DESIGN OF PULSE SHAPING OFDM/OQAM SYSTEMS FOR HIGH DATA-RATE TRANSMISSION OVER WIRELESS CHANNELS

Helmut Bölcskei[a], *Pierre Duhamel*[b], *and Rima Hleiss*[b]

[a]Department of Communications, Vienna University of Technology,
Gusshausstrasse 25/389, A-1040 Vienna, Austria
Phone: +43-1-58801-38927, Fax: +43-1-5870583, email: hboelcsk@aurora.nt.tuwien.ac.at

[b]Ecole Nationale Supérieure des Télécommunications, Département Signal,
46 rue Barrault, 75634 Paris Cedex 13, France

Abstract—Orthogonal frequency division multiplexing (OFDM) is a promising technique for high data-rate transmission over wireless channels. In general, wireless channels are time-frequency dispersive. The performance of wireless OFDM therefore depends critically on the time-frequency localization of the pulse shaping filter used. It has been pointed out in [1] that OFDM systems based on offset QAM (OFDM/OQAM) bypass a major disadvantage of OFDM schemes based on ordinary QAM, namely the fact that well-localized pulse shaping filters are prohibited in the case of a critical time-frequency grid where spectral efficiency is maximal. In this paper, we derive general orthogonality conditions for OFDM/OQAM systems and we propose efficient (FFT-based) design procedures for time-frequency well-localized OFDM/OQAM pulse shaping filters with arbitrary length and arbitrary overlapping factors. Finally, we present design examples.

1 INTRODUCTION AND OUTLINE

Orthogonal frequency division multiplexing (OFDM) [2]-[10],[1] has meanwhile become part of several telecommunications standards, such as the European standard for terrestrial Digital Audio Broadcasting (DAB), and asymmetric digital subscriber line (ADSL) for high-bit-rate digital subscriber services on twisted-pair channels. It is furthermore under investigation for digital terrestrial TV broadcasting [11]. Important features of OFDM systems include high spectral efficiency and immunity to multipath fading and impulse noise.

Recently, there has been increased interest in wireless OFDM. In general, wireless channels (such as the mobile radio channel for example) may be modeled as a linear time-varying filter [12, 13]. Consequently, transmission over wireless channels will be subject to time-frequency dispersion. In OFDM systems time-frequency dispersion causes a loss of orthogonality between the transmitted pulses [1]. Since the error resulting from channel dispersion depends critically on the time-frequency localization of the transmitter basis functions, optimum design of OFDM pulse shaping filters is an important topic [2, 9, 1]. OFDM systems employing QAM (OFDM/QAM) prohibit well-localized basis functions in the case of critical time-frequency density (i.e. maximum spectral efficiency [14, 9, 1]). In signal processing this phenomenon is known as Balian-Low theorem [15]. Two approaches to circumvent this problem have been suggested in the past [1, 14, 16, 17]. However, both methods result in a loss of spectral efficiency, which is undesirable in high data-rate applications.

In [1] it has been pointed out that OFDM systems based on offset QAM (OFDM/OQAM) [3, 2, 18, 19] allow time-frequency well-localized basis functions even in the case of critical time-frequency density (i.e. maximum spectral efficiency). For high data-rate applications OFDM/OQAM therefore seems to be an attractive alternative to OFDM/QAM.

Our research was motivated by the need for efficient systematic design procedures for time-frequency well-localized discrete-time OFDM/OQAM pulse shaping filters. In this paper, we partly build on the work reported in [20]. Our main contributions are summarized below:

- We derive general orthogonality conditions for OFDM/OQAM pulse shaping filters including discrete-time versions of the conditions found previously in [20, 2, 3, 9] as special cases.
- We provide computationally efficient (FFT-based) methods for the design of time-frequency well-localized discrete-time OFDM/OQAM pulse shaping filters with arbitrary length (up to several thousand taps) and arbitrary overlapping factors.

This paper is organized as follows. Section 2 discusses the tradeoff between dispersion robustness and spectral efficiency arising in OFDM/QAM systems and motivates the use of OFDM/OQAM. Section 3 provides general orthogonality conditions for OFDM/OQAM pulse shaping filters. In Section 4 we introduce an FFT-based orthogonalization approach for the design of time-frequency well-localized pulse shaping filters. Finally, Section 5 provides design examples, and Section 6 concludes the paper.

2 SPECTRAL EFFICIENCY AND DISPERSION ROBUSTNESS IN OFDM SYSTEMS

The baseband equivalent of an OFDM/QAM system is given by the sum of M parallel pulse shaped channels with the subchannel filters obtained by complex modulations of a pulse shaping filter $g(t)$. The transmitted signal can therefore be written as

$$x(t) = \sum_{k=0}^{M-1} \sum_{l=-\infty}^{\infty} c_{k,l}\, g(t-lT)\, e^{j2\pi kF(t-lT)},$$

where T is the symbol duration, F denotes the subcarrier spacing, M is the number of subcarriers, and $c_{k,l}$ ($k = 0, 1, ..., M-1$, $l \in \mathbb{Z}$) denotes the complex-valued data symbols.

