MIMO-OFDM Multiple Access with Variable Amount of Collision

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***Abstract*— We consider frequency selective Multiple-Input Multiple-Output (MIMO) multiple access fading channels with perfect channel knowledge in the receiver and no channel state information in the transmitters. Assuming that each of the users employs Orthogonal Frequency Division Multiplexing (OFDM), we introduce a multiple access scheme which allows to gradually vary the amount of user collision in signal space by assigning different subsets of the available OFDM tones to different users. The corresponding multiple access schemes range from FDMA (each OFDM tone is assigned to at most one user) to CDMA (each OFDM tone is assigned to all the users). We quantify the effect of signal space collision between users by computing ergodic capacity regions for joint and single user decoding. In the case of joint decoding, we prove that irrespectively of the spatial receive fading correlation, the ergodic capacity region obtained by a fully collision-based scheme is an outer bound to any other OFDM-based multiple-access strategy, where the users collide only on subsets of the available tones. For single user decoding, we find that the amount of collision maximizing the capacity region depends critically on the number of transmit and receive antennas and the users' receive fading correlation.**

I. Introduction

The use of multiple-input multiple-output (MIMO) wireless systems has recently been shown to significantly increase the spectral efficiency of point-to-point links [1]–[5]. The impact of MIMO technology on the capacity of multi-user systems is considerably less understood.

Contributions: In this paper, we focus on MIMO multiple access channels with frequency-selective fading (spatially correlated at the receiver) assuming perfect channel state information in the receiver and no channel knowledge in the transmitters. Assuming that each of the users employs Orthogonal Frequency Division Multiplexing (OFDM) [6], we propose a multiple access scheme which implements a variable amount of user collision in signal space by assigning subsets of the available OFDM tones to different users. The corresponding family of multiple access schemes encompasses the extreme cases of FDMA where each tone is assigned to at most one user and CDMA where each tone is assigned to all the users. For the resulting family of multiple access schemes, we discuss the impact of user collision on the ergodic capacity regions for joint and single user decoding.

In the case of joint decoding, we prove that irrespectively of spatial receive fading correlation, the ergodic capacity region obtained by a fully collision-based scheme is an outer bound to any other multiple access strategy, where users collide only on subsets of the available tones. This result generalizes the well known result by Gallager [7] showing that FDMA is strictly inferior to CDMA in single-antenna multipath fading multiple access channels. We then show using a simple two user example, that for rich scattering and a small number of receive antennas, under joint decoding in practice very little collision in frequency is needed to realize a significant fraction of the available sum capacity. Minimizing the amount of collision is desirable as it minimizes the receiver complexity incurred by having to separate the interfering signals. For single user decoding, we find that the amount of collision maximizing the capacity region depends critically on the number of transmit and receive antennas and spatial receive fading correlation.

Previous work: Work on SIMO (i.e., the individual users are equipped with a single transmit antenna and the receiver employs multiple antennas) and MIMO multiple access fading channels has been reported previously in [8]–[11]. Results comparing CDMA and FDMA (two extremes of our multiple access scheme) in single antenna frequency selective fading multiple access channels can be found in [7], [12]–[14]. Finally, [15] discusses the capacity of frequency selective multiple access fading channels in the case of single-antenna systems with perfect channel knowledge both at the transmitters and the receiver.

Organization of the paper: The remainder of this paper is organized as follows. Section II introduces the channel and signal models. In Section III, we compute the ergodic capacity regions for joint and single user decoding as a function of the amount of collision between users. Sections IV and V present quantitative results for the two-user case under joint and single-user decoding, respectively. We conclude in Section VI.

Notation: $\mathcal{E}$ denotes the expectation operator. The superscripts ^{T,H} and * stand for transposition, conjugate transposition and elementwise conjugation, respectively. $r(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$,

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and $\lambda_i(\mathbf{A})$ denote the rank, trace, and i-th eigenvalue[1] of the matrix $\mathbf{A}$, respectively. $\mathbf{I}_m$ stands for the $m \times m$ identity matrix. Let $\mathcal{C}$ denote a set, then $|\mathcal{C}|$ stands for the size of this set. If A and B are random variables, $A \sim B$ denotes equivalence in distribution. An m-variate circularly symmetric zero-mean complex Gaussian random vector is a random vector $\mathbf{z} = \mathbf{x} + j\mathbf{y} \sim \mathcal{CN}_m(\mathbf{0}, \mathbf{\Sigma})$, where the real-valued random vectors $\mathbf{x}$ and $\mathbf{y}$ are jointly Gaussian, $\mathcal{E}\{\mathbf{z}\} = \mathbf{0}$, $\mathcal{E}\{\mathbf{z}\mathbf{z}^H\} = \mathbf{\Sigma}$, and $\mathcal{E}\{\mathbf{z}\mathbf{z}^T\} = \mathbf{0}$.

