Influence of Propagation Conditions on the Outage Capacity of Space-Time Block Codes

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ABSTRACT

For a narrowband flat-fading channel taking into account Ricean K-factor, spatial fading correlation, and gain imbalances, we derive analytic expressions for the outage capacity of space-time block codes. Our results are then used to study the impact of these propagation parameters on the outage capacity of space-time block codes. Furthermore, we introduce an outage capacity motivated measure of diversity gain which reveals that the diversity gain offered by space-time fading channels depends strongly on the outage rate and propagation conditions. Finally, we verify the accuracy of our analytic results through comparison with numerically obtained results.

1 INTRODUCTION AND OUTLINE

Wireless channels exhibit fluctuations in signal level known as fading. The use of multiple-input multiple-output (MIMO) antenna systems combined with space-time coding can significantly improve link quality [1]-[4]. Space-time coding is capable of extracting full diversity gain without knowing the channel in the transmitter. Most of the previous work on the performance of space-time codes assumes the idealistic i.i.d. Rayleigh fading channel model [1]-[4]. In practice, however, the presence of a Ricean component, spatial fading correlation between the antenna elements due to insufficient antenna spacing and/or lack of scattering, and gain imbalances, will influence the performance of any space-time signaling scheme.

Contributions. In this paper, taking into account realistic propagation conditions, we study the outage capacity behavior of space-time block codes. Space-time block codes are particularly appealing as they drastically simplify maximum-likelihood decoding. Our detailed contributions are as follows.

- For a flat-fading MIMO channel taking into account Ricean K-factor, spatial fading correlation, and gain imbalances, we derive *analytic expressions for the cumulative distribution function (CDF)* of the mutual information achieved by space-time block codes employing scalar Gaussian code books. We verify the accuracy of our analytic expressions through comparison with numerical results.

- We apply our results to *quantify the impact of the above mentioned propagation parameters* on the outage capacity of space-time block codes in conjunction with scalar Gaussian code books.

- We introduce an outage capacity motivated *measure of diversity gain*, which reveals a strong dependence of the diversity gain offered by space-time fading channels (in conjunction with space-time block codes) on outage rate.

- For the i.i.d. Rayleigh fading channel, we *quantify the reduction in receive signal level fluctuation* due to the presence of spatial diversity, as a function of the number of transmit and receive antennas, respectively.

Relation to previous work. The impact of spatial fading correlation on the *ergodic capacity* of flat-fading and frequency-selective fading MIMO channels has been studied previously in [5] and [6], respectively. A power series expansion based approach for computing the outage behavior of space-time block codes from a symbol error point of view has been suggested in [7]. The outage capacity based investigation conducted in this paper provides us with the ultimate information-theoretic limits achievable by space-time block codes (in conjunction with scalar Gaussian code books) for arbitrary channel characteristics. Furthermore, we will show that the Laguerre series based approach for computing the outage capacity exhibits significantly superior numerical performance when compared to the power series expansion based approach of [7].

Organization of the paper. The rest of this paper is organized as follows. Sec. 2 introduces the channel

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model. In Sec. 3, we derive the CDF of mutual information of space-time block codes in conjunction with scalar Gaussian code books. In Sec. 4, we quantify the impact of varying channel characteristics on outage capacity behavior, and we provide an expression for the reduction of receive signal level fluctuation as a function of the number of transmit and receive antennas assuming i.i.d. Rayleigh fading. We present our simulation results in Sec. 5, and conclude in Sec. 6.

