

Distributed Orthogonalization in Large Interference Relay Networks

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Abstract—We study fading interference relay networks where M single-antenna source-destination terminal pairs communicate through a set of K relays using half-duplex two-hop relaying. Two specific protocols are considered, P1 introduced in [1], [2] and P2 introduced in [3]. P1 relies on the idea of relay partitioning and requires each relay terminal to know one backward and one forward fading coefficient only. P2 requires each relay terminal to know all M backward and M forward fading coefficients and does not need relay partitioning. We prove that in the large- M limit the minimum rate of growth of K for P1 to achieve a strictly positive per source-destination terminal pair capacity is $K \propto M^3$ whereas in P2 it is $K \propto M^2$. The protocols P1 and P2 are thus found to trade off the number of relay terminals for channel state information (CSI) at the relays; more CSI at the relays reduces the total number of relays needed to achieve a strictly positive per source-destination terminal pair capacity in the large- M limit.

I. INTRODUCTION

Sparked by [4], [5] there has been significant recent interest in capacity scaling and space-time coding in wireless networks with a plethora of results established under different assumptions on the setup [3], [6]–[15]. In this paper, we consider interference fading relay networks where M single-antenna source-destination terminal pairs communicate concurrently through half-duplex two-hop relaying over a common set of K single-antenna relay terminals (see Fig. 1). The network operates in a completely distributed fashion, i.e., there is no cooperation between terminals (not even the receive terminals).

It was shown in [1], [2] that for fixed M , assuming a perfectly synchronized network¹ and perfect channel state information (CSI) at the destination terminals, network capacity scales as $C = (M/2) \log(K) + O(1)$. The relaying scheme proposed in [1], [2] (referred to as Protocol 1 or short P1 in the following) requires that the relays are partitioned into M clusters with each cluster assigned to one of the M source-destination terminal pairs. The relays in a given cluster are assumed to maintain perfect knowledge of the corresponding scalar backward and forward channels to their assigned source and destination terminals, respectively. In the large K limit,

assuming that the number of relays in each of the clusters grows linearly with K , network capacity is achieved through matched-filtering at the relays and independent (coherent) decoding at the destination terminals. This result implies that distributed array gain [17] and spatial multiplexing gain [17]–[19] can be obtained in a completely distributed fashion, i.e., without cooperation between any of the terminals (not even the destination terminals). In [3] an alternative protocol (referred to as Protocol 2 or short P2 in the following) for half-duplex two-hop relaying based on matched-filtering at the relays is proposed. P2 requires that each of the relays knows all its M scalar backward and M scalar forward channels to the M source and M destination terminals, respectively. There is no need to perform relay partitioning (as in P1) and each relay assists all M source-destination terminal pairs.

Contributions and relation to previous work: The main contributions in this paper can be summarized as follows:

- The results in [1], [2] and the corresponding proof techniques rely heavily on M being fixed when $K \rightarrow \infty$. Based on a slightly modified version of a technique introduced in [20] in a completely different context and used in [3] to establish the power efficiency scaling of P2, we analyze P1 [1], [2] in the case where both M and K grow large. We establish that for $M \rightarrow \infty$ with $K \propto M^{3+\alpha}$ the per source-destination terminal pair capacity achieved by P1 scales at least as fast as $\log(M^\alpha)$. A converse result states that the per source-destination terminal pair capacity of P1 approaches zero, for $M \rightarrow \infty$, if K grows slower than M^3 .
- The focus in [3] is on establishing the power efficiency scaling behavior of P2. Interpreting the findings in [3] in terms of spectral efficiency, it is readily seen that P2, like P1, realizes distributed array gain and spatial multiplexing gain in a completely distributed fashion. In particular, the results in [3] show that for $M \rightarrow \infty$ with $K \propto M^{2+\alpha}$ P2 achieves a per source-destination terminal pair capacity, which scales at least as fast as $\log(M^\alpha)$. Using the same technique as in the proof of the converse result for P1, we then conclude that the per source-destination terminal pair capacity of P2 approaches zero, for $M \rightarrow \infty$, if K grows slower than M^2 . In summary, we conclude that

¹This assumption can be significantly relaxed without changing the capacity scaling law [16].