II. Signal and Channel Models

In this section, we shall first introduce the multiple access MIMO channel model, and then describe our signal model.

A. Multiple Access MIMO Channel Model

We consider a multiple access MIMO channel with P users each of which is equipped with M_T transmit antennas, the receiver employs M_R antennas. The individual users' channels are assumed frequency selective with the i-th user's matrix-valued transfer function given by

$$\mathbf{H}_i(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_{i,l} e^{-j2\pi l\theta}, \quad 0 \leq \theta < 1. \tag{1}$$

We restrict ourselves to purely Rayleigh block-fading channels with the elements of $\mathbf{H}_{i,l}$ $(i = 0, 1, \ldots, P-1; l = 0, 1, \ldots, L-1)$ being circularly symmetric zero-mean complex Gaussian random variables, and changing in an independent fashion from block to block [16]. Furthermore, the matrices $\mathbf{H}_{i,l}$ are assumed to be uncorrelated across users (indexed by i) and across taps (indexed by l). Moreover, we assume spatially uncorrelated fading at the transmit arrays. Spatial fading correlation at the receive array is modeled by decomposing the taps $\mathbf{H}_{i,l}$ according to $\mathbf{H}_{i,l} = \mathbf{R}_{i,l}^{1/2}\mathbf{H}_{w,i,l}$ with $\mathbf{H}_{w,i,l}$ denoting a random matrix with i.i.d. $\mathcal{CN}_1(0,1)$ entries and $\mathbf{R}_{i,l} = \mathbf{R}_{i,l}^{1/2}\mathbf{R}_{i,l}^{1/2}$ is the receive correlation matrix for the l-th tap of the i-th user. We note that the power delay profiles of the individual channels are incorporated into the correlation matrices $\mathbf{R}_{i,l}$. Finally, we assume that the receiver knows all the channels perfectly whereas the transmitters have no channel state information.

B. Signal Model

We assume that each of the users employs OFDM [6] with N tones and the length of the cyclic prefix (CP) satisfies $L_{cp} \geq L$. The latter assumption guarantees that each of the frequency-selective fading channels decouples into a set of parallel frequency-flat fading channels. The receive signal vector for the k-th tone is consequently given by

$$\mathbf{r}_k = \sum_{i=0}^{P-1} \mathbf{H}_i(e^{j2\pi\frac{k}{N}})\mathbf{c}_{i,k} + \mathbf{n}_k, \quad k = 0, 1, \ldots, N-1, \tag{2}$$

where $\mathbf{c}_{i,k} = [c_{i,k}^{(0)}\ c_{i,k}^{(1)}\ \ldots\ c_{i,k}^{(M_T-1)}]^T$ with $c_{i,k}^{(l)}$ denoting the data symbol transmitted by the i-th user from the l-th antenna on the k-th tone and $\mathbf{n}_k \sim \mathcal{CN}_{M_R}(\mathbf{0}, \mathbf{I}_{M_R})$ is white noise satisfying

$$\mathcal{E}\{\mathbf{n}_k\mathbf{n}_{k'}^H\} = \mathbf{I}_{M_R}\delta[k-k'].$$

We assume that the $\mathbf{c}_{i,k} \sim \mathcal{CN}_{M_T}\left(\mathbf{0}, \frac{P_{i,k}}{M_T}\mathbf{I}_{M_T}\right)$ are mutually independent (across users and tones) with $P_{i,k}$ denoting the power allocated to the k-th tone of the i-th user.

We shall next state an important property which will be used frequently in what follows. Under the assumptions stated in Sec. II-A, using (1) we can conclude that the channel matrices for user i are identically distributed for tones $k = 0, 1, \ldots, N-1$ [5], i.e.,

$$\mathbf{H}_i(e^{j2\pi\frac{k}{N}}) \sim \mathbf{H}_i \tag{3}$$

for $i = 0, 1, \ldots, P-1,\ k = 0, 1, \ldots, N-1$. In particular, we have $\mathbf{H}_i = \mathbf{R}_i^{1/2}\mathbf{H}_{i,w}$, where $\mathbf{R}_i = \sum_{l=0}^{L-1}\mathbf{R}_{i,l}$ and $\mathbf{H}_{i,w}$ is a random matrix with i.i.d. $\mathcal{CN}_1(0,1)$ entries.