2 CHANNEL MODEL

We consider a MIMO wireless system with M_T transmit and M_R receive antennas, and restrict our analysis to the case of frequency-flat fading. The input-output relation of such a channel is characterized by the $M_R \times M_T$ channel matrix $\boldsymbol{H}$, whose elements $H_{i,j}$ $(i = 1, 2, ..., M_R,\ j = 1, 2, ..., M_T)$ are (possibly correlated) complex Gaussian random variables. We decompose $\boldsymbol{H}$ into the sum of a fixed (possibly line-of-sight) component[1] $\overline{\boldsymbol{H}} = \mathcal{E}\{\boldsymbol{H}\}$ and a variable component $\widetilde{\boldsymbol{H}}$ consisting of zero-mean circularly symmetric complex Gaussian random variables. In the case of pure Rayleigh fading, we have[2] $\overline{\boldsymbol{H}} = \boldsymbol{0}$, while $\overline{\boldsymbol{H}} \neq \boldsymbol{0}$ in the presence of Ricean fading. We assume the standard block-fading channel model [8] where the channel remains constant over the duration of one block and then changes in an independent fashion to a new realization. Throughout the paper, we assume that the transmitter has no channel knowledge whereas the receiver knows the channel perfectly.

In practice, as a result of insufficient antenna spacing and/or lack of scattering the elements of $\widetilde{\boldsymbol{H}}$ will in general be correlated. This correlation may be concisely expressed through the covariance matrix[3]

$$\boldsymbol{R} = \mathcal{E}\left\{\widetilde{\boldsymbol{h}}\widetilde{\boldsymbol{h}}^H\right\}, \tag{1}$$

where $\widetilde{\boldsymbol{h}} = \mathrm{vec}\{\widetilde{\boldsymbol{H}}\}$. Note that $\boldsymbol{R}$ and $\overline{\boldsymbol{H}}$ completely characterize the statistical behavior of the channel and are capable of describing potential power imbalance between the elements of $\boldsymbol{H}$ say due to the use of polarization diversity [9]. In the following, $\boldsymbol{h} = \mathrm{vec}\{\boldsymbol{H}\}$, $\overline{\boldsymbol{h}} = \mathrm{vec}\{\overline{\boldsymbol{H}}\}$, and $\boldsymbol{R} = \boldsymbol{U\Sigma U}^H$ denotes the eigendecomposition of the covariance matrix $\boldsymbol{R}$ with $\boldsymbol{\Sigma} = \mathrm{diag}\{\sigma_j\}_{j=1}^{M_T M_R}$. Finally, the squared Frobenius norm of the channel is $\|\boldsymbol{H}\|_F^2 = \boldsymbol{h}^H\boldsymbol{h}$. For the sake of simplicity, we assume $\boldsymbol{R}$ to be non-singular in the case of Ricean fading.

3 OUTAGE CAPACITY OF SPACE-TIME BLOCK CODES

The use of space-time block codes [10] combined with appropriate processing at the receiver turns the matrix channel into an effective scalar channel with mutual information [11]

$$I = \log_2\left(1 + \frac{E_s\|\boldsymbol{H}\|_F^2}{M_T\sigma_n^2}\right) \quad \text{bps/Hz}, \tag{2}$$

where E_s is the total average energy available at the transmitter over a symbol period and σ_n^2 is the variance of the additive spatio-temporally white complex Gaussian noise at each of the receive antennas. Note that we have assumed an i.i.d. Gaussian code book. So far we have restricted our attention to space-time block codes. We note, however, that the analysis performed above may also be applied in analyzing the mutual information of MIMO channels in the low SNR case. Assuming that the transmit signal is circularly symmetric i.i.d. complex Gaussian [12] the mutual information is given by[4]

$$I = \log_2 \det\left(\boldsymbol{I}_{M_R} + \frac{E_s}{M_T\sigma_n^2}\boldsymbol{H}\boldsymbol{H}^H\right). \tag{3}$$

In the low SNR regime, i.e., $\frac{E_s}{M_T\sigma_n^2} \ll 1$, (3) reduces to

$$I \approx \log_2\left(1 + \frac{E_s\|\boldsymbol{H}\|_F^2}{M_T\sigma_n^2}\right), \tag{4}$$

which is identical to the mutual information achieved by space-time block codes.