0-7803-5284-X/99/$10.00 © 1999 IEEE.

Spectral efficiency. It has been shown in [1, 14] that the spectral efficiency of an OFDM system can be approximated by

$$\eta = \frac{\beta}{TF} \quad \left[\frac{\text{bit/s}}{Hz}\right], \tag{1}$$

where β is the number of bits per symbol. The spectral efficiency η is therefore maximized if the time-frequency density is critical[1], i.e., $TF = 1$.

Time-frequency localization and dispersion robustness. The OFDM transmit signal $x(t)$ is given by a linear combination of the basis functions $g_{k,l}(t) = g(t-lT)e^{j2\pi kF(t-lT)}$ ($k = 0,1,...,M-1$, $l \in \mathbb{Z}$). If the $g_{k,l}(t)$ constitute an orthogonal basis, demodulation is accomplished by projecting the received signal onto the basis functions $g_{k,l}(t)$, i.e., $\hat{c}_{k,l} = \langle x, g_{k,l}\rangle$, where $\hat{c}_{k,l}$ denotes the reconstructed data symbols. In the case of a perfect channel orthogonality between the basis functions guarantees intersymbol interference (ISI)-free and intercarrier interference (ICI)-free transmission, i.e., $\hat{c}_{k,l} = c_{k,l}$. However, if the channel is time-frequency dispersive such as the mobile radio channel for example [12, 13], good time-frequency localization of the basis functions is necessary in order to avoid the symbol energy "spreading out" and perturbing neighboring symbols in the time-frequency plane. In general, there will be both ISI and ICI due to the lack of orthogonality between the perturbed transmitter basis functions and the receiver basis functions [1]. The amount of the resulting interference depends critically on the time-frequency localization of the transmitter basis functions. OFDM systems employing a cyclic prefix (CP) [5] use a rectangular $g(t)$ which has poor frequency localization and therefore causes severe ICI in the case of frequency-dispersive channels. Furthermore, due to the CP we have $TF > 1$ with the exact value of TF depending on the length of the CP. Therefore, according to (1) the use of a CP leads to reduced spectral efficiency.

Spectral efficiency versus time-frequency localization. It is well known from the theory of Weyl-Heisenberg frames [15, 21] that well-localized (orthogonal or biorthogonal) basis functions $g_{k,l}(t)$ are possible only if $TF > 1$. More specifically, it can be shown that a function $g(t)$ generating an orthogonal or biorthogonal basis $\{g_{k,l}(t)\}$ for $TF = 1$ can not have finite time- *and* frequency dispersion (i.e. finite second moments) [15, 21]. A recent proposal of a time-frequency well-localized OFDM/QAM pulse shaping filter [1, 14] uses a time-frequency density of $TF = 2$.

In summary, a major drawback of OFDM/QAM is the fact that well-localized pulse shaping filters are prohibited in the case of critical time-frequency density $TF = 1$, i.e., maximum spectral efficiency $\eta_{max} = \beta$. Therefore, in practice, when using OFDM/QAM systems a compromise between dispersion robustness and spectral efficiency has to be sought.

Offset QAM modulation in OFDM systems. In [1] it has been pointed out that OFDM/OQAM systems allow well-localized basis functions (with finite time- and frequency-dispersion) even for critical time-frequency density $TF = 1$ (i.e., maximum spectral efficiency.) In this paper, we provide an explanation for this "little magic". For high data-rate applications OFDM/OQAM seems to be an attractive alternative to OFDM/QAM.

[1]For $TF < 1$ perfect reconstruction (even in the absence of a channel) is not possible. This case will therefore be excluded in the following.

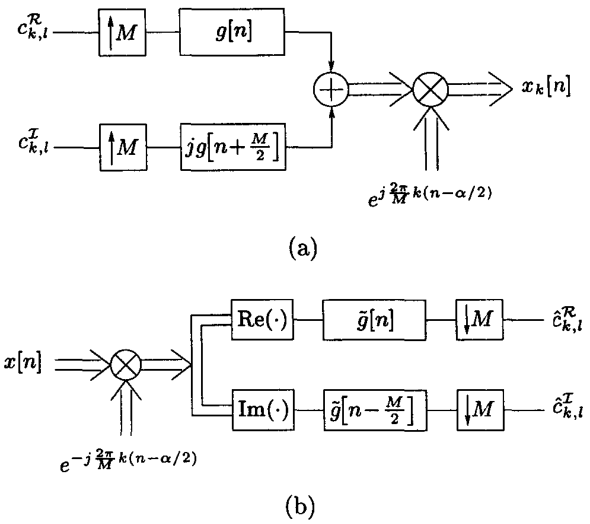

Fig. 1. Discrete-time multirate model of OFDM/OQAM system ($\tilde{g}[n] = g[-n]$): (a) k-th transmitter subchannel, (b) k-th receiver subchannel.