We conclude this section by noting that the assumption of the users employing OFDM essentially results in a periodic signal model, or more precisely the action of the channel on the transmitted signal is described by circular convolution rather than linear convolution. Our results are therefore not restricted to OFDM modulation, but hold more generally.

III. Multiple Access with Variable Amount of Collision

We consider a general multiple access scheme by assigning each OFDM tone $k = 0, 1, \ldots, N-1$ to a subset of users $\mathcal{P}_k$. A fully collision-based multiple access scheme where all tones are assigned to each user (i.e., $|\mathcal{P}_k| = P$ for $k = 0, 1, \ldots, N-1$) is referred to as CDMA. FDMA is obtained for $|\mathcal{P}_k| \leq 1,\ k = 0, 1, \ldots, N-1$. We emphasize that as in [7], the capacity region obtained for a fully collision-based scheme (denoted CDMA in this paper) provides an outer bound to the capacity region of CDMA systems employing spreading, such as multi-carrier CDMA [17]. Following [5], in our capacity calculations, we ignore the loss in spectral efficiency due to the presence of a CP. Throughout the paper rates are specified in bps/Hz.

A. Ergodic Capacity Region for Joint Decoding

For joint decoding, the ergodic capacity region is given by the closure of the set of rates satisfying [18]

$$\sum_{i\in\mathcal{S}} R_i \leq \mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} \tag{4}$$

for all $\mathcal{S} \subseteq \{0, 1, \ldots, P-1\}$, where R_i denotes the rate for user i, $\rho_{i,k} = \frac{P_{i,k}}{M_T}$, and $\mathbf{H}_i$ was defined in (3). It can be shown that under the power constraints $\mathrm{Tr}(\mathcal{E}\{\mathbf{c}_{i,k}\mathbf{c}_{i,k}^H\}) = P_{i,k}$, in the case of joint decoding, the assumptions on $\mathbf{c}_{i,k}$ made in Sec. II-B correspond to the ergodic capacity achieving strategy [18]. We shall next prove that for joint decoding, the ergodic capacity region for a variable amount of collision is always outer bounded by the capacity region for a fully collision-based scheme (CDMA).

[1] Eigenvalues of hermitian matrices are given in descending order.

Theorem 1: Let $\boldsymbol{\rho}_k = [\rho_{0,k} \quad \rho_{1,k} \quad \dots \quad \rho_{P-1,k}]^T$ for $k = 0, 1, \dots, N-1$ with the $\rho_{i,k}$ defined in (4). Assuming the power normalization $\sum_{k=0}^{N-1} \boldsymbol{\rho}_k = \mathbf{c}$, the ergodic capacity region bounds in (4)

$$\mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\det\left(\mathbf{I}_{M_R}+\sum_{i\in\mathcal{S}}\rho_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}$$

are jointly maximized for all $\mathcal{S} \subseteq \{0, 1, \dots, P-1\}$ if and only if $\boldsymbol{\rho}_k = \frac{1}{N}\mathbf{c} \quad (k = 0, 1, \dots, N-1)$.

Proof: Let $\mathcal{S} = \{0, 1, \dots, P-1\} = \mathcal{P}$, and define the function $\phi_{\mathcal{P}}(\boldsymbol{\rho}) : \mathbb{R}_+^P \to \mathbb{R}_+$

$$\phi_{\mathcal{P}}(\boldsymbol{\rho}) = \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R}+\sum_{i=0}^{P-1}\rho_i\mathbf{H}_i\mathbf{H}_i^H\right)\right\},$$

where $\boldsymbol{\rho} = [\rho_0 \quad \rho_1 \quad \dots \quad \rho_{P-1}]^T$ and $\mathbf{H}_i$ was defined in (3). It can be shown that $\phi_{\mathcal{P}}(\boldsymbol{\rho})$ is strictly concave in $\boldsymbol{\rho}$ [18]. The capacity region bound in (4) for $\mathcal{S} = \mathcal{P}$ satisfies

$$\mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\det\left(\mathbf{I}_{M_R}+\sum_{i=0}^{P-1}\rho_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} = \frac{1}{N}\sum_{k=0}^{N-1}\phi_{\mathcal{P}}(\boldsymbol{\rho}_k).$$