P1 and P2 trade off channel knowledge at the relays for the number of relays needed to achieve a given source-destination terminal pair capacity.

Notation: $|\mathcal{X}|$ is the cardinality of the set \mathcal{X} . All logarithms are to the base 2. \mathcal{E} denotes the expectation operator. $\text{VAR}(X)$ stands for the variance of the random variable (RV) X . A circularly symmetric zero-mean complex Gaussian RV is a RV $Z = X + jY \sim \mathcal{CN}(0, \sigma^2)$, where X and Y are i.i.d. $\mathcal{N}(0, \sigma^2/2)$. $\delta[k] = 1$ for $k = 0$ and 0 otherwise. For two functions $f(x)$ and $g(x)$, the notation $f(x) = O(g(x))$ means that $|f(x)/g(x)|$ remains bounded as $x \rightarrow \infty$. We write $g(x) = \Theta(f(x))$ to denote that $f(x) = O(g(x))$ and $g(x) = O(f(x))$. Finally, $f(x) = o(g(x))$ stands for $\lim_{x \rightarrow \infty} f(x)/g(x) = 0$.

Organization of the paper: The remainder of this paper is organized as follows. Section II introduces the channel and signal model. In Section III, we establish the large- M scaling behavior of P1. In Section IV, we state the converse result for P2 and discuss the relation between P1 and P2.

II. CHANNEL AND SIGNAL MODEL

In this section, we present the channel and signal model and additional basic assumptions. The discussion is general and applies to both protocols under consideration. The specifics of P1 and P2 are described in Sections III and IV, where our main results are stated.

General assumptions: We consider an interference relay network (see Fig. 1) consisting of $K + 2M$ single-antenna terminals with M designated source-destination terminal pairs $\{\mathcal{S}_l, \mathcal{D}_l\}$ ($l = 1, 2, \dots, M$) and K relays \mathcal{R}_k ($k = 1, 2, \dots, K$). Source terminal \mathcal{S}_l intends to communicate solely with destination terminal \mathcal{D}_l , a dead-zone of non-zero radius around each \mathcal{S}_l and \mathcal{D}_l is free of relay terminals, and no cooperation between terminals (not even between the destination terminals) is allowed. Furthermore, we assume that no direct link between the individual source-destination terminal pairs exists (e.g., caused by large separation), transmission takes place in half-duplex (the terminals cannot transmit and receive simultaneously) fashion in two hops (a.k.a. two-hop relaying) over two separate time slots. In the first time slot the source terminals \mathcal{S}_l broadcast their information to all the relay terminals (i.e., each relay terminal receives a superposition of all source signals). After processing the received signals, the relay terminals simultaneously broadcast the processed data to all the destination terminals during the second time slot. Finally, we assume that all terminals are located within a domain of fixed area (dense network assumption).

Channel and signal model: Throughout the paper, frequency-flat fading over the bandwidth of interest and perfectly synchronized transmission/reception between the terminals is assumed. The input-output relation for the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link during the first time slot is given by

$$r_k = \sum_{l=1}^M \sqrt{E_{k,l}} h_{k,l} s_l + n_k, \quad k = 1, 2, \dots, K \quad (1)$$

where r_k denotes the received signal at the k th relay terminal, $E_{k,l}$ is the average energy received at \mathcal{R}_k through the

$\mathcal{S}_l \rightarrow \mathcal{R}_k$ link² (having accounted for path loss and shadowing in the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link), $h_{k,l}$ denotes the corresponding $\mathcal{CN}(0, 1)$ complex-valued channel gain, s_l is the temporally i.i.d. $\mathcal{CN}(0, 1)$ data signal transmitted by \mathcal{S}_l and satisfying $\mathcal{E}\{s_l s_k^*\} = \delta[l - k]$, and n_k is $\mathcal{CN}(0, \sigma^2)$ temporally and spatially (across relay terminals) white noise.