3 ORTHOGONALITY CONDITIONS

The discrete-time OFDM/OQAM transmit signal is given by $x[n] = \sum_{k=0}^{M-1} x_k[n]$ with

$$\begin{aligned} x_k[n] &= \sum_{l=-\infty}^{\infty} c^{\mathcal{R}}_{k,l} g[n-lM]\, e^{j\frac{2\pi}{M}k(n-\alpha/2)} \\ &+ \sum_{l=-\infty}^{\infty} jc^{\mathcal{I}}_{k,l} g[n+M/2-lM]\, e^{j\frac{2\pi}{M}k(n-\alpha/2)}, \end{aligned}$$

where $c^{\mathcal{R}}_{k,l} = \mathrm{Re}\{c_{k,l}\}$ and $c^{\mathcal{I}}_{k,l} = \mathrm{Im}\{c_{k,l}\}$ denote the real and imaginary parts of the data symbols $c_{k,l}$, respectively, $g[n]$ is the real-valued transmitter pulse shaping filter, and $\alpha \in [0, M-1]$. Throughout this paper we assume that the number of subcarriers M is even. Fig. 1 shows the baseband model of a discrete-time OFDM/OQAM system.

We shall next derive orthogonality conditions for discrete-time OFDM/OQAM pulse shaping filters. We say that $g[n]$ in Fig. 1 is orthogonal if in the absence of a channel $\hat{c}_{k,l} = c_{k,l}$, i.e., if there is neither ISI nor ICI. Considering the equivalent path from the $(k+m)$-th transmitter subchannel to the k-th receiver subchannel, it follows that $g[n]$ is orthogonal if the following conditions are satisfied[2] for $m \in [0, M-1]$, $l \in \mathbb{Z}$:

$$[\mathrm{Re}\{g[n-lM]e^{j\frac{2\pi}{M}m(n-\alpha/2)}\} * \tilde{g}[n]]_{n=0} = \delta[l]\delta[m] \tag{2}$$

$$[\mathrm{Re}\{jg[n+M/2-lM]e^{j\frac{2\pi}{M}m(n-\alpha/2)}\} * \tilde{g}[n]]_{n=0} = 0 \tag{3}$$

$$[\mathrm{Im}\{g[n-lM]e^{j\frac{2\pi}{M}m(n-\alpha/2)}\} * \tilde{g}[n-M/2]]_{n=0} = 0 \tag{4}$$

$$\begin{aligned} &[\mathrm{Im}\{jg[n+M/2-lM]e^{j\frac{2\pi}{M}m(n-\alpha/2)}\} \\ &* \tilde{g}[n-M/2]]_{n=0} = \delta[l]\delta[m], \end{aligned} \tag{5}$$

where $\tilde{g}[n] = g[-n]$. For $m \neq 0$ Eqs. (2)-(5) guarantee that ICI is perfectly cancelled, whereas for $m = 0$ (2)-(5) reduces to the conditions for ISI cancellation. More

[2]$*$ stands for convolution.

specifically, for $m = 0$, Eqs. (3) and (4) guarantee that there is no interference between the real and imaginary parts of $c_{k,l}$, whereas (2) and (5) guarantee zero ISI for $c^{\mathcal{R}}_{k,l}$ and $c^{\mathcal{I}}_{k,l}$, respectively. Rewriting (2)-(5) as

$$\sum_{n=-\infty}^{\infty} g[n-lM]g[n]\cos\left(\frac{2\pi}{M}m(n-\alpha/2)\right)=\delta[l]\delta[m] \quad (6)$$

$$\sum_{n=-\infty}^{\infty} g[n+M/2-lM]g[n]\sin\left(\frac{2\pi}{M}m(n-\alpha/2)\right)=0 \quad (7)$$

$$\sum_{n=-\infty}^{\infty} g[n-lM]g[n+M/2]\sin\left(\frac{2\pi}{M}m(n-\alpha/2)\right)=0 \quad (8)$$

$$\sum_{n=-\infty}^{\infty} g[n+M/2-lM]g[n+M/2]$$
$$\cos\left(\frac{2\pi}{M}m(n-\alpha/2)\right)=\delta[l]\delta[m], \quad (9)$$

it is shown in [22] that (7) and (8) can be satisfied by choosing $g[n]$ to be an even function, i.e.,

$$g[n] = g[\alpha + (2r+1)M/2 - n] \quad (10)$$

with $r \in \mathbb{Z}$ and $\alpha \in [0, M-1]$. The parameters r and α allow a flexible choice of the center of symmetry of the pulse shaping filter $g[n]$. We note that α has to be odd (even) for even (odd) filter length L_g. Furthermore, from (10) it follows[3] that for a $g[n]$ supported in $[0, L_g - 1]$ the parameter α has to be chosen as $\alpha = \left(L_g + \frac{M}{2} - 1\right) \bmod M$. Substituting $n \to n + M/2$ in (9) it is readily seen that (9) is equivalent to (6). We are thus left with condition (6), which is satisfied for m odd [22] if $g[n]$ is chosen according to (10). Note that so far we only made use of the symmetry of $g[n]$. The real design task lies in satisfying (6) for m even. Setting $m \to 2m$ in (6) we obtain