Since $\phi_{\mathcal{P}}(\boldsymbol{\rho})$ is strictly concave in $\boldsymbol{\rho}$, Jensen's inequality [19] states that

$$\frac{1}{N}\sum_{k=0}^{N-1}\phi_{\mathcal{P}}(\boldsymbol{\rho}_k) \le \phi_{\mathcal{P}}\left(\frac{1}{N}\sum_{k=0}^{N-1}\boldsymbol{\rho}_k\right) = \phi_{\mathcal{P}}\left(\frac{1}{N}\mathbf{c}\right),$$

where equality is achieved if and only if $\boldsymbol{\rho}_k = \frac{1}{N}\mathbf{c}$ for $k = 0, 1, \dots, P-1$. The case $\mathcal{S} \subset \mathcal{P}$ follows similarly [18]. ■

Besides applying to MIMO channels, Theorem 1 generalizes the well known result by Gallager [7] in two additional ways: i) it is not restricted to "block-fading" in frequency, and ii) it establishes the superiority of a fully collision-based multiple access scheme (CDMA) over any other (frequency) collision-based multiple access scheme including the collision-free FDMA scheme as a special case.

B. Ergodic Capacity Region for Single User Decoding

In the case of single user decoding, each user is decoded separately with the other users being treated as noise. The resulting ergodic capacity region for any amount of collision is given by the closure of the set of rates satisfying [18]

$$R_p \le \mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\left(\frac{\det\left(\mathbf{I}_{M_R}+\sum_{i=0}^{P-1}\rho_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)}{\det\left(\mathbf{I}_{M_R}+\sum_{i\neq p}\rho_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)}\right)\right\} \tag{5}$$

for $p = 0, 1, \dots, P-1$. In contrast to the case of joint decoding, there is no general collision strategy for single user decoding that would maximize the capacity region in (5). In Section V, we show for the two-user case that the optimum collision strategy depends critically on the number of transmit and receive antennas and on the spatial receive fading correlation.

IV. The Two-User Case Under Joint Decoding

The purpose of this section is to analyze the ergodic capacity region under joint decoding in detail and to provide quantitative results. In particular, we shall quantify the impact of the amount of collision in frequency and spatial receive fading correlation on the capacity region. For the sake of simplicity of exposition, throughout the section, we restrict ourselves to the two-user case.

We assume that each user employs the same number of tones $N_u \ge N/2 \in \mathbb{N}$ out of which bN tones with $b = \frac{2i}{N}$, $i \in \{0, 1, \dots, N/2\}$, are assigned to both users. Consequently, $b = 0$ corresponds to FDMA and $b = 1$ yields a fully collision-based multiple access scheme. The total power is the same for both users, i.e., $\sum_{k=0}^{N-1} P_{1,k} = \sum_{k=0}^{N-1} P_{2,k} = \bar{P}$. It is also assumed that each user allocates the total power uniformly over the tones it has been assigned. Finally, using the above assumptions, the ergodic capacity region is given by the closure of the set of rates satisfying

$$\begin{aligned}R_0+R_1 \le\ & b\ \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R}+\frac{2\rho}{b+1}(\mathbf{H}_0\mathbf{H}_0^H+\mathbf{H}_1\mathbf{H}_1^H)\right)\right\}\\ &+\frac{(1-b)}{2}\ \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R}+\frac{2\rho}{b+1}\mathbf{H}_0\mathbf{H}_0^H\right)\right\}\\ &+\frac{(1-b)}{2}\ \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R}+\frac{2\rho}{b+1}\mathbf{H}_1\mathbf{H}_1^H\right)\right\}\\ R_i \le\ & \frac{1+b}{2}\ \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R}+\frac{2\rho}{b+1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}, i=0,1,\end{aligned}$$

where $\mathbf{H}_0$ and $\mathbf{H}_1$ have been defined in (3) and $\rho = \frac{\bar{P}}{NM_T}$. In the following, we discuss the low and high-SNR cases separately since the corresponding conclusions are fundamentally different.