Since $\boldsymbol{H}$ is random, the mutual information I will be random as well. The rate-dependent outage probability is defined as $P_{out,R} = P(I \leq R)$, where R denotes rate [12, 13, 8]. Equivalently, one can define the $q\%$ outage capacity $C_{out,q}$ as the capacity that is guaranteed in $(100 - q)\%$ of the cases, i.e., $P(I \leq C_{out,q}) = q\%$ [13, 8]. For details on the operational meaning of these definitions the reader is referred to [12, 13, 8]. From (2) and (4) it is clear that the statistics of I and hence the outage capacity depend on the statistics of $\|\boldsymbol{H}\|_F^2$. Furthermore, it is easy to verify that the CDF of I is given by

$$F_I(y) = F\left(\frac{(2^y - 1)M_T\sigma_n^2}{E_s}\right), \tag{5}$$

where $F(y)$ stands for the CDF of $\|\boldsymbol{H}\|_F^2$. Since $\|\boldsymbol{H}\|_F^2$ is a quadratic form in complex Gaussian random variables, the Laplace transform of the probability density function (PDF) of $\|\boldsymbol{H}\|_F^2$, $\psi(s) = \mathcal{E}\{e^{-s\|\boldsymbol{H}\|_F^2}\}$, is given by [14]

$$\psi(s) = \frac{\exp\left(-\sum_{j=1}^{M_T M_R}|b_j|^2 + \sum_{j=1}^{M_T M_R}\frac{|b_j|^2}{1+s\sigma_j}\right)}{\prod_{j=1}^{M_T M_R}(1 + s\sigma_j)}, \tag{6}$$

where b_j represents the j-th element of the $M_T M_R \times 1$ vector $\boldsymbol{b} = \boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^H\overline{\boldsymbol{h}}$. In the following, we consider the cases of Rayleigh and Ricean fading separately in deriving $F(y)$.

Rayleigh fading. In the case of Rayleigh fading, $\overline{\boldsymbol{h}} = \boldsymbol{0}$ which translates to $\boldsymbol{b} = \boldsymbol{0}$. Expressing distinct non-zero eigenvalues of $\boldsymbol{R}$ by $\widetilde{\sigma}_j(j = 1, 2, ..., L,\ 1 \leq L \leq M_T M_R)$ and denoting their respective multiplicities by $m_j(j = 1, 2, ..., L)$, we can express $\psi(s)$ in (6) via partial fraction expansion [15] as

[1] $\mathcal{E}$ denotes the expectation operator.

[2] $\boldsymbol{0}$ denotes the all zeros matrix of appropriate size.

[3] The superscript ^{H} stands for conjugate transpose.

[4] $\boldsymbol{I}_m$ is the $m \times m$ identity matrix.

$$\psi(s) = \sum_{j=1}^{L}\sum_{k=1}^{m_j} \frac{A_{jk}}{(1+s\widetilde{\sigma}_j)^k}, \quad (7)$$

where the $A_{jk}(j = 1, 2, ..., L, k = 1, 2, ..., m_j)$ are determined by solving a system of linear equations. Note that $\psi(0) = 1$ implies that $\sum_{j=1}^{L}\sum_{k=1}^{m_j} A_{jk} = 1$. The PDF of $\|\boldsymbol{H}\|_F^2$ which is simply the inverse Laplace transform of (7) can then be expressed as[5]

$$f(x) = \sum_{j=1}^{L}\sum_{k=1}^{m_j} A_{jk} \frac{x^{k-1}}{(k-1)!\,\widetilde{\sigma}_j^k} e^{-\frac{x}{\widetilde{\sigma}_j}} u(x). \quad (8)$$

From (8), the CDF of $\|\boldsymbol{H}\|_F^2$ can be derived as

$$F(y) = \left(1 - \sum_{j=1}^{L}\sum_{k=1}^{m_j} A_{jk} \sum_{i=0}^{k-1} \frac{\widetilde{\sigma}_j^{i+1-k} y^{k-1-i}}{(k-1-i)!} e^{-\frac{y}{\widetilde{\sigma}_j}}\right) u(y). \quad (9)$$

A power series expansion of $F(y)$ has been provided previously in [7].