Each relay terminal processes its received signal r_k to produce the output signal t_k , which is then broadcast to the destination terminals during the second time slot while the source terminals are silent. The l th destination terminal receives the signal

$$y_l = \sum_{k=1}^K \sqrt{P_{l,k}} f_{l,k} t_k + z_l, \quad l = 1, 2, \dots, M \quad (2)$$

where $P_{l,k}$ denotes the average energy received at \mathcal{D}_l through the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link (having accounted for path loss and shadowing in the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link), $f_{l,k}$ is the corresponding $\mathcal{CN}(0, 1)$ complex-valued channel gain, and z_l is $\mathcal{CN}(0, \sigma^2)$ temporally and spatially (across destination terminals) white noise. The transmit signal t_k must be chosen to satisfy the average power constraint $\mathcal{E}\{|t_k|^2\} \leq 1$. Note that we impose a power constraint on a per-relay basis rather than a sum power constraint across relay terminals.

As already mentioned above, throughout the paper, path-loss and shadowing is accounted for through the $E_{k,l}$ ($k = 1, 2, \dots, K; l = 1, 2, \dots, M$) (for the first hop) and the $P_{l,k}$ ($l = 1, 2, \dots, M; k = 1, 2, \dots, K$) (for the second hop). We assume that these parameters are deterministic, uniformly bounded from above (follows from the dead-zone assumption) and below (follows from considering a domain of fixed area) so that

$$\underline{E} \leq E_{k,l} \leq \bar{E}, \quad \underline{P} \leq P_{l,k} \leq \bar{P}, \quad \forall k, l. \quad (3)$$

Additionally, we assume a block fading channel model such that the $h_{k,l}$ and $f_{l,k}$ change in an independent fashion from channel use to channel use and coding is performed over an infinite number of independent channel uses.

Throughout the paper, we assume that the source terminals \mathcal{S}_l do not have CSI. The assumptions on CSI at the relays and the destination terminals will be made specific when discussing P1 and P2 below in Sections III and IV, respectively.

III. CAPACITY SCALING FOR PROTOCOL 1

In this section, we first briefly review Protocol 1 introduced in [1], [2]. The capacity scaling law for P1 is stated in Theorem 1.

A. Protocol 1

The basic setup was introduced in the previous section. We shall next describe the specifics of P1. The K terminals are partitioned into M subsets \mathcal{M}_l ($l = 1, 2, \dots, M$) with³ $|\mathcal{M}_l| = K/M$. The relays in \mathcal{M}_l are assumed to assist the l th source-destination terminal pair $\{\mathcal{S}_l, \mathcal{D}_l\}$. For simplicity, we

² $\mathcal{A} \rightarrow \mathcal{B}$ signifies communication from terminal \mathcal{A} to terminal \mathcal{B} .

³This assumption can be relaxed to requiring that the number of relay terminals in each of these sets grows linearly with M in the large- M limit.

introduce the relay partitioning function $p : [1, K] \rightarrow [1, M]$ defined as

$$p(k) = l \Leftrightarrow \mathcal{R}_k \in \mathcal{M}_l.$$

We assume that the k th relay terminal has perfect knowledge of the phases $\arg(h_{k,p(k)})$ and $\arg(f_{p(k),k})$ of the single-input single-output (SISO) backward channel $\mathcal{S}_{p(k)} \rightarrow \mathcal{R}_k$ and the corresponding forward channel $\mathcal{R}_k \rightarrow \mathcal{D}_{p(k)}$, respectively. Based on this CSI the signal r_k received at the k th relay terminal is co-phased with respect to (w.r.t.) the assigned backward channel followed by an energy normalization so that