A. Low-SNR Regime

When $\rho \ll 1$, we can employ the first-order approximation

$$\log_2\det\left(\mathbf{I}_{M_R}+\frac{2\rho}{b+1}\mathbf{H}\mathbf{H}^H\right) \approx \frac{2\rho}{(b+1)\ln(2)}\mathrm{Tr}\left(\mathbf{H}\mathbf{H}^H\right)$$

which results in the ergodic capacity region given by the closure of the set of rates satisfying

$$\begin{aligned}R_0+R_1 &\le \frac{\bar{P}}{N\ln(2)}\left(\mathrm{Tr}\left(\mathbf{R}_0\right)+\mathrm{Tr}\left(\mathbf{R}_1\right)\right)\\ R_i &\le \frac{\bar{P}}{N\ln(2)}\ \mathrm{Tr}\left(\mathbf{R}_i\right), \quad i=0,1.\end{aligned} \tag{6}$$

This shows that in the low-SNR regime, the amount of collision between the users has no impact on the ergodic capacity region. Moreover, since (6) depends on $\mathrm{Tr}(\mathbf{R}_0)$ and $\mathrm{Tr}(\mathbf{R}_1)$ only, the amount of spatial fading correlation quantified by $r(\mathbf{R}_0)$ and $r(\mathbf{R}_1)$ will not have an impact on the ergodic capacity region. Finally, we can conclude from (6) that the capacity region is rectangular.

B. High-SNR Regime

We shall next study the impact of the amount of collision (quantified through the parameter b) and the receive correlation matrices $\mathbf{R}_i$ on the ergodic capacity region in the high-SNR regime ($\rho \gg 1$). Invoking the high-SNR approximation

$$\log_2 \det(\mathbf{I}_{M_R} + \rho \mathbf{H}\mathbf{H}^H) \approx r(\mathbf{H}) \log_2(\rho) + \sum_{l=1}^{r(\mathbf{H})} \log_2(\lambda_l(\mathbf{H}\mathbf{H}^H)),$$

and noting that $r(\mathbf{H}_i) = Q_i = \min(r(\mathbf{R}_i), M_T)$ for $i = 0, 1$ with probability 1 (w.p.1) [20], we obtain the high-SNR capacity region as the closure of the set of rates satisfying

$$\begin{aligned} R_0 + R_1 \leq\ & b \left(Q \log_2\left(\frac{2\rho}{b+1}\right) + \mathcal{E}\left\{ \sum_{l=1}^{Q} \log_2(\lambda_l) \right\} \right) \\ & + \frac{1-b}{2} \left((Q_0 + Q_1) \log_2\left(\frac{2\rho}{b+1}\right) \right. \\ & \left. + \mathcal{E}\left\{ \sum_{i=0}^{1} \sum_{l=1}^{Q_i} \log_2(\lambda_{i,l}) \right\} \right) = f(b) \\ R_i \leq\ & \frac{1+b}{2} \left(Q_i \log_2\left(\frac{2\rho}{b+1}\right) + \mathcal{E}\left\{ \sum_{l=1}^{Q_i} \log_2(\lambda_{i,l}) \right\} \right) \\ =\ & f_i(b), \quad i = 0, 1, \end{aligned}$$

where λ_l and $\lambda_{i,l}$ denote the eigenvalues of $\mathbf{H}_0\mathbf{H}_0^H + \mathbf{H}_1\mathbf{H}_1^H$ and[2] $\mathbf{H}_i\mathbf{H}_i^H$ $(i = 0, 1)$, respectively, and $r([\mathbf{H}_0\ \mathbf{H}_1]) = Q$ w.p.1. A general expression for Q cannot be given since we essentially have to specify the rank of the sum of two matrices. It can, however, be shown that [18]

$$\min(\max(r(\mathbf{R}_0), r(\mathbf{R}_1)), M_T) \leq Q \leq \min(r(\mathbf{R}_0 + \mathbf{R}_1), 2M_T).$$

If $M_R \leq M_T$, we can sharpen this result to obtain $Q = r(\mathbf{R}_0 + \mathbf{R}_1)$ [18].

As already stated in Theorem 1, for joint decoding the capacity region for any amount of collision b is always outer-bounded by the capacity region for full collision, i.e., $b = 1$. In order to develop quantitative insights into the impact of the parameter b on the capacity region, we shall next compare the limiting (in terms of SNR) behavior of $f(b)$, $f_0(b)$ and $f_1(b)$ specified above.

For the individual rates we have

$$\lim_{\rho\to\infty} \frac{f_i(b)}{f_i(1)} = \frac{1+b}{2}, \quad i = 0, 1, \tag{7}$$

which shows that in the case of orthogonal accessing where $b = 0$ (i.e. no collision), the individual rates are reduced by a factor of 2 compared to the case of full collision. This result is intuitively clear since for $\rho \to \infty$, we operate in the degree-of-freedom limited regime. We emphasize that (7) holds irrespectively of the receive correlation characteristics. The limiting behavior of the sum rate is somewhat different.

[2] The existence proof of $r([\mathbf{H}_0\ \mathbf{H}_1]) = Q$ w.p.1 is provided in [18].