Ricean fading. In the case of Ricean fading, $f(x)$ can be derived through more complicated Laplace transform inversion techniques involving series expansion of $\psi(s)$ [14, 16]. We provided a power-series expansion based approach in [7]. In this paper, we present an alternative series expansion for $f(x)$ based on Laguerre polynomials. The advantage of the Laguerre series based approach is that for a fixed residual error it requires significantly fewer terms than the power series based approach.

Taking β to be a positive constant which can be chosen freely, we get [14]

$$\begin{aligned} f(x) &= \sum_{k=0}^{\infty} c_k \left(\frac{k!\boldsymbol{L}_k^{(M_TM_R-1)}\left(\frac{x}{2\beta}\right) e^{-\frac{x}{2\beta}}}{(M_TM_R+k-1)!(2\beta)} \right. \\ &\quad \left. \times \left(\frac{x}{2\beta}\right)^{M_TM_R-1}\right) u(x), \end{aligned} \quad (10)$$

where $\boldsymbol{L}_k^{(\alpha)}(x)\,(\alpha > -1, k = 0, 1, 2, ...)$ denotes the generalized Laguerre polynomials [17].

Furthermore, $c_0 = 1$ and $c_k\ (k \geq 1)$ can be calculated recursively using the following relations [14, 16]

$$c_k = \frac{1}{k}\sum_{r=0}^{k-1} d_{k-r}c_r, \ k \geq 1 \quad (11)$$

$$\begin{aligned} d_k &= -\frac{k}{2\beta}\sum_{j=1}^{M_TM_R} |b_j|^2\sigma_j \left(1 - \frac{\sigma_j}{2\beta}\right)^{k-1} \\ &+ \sum_{j=1}^{M_TM_R}\left(1 - \frac{\sigma_j}{2\beta}\right)^k, \ k \geq 1. \end{aligned} \quad (12)$$

[5] $u(x)$ is the unit step function defined as $u(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$.

The convergence properties of the series expansion (10) depend strongly on the choice of the parameter β. Typically, choosing $1 \leq \beta \leq 4$ enables us to compute $f(x)$ accurately by summing a reasonable number of terms (up to 50). Using (10) it is shown in [14] that $F(y)$ for the Ricean case is given by

$$\begin{aligned} F(y) = u(y) + \\ \left(\sum_{k=1}^{\infty} c_k \frac{(k-1)!\,\boldsymbol{L}_{k-1}^{M_TM_R}\left(\frac{y}{2\beta}\right) e^{-\frac{y}{2\beta}}}{(M_TM_R+k-1)!} \left(\frac{y}{2\beta}\right)^{M_TM_R} - \right. \\ \left. \sum_{i=0}^{M_TM_R-1} \frac{\left(\frac{y}{2\beta}\right)^{M_TM_R-1-i}}{(M_TM_R-1-i)!} e^{-\frac{y}{2\beta}} \right) u(y). \end{aligned} \quad (13)$$

Using (9) and (13) together with (5) yields the CDF of I in the Rayleigh and Ricean cases, respectively. Numerical results in Sec. 5 reveal an almost identical match between the so obtained CDF of the mutual information and the empirically obtained CDF (through Monte Carlo methods). For $\overline{\boldsymbol{h}} = \boldsymbol{0}$, the expression for $F(y)$ for the Ricean case simplifies to the Rayleigh case. We chose to present the Rayleigh fading case separately as a closed-form expression of $F(y)$ can be obtained.

4 A NEW MEASURE OF DIVERSITY

In this section, we shall introduce an outage capacity motivated measure of diversity gain, which shows that the effective diversity gain offered by space-time channels in conjunction with space-time block codes depends strongly on the outage rate. For the i.i.d. Rayleigh fading case, we shall furthermore quantify the reduction in receive signal level fluctuation due to the presence of multiple transmit and/or receive antennas by computing the standard deviation of mutual information.