$$\frac{1}{2}\sum_{n=-\infty}^{\infty} g[n]g[n-lM]e^{j\frac{2\pi}{M/2}m(n-\alpha/2)}$$
$$+\frac{1}{2}\sum_{n=-\infty}^{\infty} g[n]g[n-lM]e^{-j\frac{2\pi}{M/2}m(n-\alpha/2)} = \delta[l]\delta[m],$$

which upon defining

$$g_{l,m}[n] = g[n-lM]e^{j\frac{2\pi}{M/2}m(n-\alpha/2)}$$

can be rewritten as

$$\frac{1}{2}\langle g_{0,0}, g_{l,-m}\rangle + \frac{1}{2}\langle g_{0,0}, g_{l,m}\rangle = \delta[l]\delta[m]. \quad (11)$$

It therefore follows that a $g[n]$ satisfying the symmetry property (10) is orthogonal if the time-frequency shifted versions $g_{l,m}[n]$ of $g[n]$ constitute an orthonormal basis, i.e., $\langle g_{l,m}, g_{l',m'}\rangle = \delta[l-l']\delta[m-m']$. We have thus reduced the design of an OFDM/OQAM pulse shaping filter to the design of an orthogonal basis consisting of time-frequency shifted versions of $g[n]$. Since orthonormality of the function set $\left\{g[n-lM]e^{j\frac{2\pi}{M/2}m(n-\alpha/2)}\right\}$ is equivalent to orthonormality of $\left\{g[n-lM]e^{j\frac{2\pi}{M/2}mn}\right\}$, we set $g_{l,m}[n] = g[n-lM]e^{j\frac{2\pi}{M/2}mn}$ in the following.

In [22] it is shown that $\{g_{l,m}[n]\}$ is orthogonal if and only if the following equivalent condition is satisfied:

$$\sum_{r=-\infty}^{\infty} g\left[n - r\frac{M}{2}\right] g\left[n - r\frac{M}{2} - lM\right] = \frac{2}{M}\delta[l]. \quad (12)$$

The design of an OFDM/OQAM pulse shaping filter therefore reduces to the design of a symmetric function $g[n]$ satisfying (12). Such functions $g[n]$ can be obtained by performing a constrained optimization with the side constraints given by (12). However, for large filter lengths this approach will be computationally expensive and might furthermore have convergence problems. In the following section, we shall therefore introduce a computationally efficient method based on an orthogonalization procedure. For this purpose we need the discrete-time Zak transform (DZT) [23] of the pulse shaping filter $g[n]$ defined as[4]

$$\mathcal{Z}_g(n,\theta) = \sum_{r=-\infty}^{\infty} g\left[n + r\frac{M}{2}\right] e^{-j2\pi r\theta} \quad (13)$$

with its inverse given by $g[n] = \int_0^1 \mathcal{Z}_g(n,\theta)\, d\theta$. In [22] it is shown that (12) is equivalent to

$$|\mathcal{Z}_g(n,\theta)|^2 + |\mathcal{Z}_g(n,\theta - 1/2)|^2 = \frac{4}{M} \quad (14)$$

for $n = 0, 1, ..., M/2 - 1$. Using an alternative definition of the DZT, it is shown in [22] that the perfect reconstruction condition in an OFDM/OQAM system is equivalent to the perfect reconstruction condition in a critically sampled cosine-modulated filter bank. In the frequency domain, the orthogonality condition (12) can be formulated as

$$\frac{1}{M}\sum_{k=0}^{M-1} G\left(e^{j2\pi(\theta - \frac{k}{M} - \frac{l}{M/2})}\right) G^*\left(e^{j2\pi(\theta - \frac{k}{M})}\right) = \delta[l]. \quad (15)$$

We conclude this section with some important special cases.

Time-limited pulse shaping filter. If the pulse shaping filter $g[n]$ is compactly supported in an interval of length[5] M the orthogonality condition (12) simplifies to

$$g^2[n] + g^2\left[n + \frac{M}{2}\right] = \frac{2}{M} \qquad \text{for} \quad n = 0, 1, ..., M/2 - 1. \quad (16)$$

The design of orthogonal pulse shaping filters supported in an interval of length M is therefore equivalent to the design of a function $g[n]$ whose square $g^2[n]$ satisfies a Nyquist criterion in the time-domain. The rectangular function

$$g[n] = \begin{cases} \frac{1}{\sqrt{M}}, & n = 0, 1, ..., M-1 \\ 0, & \text{else} \end{cases}$$

[3] mod stands for the modulo operation.

[4] The DZT is the polyphase decomposition [24] evaluated on the unit circle. Throughout this paper, we shall use the term DZT since in our context it is rather a signal transformation.

[5] For the sake of simplicity we assume that $g[n]$ is supported in $n \in [0, M-1]$.

trivially satisfies (16). We note that the time-limited continuous-time OFDM/OQAM pulse shaping filter proposed in [9] satisfies a continuous-time version of (16).

