzetta, "Cutoff rate and signal design for the quasi-static Rayleigh-fading space-time channel," *IEEE Trans. Inf. Theory*, vol. 47, no. 6, pp. 2400–2416, Sept. 2001.

[11] D. Gesbert, H. Bölcskei, D. Gore, and A. J. Paulraj, "MIMO wireless channels: Capacity and performance prediction," in *Proc. IEEE GLOBECOM*, San Francisco, CA, Nov./Dec. 2000, vol. 2, pp. 1083–1088.

[12] S. H. Jamali and T. Le-Ngoc, *Coded-modulation techniques for fading channels*, Kluwer, Norwell, MA, 1994.

[13] R. U. Nabar, H. Bölcskei, and A. J. Paulraj, "Outage performance of space-time block codes for generalized MIMO channels," *IEEE Trans. Inf. Theory*, March 2002, submitted.