$$u_k = \tau_k^{(1)} e^{-j \arg(h_{k,p(k)})} r_k \quad (4)$$

where $\tau_k^{(1)} = (\sum_{l=1}^M E_{k,l} + \sigma^2)^{-1/2}$ ensures $\mathcal{E}\{|u_k|^2\} = 1$. The relay terminal \mathcal{R}_k then computes the transmit signal t_k by co-phasing with respect to its assigned forward channel, i.e.,

$$t_k = e^{-j \arg(f_{p(k),k})} u_k \quad (5)$$

so that the power constraint $\mathcal{E}\{|t_k|^2\} = 1$ is satisfied. In summary, P1 ensures that $|\mathcal{M}_l|$ of the relay terminals forward the signal intended for \mathcal{D}_l in a ‘‘doubly coherent’’ (w.r.t. backward and forward channel) fashion whereas the signals transmitted by the source terminals \mathcal{S}_m with $m \neq l$ are forwarded to \mathcal{D}_l in a ‘‘noncoherent’’ fashion (i.e., we have phase incoherence either on the backward or the forward link or on both links). A more detailed description of P1 can be found in [1].

B. Bounds on Network Capacity

We are now ready to state our main result on the capacity scaling law pertaining to P1.

Theorem 1: Suppose that destination terminal \mathcal{D}_l ($l = 1, 2, \dots, M$) has perfect knowledge of the effective channel gain $(\pi/4) \sum_{k \in \mathcal{M}_l} \tau_k^{(1)} (E_{k,l} P_{l,k})^{1/2}$ of the $\mathcal{S}_l \rightarrow \mathcal{D}_l$ link. Then, for any $\epsilon > 0$ there exist M_0, K_0 , such that for all $M \geq M_0, K \geq K_0$, the per source-destination terminal pair capacity⁴ achieved by P1 is lower-bounded according to

$$C_{P1} \geq \frac{1}{2} \log \left(1 + \frac{\pi^2}{16} \frac{P}{\bar{P}} \frac{E}{\bar{E}} \frac{K}{M^3} \right) - \epsilon. \quad (6)$$

Furthermore, if $K = o(M^3)$, then

$$C_{P1} \rightarrow 0 \text{ for } M \rightarrow \infty. \quad (7)$$

Proof: Here, we derive the lower bound (6) only. The proof of the converse statement (7) requires a completely different approach and is reported in detail in [21].

Consider the SISO channel between the terminals \mathcal{S}_l and \mathcal{D}_l ($l = 1, 2, \dots, M$). The destination terminal \mathcal{D}_l receives doubly (backward and forward link) coherently combined contributions corresponding to the data signal s_l , interfering terms containing contributions from the signals s_m with $m \neq l$ as well as noise, forwarded by the relays. Combining (1), (4), (5)

⁴ The lower bound in (6) is obtained by invoking the uniform bounds (3) and is hence uniform over all $\{\mathcal{S}_l, \mathcal{D}_l\}$ pairs. The exact (asymptotic) capacity expressions for the $\{\mathcal{S}_l, \mathcal{D}_l\}$ pairs differ (across l) in the constant factor multiplying the $\frac{K}{M^3}$ term.

and (2), it follows (after some straightforward algebra) that the signal received at \mathcal{D}_l is given by ($l = 1, 2, \dots, M$)

$$y_l = s_l \sum_{k=1}^K d_k^{l,l} + \sum_{m \neq l} s_m \sum_{k=1}^K d_k^{l,m} + \sum_{k=1}^K b_k^l n_k + z_l,$$

where

$$d_k^{l,m} = \tau_k^{(1)} \sqrt{E_{k,m} P_{l,k}} h_{k,m} e^{-j \arg(h_{k,p(k)})} f_{l,k} e^{-j \arg(f_{p(k),k})}$$

$$b_k^l = \tau_k^{(1)} \sqrt{P_{l,k}} f_{l,k} e^{-j \arg(f_{p(k),k})} e^{-j \arg(h_{k,p(k)})}.$$