Band-limited pulse shaping filter. If $g[n]$ is band-limited to an interval of length[6] $2/M$ the orthogonality condition (15) simplifies to

$$\frac{1}{M}\left[\left|G(e^{j2\pi\theta})\right|^2+\left|G\left(e^{j2\pi(\theta-\frac{1}{M})}\right)\right|^2\right]=1 \qquad (17)$$

for $\theta \in [0,1/M)$. The design of orthogonal filters with bandwidth $\leq 2/M$ is therefore equivalent to the design of a filter $g[n]$ whose squared transfer function satisfies a Nyquist property. We finally note that the square-root Nyquist filter proposed in [18, 19] and the filters proposed in [2] satisfy (17). A continuous-time version of (17) has been reported previously in [2, 9].

4 ORTHOGONALIZATION PROCEDURE

In this section we shall introduce a computationally efficient procedure for the design of OFDM/OQAM pulse shaping filters of arbitrary length and arbitrary overlapping factors. The proposed design method starts from an arbitrary (nonorthogonal) symmetric filter which is modified to obtain a symmetric orthogonal pulse shaping filter. In general, this orthogonalization procedure has to be formulated in the DZT domain. For properly time-limited or band-limited filters, a time-domain or frequency-domain formulation, respectively, can be given. We note that the OFDM/IOTA approach proposed in [9] and further elaborated in [25, 26] is based on ideas similar to those underlying our orthogonalization method. However, in the case of OFDM/IOTA the initial filter has to be a gaussian and an orthogonalization is performed in the time-domain and in the frequency-domain separately leading to what is called extended gaussian functions in [25, 26].

Orthogonalization using the DZT. Starting from an arbitrary (nonorthogonal) filter $g[n]$ satisfying (10), an orthogonal pulse shaping filter $g_o[n]$ is obtained according to [22]

$$\mathcal{Z}_{g_o}(n,\theta)=\frac{2\mathcal{Z}_g(n,\theta)}{\sqrt{M|\mathcal{Z}_g(n,\theta)|^2+M\left|\mathcal{Z}_g(n,\theta-\frac{1}{2})\right|^2}}. \qquad (18)$$

Inserting (18) in (14) and using $\mathcal{Z}_g(n,\theta-1)=\mathcal{Z}_g(n,\theta)$ it is easily seen that the DZT of $g_o[n]$ satisfies

$$|\mathcal{Z}_{g_o}(n,\theta)|^2+|\mathcal{Z}_{g_o}(n,\theta-1/2)|^2=\frac{4}{M},$$

which proves that $g_o[n]$ is orthogonal. It can furthermore be shown that $g_o[n]$ satisfies the same symmetry property as the initial filter $g[n]$ [22]. However, $g_o[n]$ need not have the same length as $g[n]$. As we shall see later, in general it will be longer than $g[n]$.

Implementation of the Algorithm. We shall next describe how the proposed orthogonalization procedure can be implemented in practice. As already mentioned, in general the resulting orthogonal filter $g_o[n]$ will be longer than the initial filter $g[n]$. Therefore, $g[n]$ has to be zero-padded before orthogonalization. The DZT has to be evaluated on a discrete time-frequency grid according to [23]

$$\mathcal{Z}_g^{(M/2,K)}[n,k]=\sum_{r=0}^{K-1} g\left[n+r\frac{M}{2}\right]e^{-j2\pi\frac{k}{K}r},$$
$$n=0,1,...,M/2-1,\; k=0,1,...,K-1$$

with the length of the zero-padded filter satisfying $L_g=\frac{M}{2}K$. Note that this implies that the filter length L_g has to be even. It is easily seen that the computation of $\mathcal{Z}_g^{(M/2,K)}[n,k]$ reduces to the column-wise FFT of the $K\times\frac{M}{2}$ matrix $[\mathbf{G}]_{n,r}=g\left[r+n\frac{M}{2}\right]$. The orthogonalization procedure can now be summarized as follows:

- Design an initial (lowpass) filter $g[n]$ satisfying the symmetry condition (10).
- Perform zero-padding of $g[n]$ to obtain a filter of length L_g.
- Compute the DZT of the orthogonal filter $g_o[n]$ according to[7] $\mathcal{Z}_{g_o}^{(M/2,K)}[n,k]=$

$$\frac{2\mathcal{Z}_g^{(M/2,K)}[n,k]}{\sqrt{M\left|\mathcal{Z}_g^{(M/2,K)}[n,k]\right|^2+M\left|\mathcal{Z}_g^{(M/2,K)}\left[n,k-\frac{K}{2}\right]\right|^2}}. \qquad (19)$$

- Compute the inverse DZT to obtain the orthogonal filter $g_o[n]$, i.e., compute the inverse FFT of the columns of the $K\times\frac{M}{2}$ matrix $\mathcal{Z}_{g_o}^{(M/2,K)}[n,k]$.