An outage rate dependent measure of diversity. Noting that $\log_2(x)$ is a monotonically increasing function, we can specify the following outage rate dependent measure of diversity. Assume that $F_1(y)$ and $F_2(y)$ are the CDFs of $\|\boldsymbol{H}\|_F^2$ for two different channels with different statistics and $F_1(y1) = F_2(y2) = a\%$. It is clear that the channel with higher $y_i(i = 1, 2)$ will have a higher $a\%$ outage capacity. We can therefore quantify the gain (or loss) in diversity at the $a\%$ outage level offered by channel 1 over channel 2 as follows

$$\eta(a\%) = 10\log_{10}\left(\frac{y_1}{y_2}\right) (\text{dB}). \quad (14)$$

This definition allows to quantify the gain/loss in effective diversity order due to a Ricean component, fading signal correlation, and/or gain imbalance by using the classical i.i.d Rayleigh fading MIMO channel as a reference. We note that diversity gain measured from an outage point of view is identical whether the performance criterion is uncoded symbol error rate [7] or outage capacity.

Quantifying channel fluctuations. The idea of diversity gain is inherently connected to the receive signal level fluctuations in the wireless link. The reduction of these channel fluctuations (tightening of the channel) with an increase in diversity order has been depicted with measured data in [18]. We propose to quantify the fluctuations of a wireless link by computing the standard deviation of I achieved by space-time block codes. Let us consider the classical i.i.d. Rayleigh fading MIMO channel ($\overline{\boldsymbol{H}} = \mathbf{0}$ and $\boldsymbol{R} = \boldsymbol{I}_{M_T M_R}$). Making use of the fact that[6] $\|\boldsymbol{H}\|_F^2 = \frac{1}{2}\chi^2_{2M_T M_R}$, the variance of I in the high SNR regime is given by [14]

$$\rho_I^2 \approx \frac{(\log_2 e)^2}{M_T M_R}. \tag{15}$$

Equivalently, the standard deviation of the mutual information is given as $\rho_I \approx \frac{\log_2 e}{\sqrt{M_T M_R}}$. Thus, asymptotically, as the number of degrees of freedom $M_T M_R$ approaches infinity, we get $\rho_I \to 0$ and the link does not exhibit any fluctuations. The reduction of the receive signal level fluctuation may also be quantified by considering the standard deviation of $\|\boldsymbol{H}\|_F^2$ as done in [14], which (neglecting scaling factors) leads to the same quantitative behavior. We finally note that based on the standard deviation of $\|\boldsymbol{H}\|_F^2$ (which can be computed analytically for arbitrary channel statistics [14]) an effective diversity order for general space-time channels can be defined. The details of this concept are presented in [14].

5 Simulation Results

We consider a system with $M_T = M_R = 2$. The SNR in our simulations was defined as $10\log_{10}\left(\frac{E_s}{\sigma_n^2}\right)$(dB). We chose $\overline{\boldsymbol{H}}$ and $\widetilde{\boldsymbol{H}}$ to be of the form

$$\overline{\boldsymbol{H}} = \sqrt{\frac{K}{1+K}}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{16}$$

$$\widetilde{\boldsymbol{H}} = \sqrt{\frac{1}{1+K}}\begin{bmatrix} \widetilde{G}_{1,1} & \widetilde{G}_{1,2} \\ \widetilde{G}_{2,1} & \widetilde{G}_{2,2} \end{bmatrix}, \tag{17}$$

where $\widetilde{G}_{i,j} (i,j = 1,2)$ are (possibly correlated) zero-mean circularly symmetric complex Gaussian random variables with unit variance (i.e. no gain imbalance) and K is the Ricean K-factor. In the case of pure Rayleigh fading, we have $K = 0$. Furthermore, we define the following correlations[7]