Next, we set

$$F_l = \underbrace{\sum_{k=1}^K \mathcal{E}\{d_k^{l,l}\}}_{\bar{F}_l} + \underbrace{\sum_{k=1}^K (d_k^{l,l} - \mathcal{E}\{d_k^{l,l}\})}_{\tilde{F}_l}$$

$$W_l = \sum_{m \neq l} s_m \sum_{k=1}^K d_k^{l,m} + \sum_{k=1}^K b_k^l n_k + z_l, \quad l = 1, 2, \dots, M$$

so that $y_l = (\bar{F}_l + \tilde{F}_l) s_l + W_l$. A lower bound on the capacity of this SISO channel can be obtained following (a slightly modified version of) the approach described in [20, Sec. III]. In contrast to [20], in our case the noise term W_l is not Gaussian and W_l and \tilde{F}_l are not statistically independent. However, it is not difficult to see that despite the relaxed assumptions on the signal model, the result in [20] is still valid⁵ so that

$$I(y_l; s_l) \geq \log \left(1 + \frac{\bar{F}_l^2}{\text{VAR}(F_l) + \text{VAR}(W_l)} \right). \quad (8)$$

What remains to be done is the computation of \bar{F}_l , $\text{VAR}(F_l)$ and $\text{VAR}(W_l)$. The derivation of these quantities is tedious but straightforward and can be found in [21]. We summarize the result as [21] ($l = 1, 2, \dots, M$)

$$\bar{F}_l^2 \geq \frac{\pi^2 K^2}{16 M^2} \frac{P}{M \bar{E}} \frac{E}{\sigma^2}$$

$$\text{VAR}(F_l) \leq (M-1) \frac{K}{M} \frac{\bar{P} \bar{E}}{M \bar{E} + \sigma^2}$$

$$\text{VAR}(W_l) \leq MR(M-1 + \sigma^2) \frac{\bar{P} \bar{E}}{M \bar{E} + \sigma^2} + \sigma^2.$$

Substituting these bounds into (8) further (straightforward) manipulations⁶ yield the desired result. ■

Theorem 1 shows that in the large- M limit, for $K \propto M^{3+\alpha}$, the per source-destination terminal pair capacity scales at least as fast as $\log(M^\alpha)$. Since we have a total of M source-destination terminal pairs, the network capacity scales at least as fast as $(M/2) \log(M^\alpha)$, which shows that P1 achieves full spatial multiplexing gain in a completely distributed fashion (recall that the \mathcal{D}_l perform independent decoding). The corresponding distributed array gain is given

⁵A detailed derivation of the result in [20, Sec. III. A] under the relaxed assumptions described here can be found in [21]. Similarly to [20] here we use the fact that the average gain of the effective channel $\bar{F}_l = (\pi/4) \sum_{k \in \mathcal{M}_l} \tau_k^{(1)} (E_{k,l} P_{l,k})^{1/2}$ is known at the receiver \mathcal{D}_l .

⁶Here, we used the fact that σ^2 does not scale with M and is hence small compared to M in the large- M limit.

by M^α . Moreover, it is worthwhile to point out that in contrast to the finite M results in [1], [2], the destination terminals \mathcal{D}_l do not need any knowledge of the small-scale fading coefficients $h_{k,l}$ and $f_{l,k}$. This can be seen by noting that \bar{F}_l depends on the $E_{k,l}$ and $P_{l,k}$ only. Moreover, the coefficient $(\pi/4) \sum_{k \in \mathcal{M}_l} \tau_k^{(1)} (E_{k,l} P_{l,k})^{1/2}$ can easily be acquired through training. Finally, we note that (6) provides a rough idea of how $\underline{E}, \bar{E}, \underline{P}$ and \bar{P} impact the network capacity.