It is easy to see that

$$\lim_{\rho\to\infty} \frac{f(b)}{f(1)} = b + (1-b)\frac{Q_0 + Q_1}{2Q}. \tag{8}$$

Since $Q_i \leq Q$ for $i = 0, 1$, the limit in (8) is equal to 1 if and only if $Q_0 = Q_1 = Q$. It can be shown that this happens if and only if the signal subspaces spanned by the two correlation matrices $\mathbf{R}_0$ and $\mathbf{R}_1$ are equal, and $r(\mathbf{R}_0) = r(\mathbf{R}_1) \leq M_T$, corresponding to the case of no spatial separation between the users [18]. Since $Q_0 + Q_1 \geq Q$, we have $\frac{Q_0+Q_1}{2Q} \geq \frac{1}{2}$ with the minimum being attained at least in the following two cases [18]

i) $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$.
ii) $r(\mathbf{R}_i) \geq 2M_T$ for $i = 0, 1$.

The first case corresponds to full spatial separation (induced by the channel) since the users' spatial signatures span orthogonal subspaces. Moreover, we note that under i) the capacity region is rectangular. The second condition essentially guarantees that the receiver has enough spatial degrees of freedom to perfectly separate the two users. We can conclude that the limiting behavior of the sum capacity in the two-user case depends critically on the spatial separation of the users. If the spatial separation is poor (i.e., the subspaces spanned by $\mathbf{R}_0$ and $\mathbf{R}_1$ tend to be aligned), we obtain a significant fraction of the maximum sum capacity with any amount of collision. On the other hand, if the spatial separation of the users is good (i.e., the subspaces spanned by $\mathbf{R}_0$ and $\mathbf{R}_1$ tend to be orthogonal), the fraction of the sum capacity obtained depends critically on the amount of collision.

Let us next consider an example of two extreme cases where the impact of b can be quantified analytically using the results in [21]. Assume $M_R \geq 2$ and $r(\mathbf{R}_0) = r(\mathbf{R}_1) = 1$ (i.e., fully correlated fading at the receive array) with $\mathrm{Tr}(\mathbf{R}_0) = \mathrm{Tr}(\mathbf{R}_1) = 1$. For $\mathbf{R}_0 = \mathbf{R}_1$ (i.e., no spatial separation), we have [18]

$$\begin{aligned} f(b) =\ & \log_2\left(\frac{2\rho}{1+b}\right) + \frac{1}{\ln 2}\left(\sum_{p=1}^{M_T-1} \frac{1}{p} - \gamma \right) \\ & + \frac{b}{\ln 2} \sum_{p=M_T}^{2M_T-1} \frac{1}{p}, \end{aligned} \tag{9}$$

where $\gamma \approx 0.5722$ denotes Euler's constant. Since $\sum_{p=M_T}^{2M_T-1} \frac{1}{p}$ is strictly decreasing as a function of M_T, we can conclude that the impact of b on sum capacity reduces for increasing M_T. In fact, $\lim_{M_T\to\infty} [f(1) - f(0)] = 0$ which implies that asymptotically (in the number of transmit antennas) in the high-SNR regime a fully collision-based scheme achieves the same sum capacity as orthogonal accessing. In the case of full spatial separation, i.e., when $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$, we have [18]

$$f(b) = (1+b)\log_2\left(\frac{2\rho}{1+b}\right) + \frac{1+b}{\ln 2}\left(\sum_{p=1}^{M_T-1} \frac{1}{p} - \gamma \right). \tag{10}$$

Since $\sum_{p=1}^{M_T-1} \frac{1}{p}$ is strictly increasing in M_T it follows that increasing the collision parameter b (and hence the amount of