$$t = \mathcal{E}\{\widetilde{G}_{1,1}\,\widetilde{G}^*_{1,2}\} = \mathcal{E}\{\widetilde{G}_{2,1}\,\widetilde{G}^*_{2,2}\} \tag{18}$$

$$r = \mathcal{E}\{\widetilde{G}_{1,1}\,\widetilde{G}^*_{2,1}\} = \mathcal{E}\{\widetilde{G}_{1,2}\,\widetilde{G}^*_{2,2}\}, \tag{19}$$

where t and r are referred to as transmit and receive correlation coefficient, respectively. For the sake of simplicity, we furthermore assume $\mathcal{E}\{\widetilde{G}_{1,1}\widetilde{G}^*_{2,2}\} = \mathcal{E}\{\widetilde{G}_{1,2}\widetilde{G}^*_{2,1}\} = 0$.

Simulation Example 1. The first simulation example serves to assess the accuracy of our analytical expressions in determining the CDF of the mutual information achieved by a space-time block code such as the Alamouti scheme [4]. For $K = 2$, $r = 0.3$, $t = 0.4$, and an SNR of 10dB, Fig. 1 shows the empirical CDF (through Monte Carlo methods) of the mutual information achieved by the Alamouti scheme as well as the CDF calculated from (5) and (13). In this example we set $\beta = 1$ and used the first 20 terms in the series expansion (13). The empirically determined CDF and the analytical expression for the CDF of the mutual information closely match each other, verifying the accuracy of our analysis.

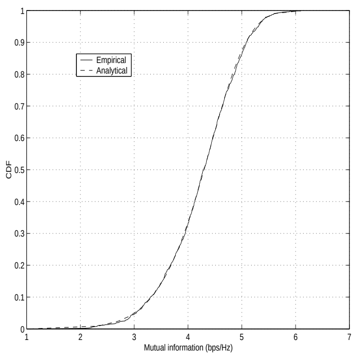

Figure 1: Comparison of empirical and analytical CDF of mutual information in the presence of Ricean fading for $K = 2$, $r = 0.3$, $t = 0.4$, and an SNR of 10dB.

Simulation Example 2. In this simulation example we demonstrate the superior computational properties of the Laguerre series expansion for the CDF of $\|\boldsymbol{H}\|_F^2$ over the power series expansion derived in [7]. In the following, $F_E(y)$ and $F_A(y)$ denote the empirically determined CDF and the analytically obtained CDF (according to (9) and (13)) of $\|\boldsymbol{H}\|_F^2$, respectively. We define the mean absolute error between $F_E(y)$ and $F_A(y)$ as

$$\tau = \frac{1}{N}\sum_{i=1}^{N} |F_E(y_i) - F_A(y_i)| \tag{20}$$

with N data points y_i $(i = 1, 2, ..., N)$. For $N = 1000$, Fig. 2a) shows the mean absolute error between the empirical CDF and the analytical CDF as a function of the number of terms summed in the Laguerre series expansion ($\beta = 1$) of $F(y)$ for a channel with $K = 2$, $r = 0.1$, and $t = 0.2$. Fig. 2b) depicts the same for the power series expansion of $F(y)$. It is clearly seen from the figures that for a given τ the Laguerre series expansion requires significantly fewer terms than the power series expansion. The error floor in Figs. 2a) and b) can be attributed to the fact that τ is obtained by averaging the absolute error over a finite number of data points, and to the uncertainty associated with the empirical estimation of $F(y)$. Beyond a certain number of terms in the series expansion, the error in empirical estimation dominates and hence the flooring.

Simulation Example 3. This simulation example studies the impact of the Ricean K-factor on the outage capacity of space-time block codes. Figs. 3a) and b) show the 5% and 25% outage capacity, respectively, of the Alamouti scheme for a channel with $r = t = 0$ and $K = 0$ or $K = 10$. It is clear from the figures that the channel experiencing Ricean fading outperforms the

[6] χ^2_N is a chi-squared random variable with N degrees of freedom.
[7] The superscript * stands for complex conjugate.