We note that this orthogonalization procedure does not automatically guarantee that $g_o[n]$ is well-localized in time and frequency. In practice, however, as we shall see in Sec. 5, starting from a time-frequency well-localized initial filter $g[n]$ with bandwidth approximately equal to $1/M$ (for example a lowpass filter or a gaussian function) in most cases results in a time-frequency well-localized $g_o[n]$. We emphasize that the above orthogonalization procedure is computationally extremely cheap, since it consists of FFTs (forward and inverse DZT) and divisions in the DZT domain (cf. (19)). We shall next discuss two important special cases of the proposed orthogonalization approach.

Time-limited pulse shaping filter. For $g[n]$ compactly supported in an interval of length M (cf. (16)), the orthogonalization procedure described above essentially reduces to divisions in the time-domain. Assuming that the initial filter $g[n]$ is supported in $n\in[0,M-1]$ and satisfies (10), an orthogonal pulse shaping filter is obtained as [22]

$$g_o[n]=\frac{\sqrt{2}g[n]}{\sqrt{Mg^2[n]+Mg^2[n-M/2]+Mg^2[n+M/2]}}. \qquad (20)$$

Consequently, $g_o[n]$ has the same support as $g[n]$.

Band-limited pulse shaping filter. For $G(e^{j2\pi\theta})$ band-limited to the interval $|\theta|\leq 1/M$, an orthogonal

[6] For the sake of simplicity we assume that $G(e^{j2\pi\theta})$ is supported within the interval $|\theta|\leq 1/M$.

[7] For the sake of simplicity we assume that K is even.

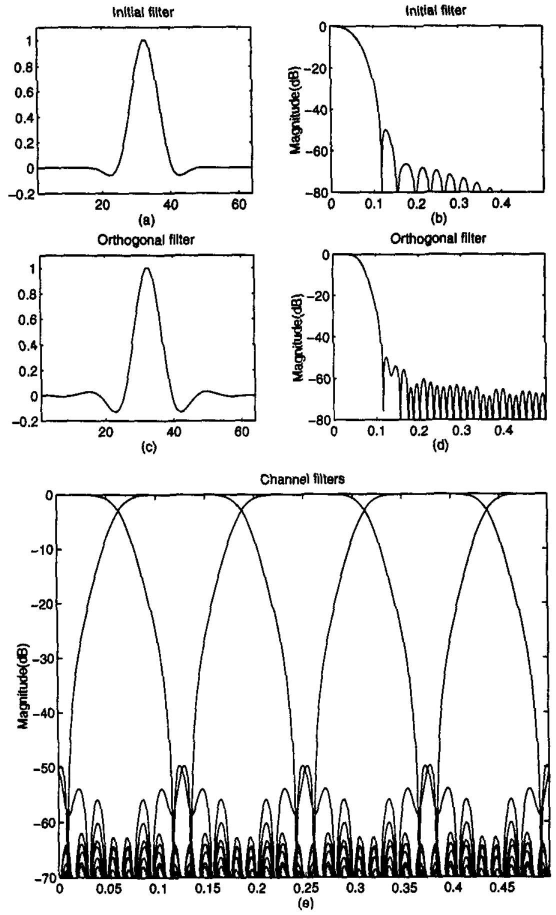

Fig. 2. 8-channel OFDM/OQAM pulse shaping filter of length 64*: (a)-(b) initial filter, (c)-(d) orthogonal filter, (e) corresponding subchannel filters.*

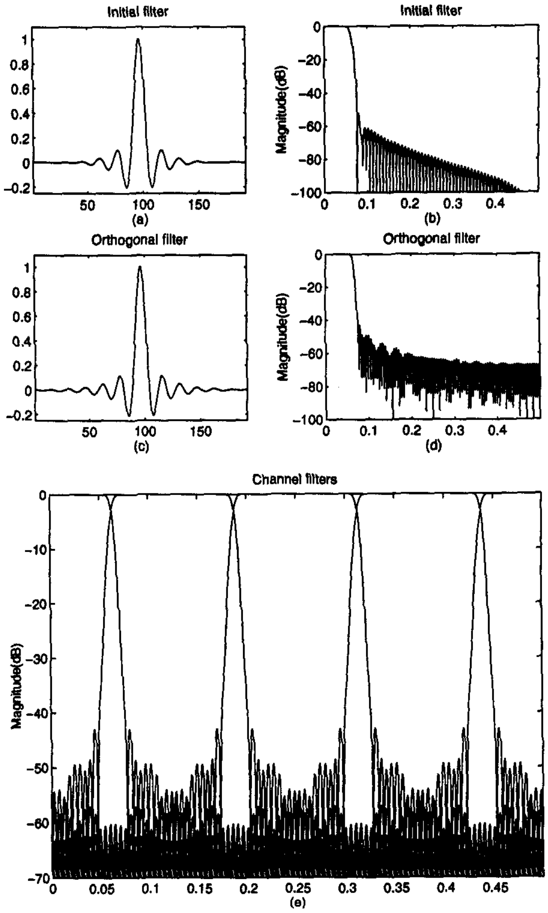

Fig. 3. 8-channel OFDM/OQAM pulse shaping filter of length 192*: (a)-(b) initial filter, (c)-(d) orthogonal filter, (e) corresponding subchannel filters.*