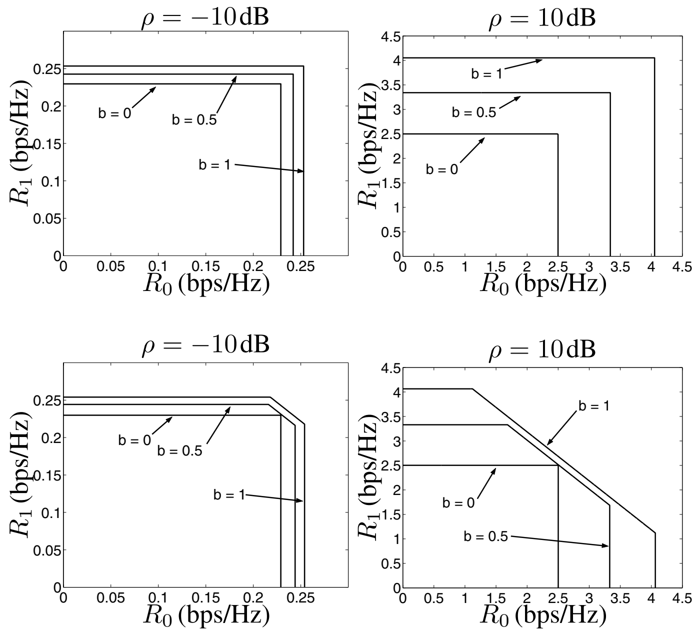

Fig. 1. Ergodic capacity regions under joint decoding for $M_T = M_R = 2$. First row: $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$ (perfect spatial separation). Second row: $\mathbf{R}_0 = \mathbf{R}_1$ (no spatial separation). First column: $\rho = -10$ dB. Second column: $\rho = 10$ dB.

collision) yields higher sum capacity for increasing M_T.

In summary, we can conclude that in the high-SNR regime, collision is required either in the spatial dimension or in frequency in order to realize a high sum capacity.

C. Simulation Results

We shall next provide numerical results describing the impact of collision on the two-user capacity region. We simulated a system with $M_T = M_R = 2$ and receive correlation matrices $\mathbf{R}_0$ and $\mathbf{R}_1$ satisfying $r(\mathbf{R}_0) = r(\mathbf{R}_1) = 1$ and normalized such that $\mathrm{Tr}(\mathbf{R}_0) = \mathrm{Tr}(\mathbf{R}_1) = 1$. Again, two different scenarios were considered, namely $\mathbf{R}_0 = \mathbf{R}_1$ (no spatial separation) and $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$ (perfect spatial separation).

Fig. 1 shows the estimated capacity regions (obtained through $10{,}000$ Monte-Carlo runs) for joint decoding and two different SNR values. In accordance with (10) the simulation results demonstrate that for the case where the users are spatially well separated, the difference between the capacity regions for full collision (CDMA) and for orthogonal multiple accessing (FDMA) becomes more pronounced for increasing SNR. Moreover, we can see that in this case the capacity region is rectangular (as noted in Sec. IV-B). In the case of poor spatial separation between the users, the orthogonal accessing scheme achieves a significant fraction of the maximum available sum capacity (as suggested by (8) and the ensuing discussion).

V. Two-user Case Under Independent Decoding

In this section, using the same setting as in the case of joint decoding, we study the impact of the amount of collision on the capacity region under single-user decoding. The capacity region is given by the closure of the set of rates satisfying

$$\begin{aligned}
R_0 \le b\, \Bigg(&\mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\left(\mathbf{H}_0\mathbf{H}_0^H + \mathbf{H}_1\mathbf{H}_1^H\right)\right)\right\} \\
&- \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_1\mathbf{H}_1^H\right)\right\}\Bigg) \\
&+ \frac{1-b}{2}\left(\mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_0\mathbf{H}_0^H\right)\right\}\right) \\
R_1 \le b\, \Bigg(&\mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\left(\mathbf{H}_0\mathbf{H}_0^H + \mathbf{H}_1\mathbf{H}_1^H\right)\right)\right\} \\
&- \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_0\mathbf{H}_0^H\right)\right\}\Bigg) \\
&+ \frac{1-b}{2}\left(\mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_1\mathbf{H}_1^H\right)\right\}\right).
\end{aligned}$$

In the low-SNR regime (i.e., $\rho \ll 1$) it is easily seen [18] that the collision parameter b does not have any impact on the capacity region. For high SNR, using the same approximations as in the previous section, the capacity region is obtained as

$$R_0 \le \log_2(\rho)\left(b(Q - Q_1) + \frac{1-b}{2}Q_0\right) + c_0(b) = g_0(b)$$

$$R_1 \le \log_2(\rho)\left(b(Q - Q_0) + \frac{1-b}{2}Q_1\right) + c_1(b) = g_1(b),$$