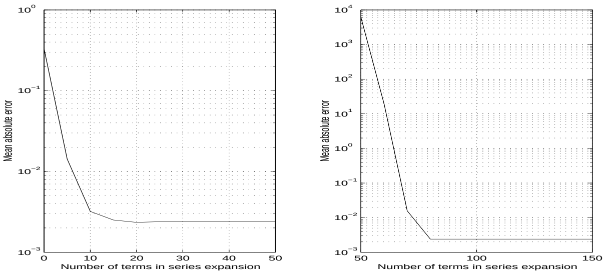

Figure 2: Comparison of the convergence properties of the Laguerre series and power series expansions of $F(y)$.

channel experiencing pure Rayleigh fading for both outage rates. This is due to the presence of a fixed component in the channel in the case of Ricean fading that effectively stabilizes the link. Furthermore, comparing the two figures, we can see that the diversity gain extracted by the space-time block code is clearly a function of outage rate. More specifically, using the i.i.d. Rayleigh fading channel as a reference we find that the diversity gain offered by the Ricean channel at the 5% outage level is $\eta(5\%) = 2.9$ dB and similarly, $\eta(25\%) = 1.2$ dB.

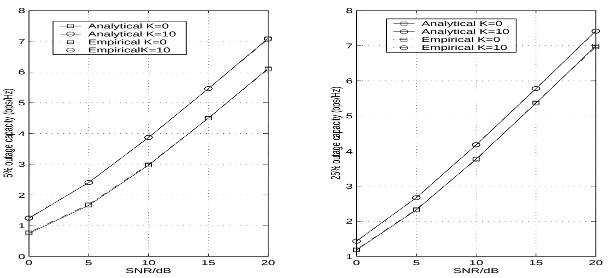

Figure 3: Effect of Ricean K-factor on outage capacity of space-time block codes.

Simulation Example 4. The last simulation example serves to demonstrate the impact of fading signal correlation on the outage capacity performance of the Alamouti scheme. Fig. 4 shows the diversity gain according to (14) offered by a channel with $t = 0$ and $K = 0$ for varying degrees of receive correlation r using the i.i.d. Rayleigh fading MIMO channel as a reference. It is evident that receive correlation is detrimental to the outage performance of a space-time block code for low outage rates. Note furthermore that the diversity loss due to spatial fading correlation is significantly higher in the 5% outage rate case. The same is true of transmit correlation. We conclude this simulation example by noting that it has been shown analytically in [14] that outage performance of space-time block codes is optimal when $\boldsymbol{R}$ is orthogonal, or in other words in the absence of fading signal correlation and energy imbalance between channel elements.

6 Conclusions

For a narrowband Rayleigh or Ricean flat-fading channel taking into account Ricean K-factor, spatial fading correlation, and gain imbalances, we derived analytic expressions for the CDF of the mutual information achieved by space-time block codes. We applied our results to *quantify the impact of propagation parameters* on the

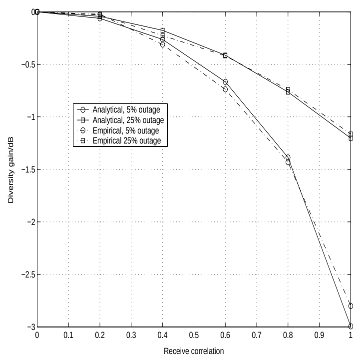

Figure 4: Diversity gain according to (14) as a function of receive correlation for $t = 0$ in the presence of Rayleigh fading.

outage capacity of space-time block codes and found that the effective diversity gain provided by a space-time channel in conjunction with space-time block codes depends strongly on the outage rate. Finally, we verified the accuracy of our analytic expressions through comparison with numerically obtained results.

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