pulse shaping filter is obtained as $G_o(e^{j2\pi\theta}) =$

$$\frac{\sqrt{M}G(e^{j2\pi\theta})}{\sqrt{|G(e^{j2\pi\theta})|^2 + \left|G\left(e^{j2\pi(\theta-\frac{1}{M})}\right)\right|^2 + \left|G\left(e^{j2\pi(\theta+\frac{1}{M})}\right)\right|^2}}. \tag{21}$$

It follows from (21) that $G_o(e^{j2\pi\theta})$ has the same support as $G(e^{j2\pi\theta})$.

5 DESIGN EXAMPLES

In this section, we demonstrate how the orthogonalization procedure introduced in Sec. 4 can be used to design time-frequency well-localized pulse shaping filters for OFDM/OQAM systems. We shall see that the basic philosophy of the orthogonalization method is to start from a well-localized initial lowpass filter with bandwidth approximately equal to $1/M$. In most cases this guarantees that the resulting orthogonal filter is again well-localized.

Design example 1. In the first example we designed an 8-channel OFDM/OQAM system. The initial filter is a 32 tap lowpass filter with bandwidth $1/M = 0.125$ (designed using the MATLAB function FIR1) zero-padded to obtain a filter of overall length 64. Figs. 2(a) and (b) show the initial filter and its transfer function. Figs. 2(c) and (d) show the resulting orthogonal filter's impulse response and transfer function, respectively. The corresponding subchannel filters are depicted in Fig. 2(e). We can see that the orthogonal filter has good time-frequency localization. Since the length of $g_o[n]$ is $L_g = 64$, we have $\alpha = 3$ in (10). The pulse shaping filter length is 8 times the symbol length.

Design example 2. In the second design example we demonstrate how the frequency localization of the pulse shaping filter can be improved by increasing the filter length. We designed an 8-channel OFDM/OQAM system using a pulse shaping filter of length 192. The initial filter is shown in Figs. 3(a) and (b). (It was designed using the MATLAB function FIR1 with nominal bandwidth 0.125). Figs. 3(c) and (d) show the resulting orthogonal filter and its transfer function, respectively. The corresponding subchannel filters are depicted in Fig. 3(e). Comparing Figs. 3(e) and 2(e), we can see that the pulse shaping filter in this example has much better frequency selectivity than that in Example 1.

Design example 3. In the last example, we demonstrate that our approach allows the design of OFDM/OQAM systems with a large number of channels and long pulse shaping filters. For $M = 256$ channels we

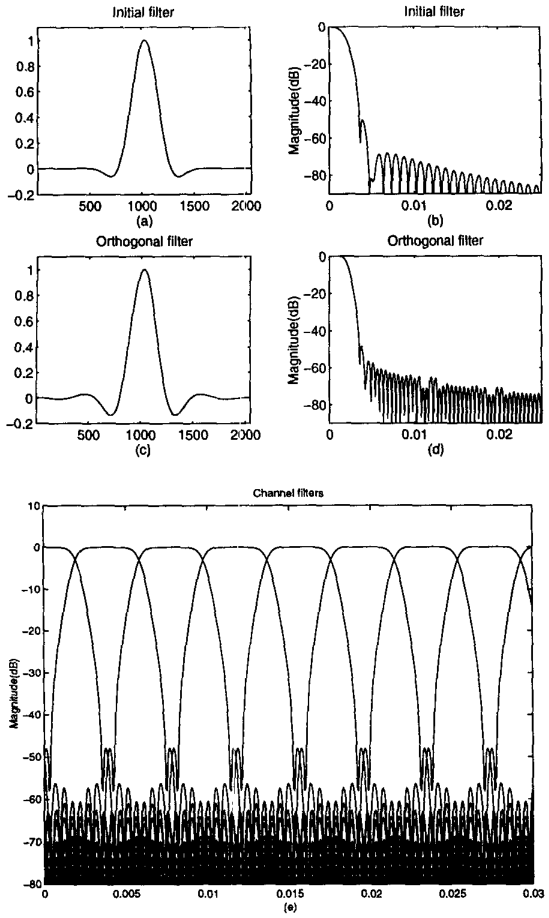

Fig. 4. 256-channel OFDM/OQAM pulse shaping filter of length 2048: (a)-(b) initial filter, (c)-(d) orthogonal filter, (e) corresponding subchannel filters.

designed a pulse shaping filter of length 2048 (i.e., the pulse shaping filter length is 8 times the symbol length). The initial filter shown in Figs. 4(a) and (b) is a zero-padded lowpass filter (again designed using the MATLAB function FIR1) with nominal bandwidth 1/256. Figs. 4(c) and (d) show the resulting orthogonal filter and its transfer function. Finally, Fig. 4(e) shows the corresponding subchannel filters.