where

$$\begin{aligned}
c_0(b) = &\log_2\left(\frac{2}{b+1}\right)\left(b(Q - Q_1) + \frac{1-b}{2}Q_0\right) \\
&+ \mathcal{E}\left\{b\left(\sum_{l=1}^{Q}\log_2(\lambda_l) - \sum_{l=1}^{Q_1}\log_2(\lambda_{1,l})\right)\right\} \\
&+ \frac{1-b}{2}\,\mathcal{E}\left\{\sum_{l=1}^{Q_0}\log_2(\lambda_{0,l})\right\} \\
c_1(b) = &\log_2\left(\frac{2}{b+1}\right)\left(b(Q - Q_0) + \frac{1-b}{2}Q_1\right) \\
&+ \mathcal{E}\left\{b\left(\sum_{l=1}^{Q}\log_2(\lambda_l) - \sum_{l=1}^{Q_0}\log_2(\lambda_{0,l})\right)\right\} \\
&+ \frac{1-b}{2}\mathcal{E}\left\{\sum_{l=1}^{Q_1}\log_2(\lambda_{1,l})\right\},
\end{aligned}$$

and λ_l, $\lambda_{i,l}$ $(i = 0, 1)$, Q, Q_0, and Q_1 are defined in Sec. IV-B. The derivatives of $g_0(b)$ and $g_1(b)$ with respect to b are given by[3]

$$g_0'(b) = \log_2(\rho)(Q - Q_1 - Q_0/2) + c_0'(b)$$
$$g_1'(b) = \log_2(\rho)(Q - Q_0 - Q_1/2) + c_1'(b)$$

which shows that in the high-SNR case (note that the $c_i(b)$ for $i = 0, 1$ do not depend on ρ) the behavior of $g_0'(b)$ and $g_1'(b)$ depends on the coefficients $Q - Q_1 - Q_0/2$ and $Q -$

[3] Note that strictly speaking $g_0(b)$, $g_1(b)$, $c_0(b)$ and $c_1(b)$ are defined for $b = \frac{2i}{N}$ with $i \in \{0, 1, \ldots, N/2\}$ only. For the discussion in the remainder of this section, with slight abuse of notation, we shall assume that $g_0(b)$, $g_1(b)$, $c_0(b)$ and $c_1(b)$ are functions of a real parameter $b \in [0, 1]$.

$Q_0 - Q_1/2$, respectively. If $Q - Q_1 - Q_0/2 > 0$ and $Q - Q_0 - Q_1/2 > 0$, $g_0'(b)$ and $g_1'(b)$ will be positive for all b and hence the capacity region is maximized for $b = 1$. On the other hand, if $Q - Q_1 - Q_0/2 < 0$ and $Q - Q_0 - Q_1/2 < 0$, the capacity region is maximized for $b = 0$ (i.e., no collision). It is straightforward to see that $Q - Q_1 - Q_0/2 < 0$ and $Q - Q_0 - Q_1/2 < 0$ if $Q = Q_0 = Q_1$, i.e., when there is no spatial separation between the users. When the spatial separation of the users is perfect, i.e., $Q = Q_0 + Q_1$, we have $Q - Q_1 - Q_0/2 > 0$ and $Q - Q_0 - Q_1/2 > 0$. Hence, in summary we can conclude that for good spatial separation, it is optimum to have the users fully collide in frequency. When the spatial separation is poor, the capacity region is maximized for orthogonal accessing. The last conclusion is fundamentally different from the case of joint decoding where full collision is always optimal even though for poor spatial separation most of the total available sum capacity is achieved by orthogonal accessing.

VI. Conclusion

We proposed an entire family of multiple access schemes allowing a flexible amount of collision in frequency and including FDMA and CDMA as special cases. The performance of the proposed schemes was assessed through the corresponding ergodic capacity regions under joint and single user decoding.

For joint decoding our conclusions are as follows. Generalizing a well known result by Gallager [7], we showed that the ergodic capacity region for any amount of collision is always outer bounded by the capacity region for a fully collision-based multiple access scheme (CDMA). Moreover, it was found that in the low-SNR regime the amount of collision has a negligible impact on the capacity region. For high SNR the spatial separation between the users governs the shape of the capacity region. When the users are spatially well separated, full collision (CDMA) should be used. When spatial separation is poor, a significant fraction of sum capacity is obtained with orthogonal accessing (FDMA). Hence, when the sum capacity is the limiting factor (e.g., when the users have equal rates), collision is not necessarily needed in the latter case. Minimizing the amount of user collision is desirable in practice, as it minimizes the receiver complexity incurred by having to separate the interfering signals.

Finally, in the case of single user decoding we showed that for low SNR the amount of collision does not have any impact on the capacity region. In the high-SNR regime for good spatial separation it is optimum to use full collision (CDMA), whereas for poor spatial separation it is optimum to use orthogonal accessing (FDMA). The latter conclusion is in strong contrast to the case of joint decoding where CDMA always outperforms FDMA, albeit the performance difference (in terms of sum rate) is small in the case of poor spatial separation.

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