6 CONCLUSION

We introduced a computationally efficient (FFT-based) method for the design of time-frequency well-localized OFDM/OQAM pulse shaping filters. The proposed approach is based on an orthogonalization of an arbitrary (nonorthogonal) initial filter. Our method is highly flexible since it allows the design of pulse shaping filters of arbitrary length and arbitrary overlapping factors. The dispersion robustness of the resulting pulse shaping OFDM/OQAM systems makes them attractive for high data-rate transmission over time-frequency dispersive channels.

References

[1] R. Haas, *Application des transmissions à porteuses multiples aux communications radio mobiles.* PhD thesis, Ecole Nationale Supérieure des Télécommunications Paris, Paris, France, Jan. 1996.

[2] R. W. Chang, "Synthesis of band-limited orthogonal signals for multi-channel data transmission," *Bell Syst. Tech. J.*, vol. 45, pp. 1775–1796, Dec. 1966.

[3] B. R. Saltzberg, "Performance of an efficient parallel data transmission system," *IEEE Trans. Comm. Technol.*, vol. 15, pp. 805–811, Dec. 1967.

[4] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency division multiplexing using the discrete Fourier transform," *IEEE Trans. Comm. Tech.*, vol. 19, pp. 628–634, Oct. 1971.

[5] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proc. IEEE ICASSP-80*, (Denver, CO), pp. 964–967, 1980.

[6] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Comm.*, vol. 33, pp. 665–675, July 1985.

[7] J. S. Chow, J. C. Tu, and J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE J. Sel. Areas Comm.*, vol. 9, pp. 895–908, Aug. 1991.

[8] W. Y. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadc.*, vol. 41, pp. 1–8, March 1995.

[9] B. LeFloch, M. Alard, and C. Berrou, "Coded orthogonal frequency division multiplex," *Proc. of IEEE*, vol. 83, pp. 982–996, June 1995.

[10] M. Sandell, *Design and analysis of estimators for multicarrier modulation and ultrasonic imaging.* PhD thesis, Lulea University of Technology, Lulea, Sweden, 1996.

[11] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," *IEEE Communications Magazine*, pp. 100–109, Feb. 1995.

[12] J. D. Parsons, *The Mobile Radio Propagation Channel.* London: Pentech Press, 1992.

[13] T. S. Rappaport, *Wireless communications: Principles & Practice.* Upper Saddle River, New Jersey: Prentice Hall, 1996.

[14] R. Haas and J. C. Belfiore, "A time-frequency well-localized pulse for multiple carrier transmission," *Wireless Personal Communications*, vol. 5, pp. 1–18, 1997.

[15] I. Daubechies, *Ten Lectures on Wavelets.* SIAM, 1992.

[16] M. de Courville, *Utilisation de bases orthogonales pour l'algorithmique adaptive et l'egalisation des systèmes multiporteuses.* PhD thesis, Ecole Nationale Supérieure des Télécommunications Paris, Paris, France, Oct. 1996.

[17] R. Hleiss, P. Duhamel, and M. Charbit, "Oversampled OFDM systems," in *Proc. of Int. Conf. on DSP*, (Santorini, Greece), pp. 329–332, July 1997.

[18] B. Hirosaki, "An orthogonally multiplexed QAM system using the discrete Fourier transform," *IEEE Trans. Comm.*, vol. 29, pp. 982–989, July 1981.

[19] B. Hirosaki, S. Hasegawa, and A. Sabato, "Advanced groupband data modem using orthogonally multiplexed QAM technique," *IEEE Trans. Comm.*, vol. 34, pp. 587–592, June 1986.

[20] A. Vahlin and N. Holte, "Optimal finite duration pulses for OFDM," *IEEE Trans. Comm.*, vol. 4, pp. 10–14, Jan. 1996.

[21] H. G. Feichtinger and T. Strohmer, eds., *Gabor Analysis and Algorithms: Theory and Applications.* Boston (MA): Birkhäuser, 1998.

[22] H. Bölcskei, P. Duhamel, and R. Hleiss, "Design of pulse shaping OFDM/OQAM systems for wireless communications with high spectral efficiency," *IEEE Trans. Signal Processing*, Nov. 1998. submitted.

[23] H. Bölcskei and F. Hlawatsch, "Discrete Zak transforms, polyphase transforms, and applications," *IEEE Trans. Signal Processing*, vol. 45, pp. 851–866, April 1997.

[24] P. P. Vaidyanathan, *Multirate Systems and Filter Banks.* Englewood Cliffs (NJ): Prentice Hall, 1993.

[25] C. Roche and P. Siohan, "Bancs de filtres modulés de type IOTA/EGF: Le cas orthogonal," Tech. Rep. 5225, CNET, Feb. 1998.

[26] P. Siohan and C. Roche, "Analytical design for a family of cosine modulated filter banks," in *Proc. IEEE ISCAS-98*, (Monterey, CA), May 1998.