The converse part (7) of Theorem 1 shows that the rate of growth $K \propto M^3$ is fundamental in the sense that $K \propto M^\beta$ with $\beta < 3$ implies that $C_{P1} \rightarrow 0$ for $M \rightarrow \infty$. Equivalently, $K \propto M^3$ is the minimum rate of growth needed to sustain a strictly positive per source-destination terminal pair capacity in the large- M limit.

IV. RELATION BETWEEN P1 AND P2

In this section, we briefly review the protocol introduced in [3] (referred to as P2 in this paper) and we discuss the relation between P1 and P2 in detail. Whereas [3] is primarily focused on the power efficiency scaling behavior of the network, we shall interpret the lower bound in [3, Sec. 5.2] in terms of spectral efficiency per source-destination terminal pair capacity.

A. Protocol 2

The only difference between P1 and P2 is in the processing at the relays. In P1 the K relays are partitioned into M groups (of equal size) with each of these groups assisting one particular source-destination terminal pair. In P2 each relay assists all the source-destination terminal pairs so that relay partitioning is not needed. The downside of P2 is that each of the relays needs to know the phases of all its M backward and M forward channels, i.e., \mathcal{R}_k needs knowledge of the phases of $\mathcal{S}_l \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}_l$, respectively, for $l = 1, 2, \dots, M$. Consequently, P2 requires significantly more CSI at the relays than P1. The relay processing stage in P2 computes

$$t_k = \tau_k^{(2)} \left(\sum_{l=1}^M e^{-j \arg(h_{k,l})} e^{-j \arg(f_{l,k})} \right) r_k$$

where $\tau_k^{(2)} = (M \sum_{l=1}^M E_{k,l} + M\sigma^2)^{-1/2}$ ensures that the power constraint $\mathcal{E}\{|t_k|^2\} = 1$ is satisfied.

B. Bounds on Network Capacity

In this section, we restate the lower bound in [3, Sec. 5.2] in our setup and provide a corresponding converse result. We hasten to add that we provide the lower bound for completeness only and reformulate it in our framework for the sake of having a basis for comparison of P1 and P2.

Theorem 2: Suppose that destination terminal \mathcal{D}_l ($l = 1, 2, \dots, M$) has perfect knowledge of the effective channel gain $(\pi/4) \sum_{k \in \mathcal{M}_l} \tau_k^{(2)} (E_{k,l} P_{l,k})^{1/2}$ of the $\mathcal{S}_l \rightarrow \mathcal{D}_l$ link. Then, for any $\epsilon > 0$ there exist M_0, K_0 , such that for all $M \geq M_0, K \geq K_0$, the per source-destination terminal pair capacity⁷ achieved by P2 is lower-bounded according to

$$C_{P2} \geq \frac{1}{2} \log \left(1 + \frac{\pi^2}{16} \frac{\underline{P} \bar{E}^2}{\bar{P} \underline{E}^2} \frac{K}{M^2} \right) - \epsilon. \quad (9)$$

⁷The same comment as in footnote 4 applies.

Furthermore, if $K = o(M^2)$, then

$$C_{P2} \rightarrow 0 \text{ for } M \rightarrow \infty.$$

Proof: The proof of both parts of this theorem follows exactly the same idea as the proof of Theorem 1 and is provided in detail in [21]. ■

Theorem 2 shows that for $M \rightarrow \infty$ with $K \propto M^{2+\alpha}$ the per source-destination terminal pair capacity scales at least as fast as $\log(M^\alpha)$. Again, since there is a total of M source-destination terminal pairs, the network capacity scales at least as fast as $(M/2) \log(M^\alpha)$, which proves that P2 achieves full spatial multiplexing gain in a completely distributed fashion (recall that the \mathcal{D}_l perform independent decoding). The corresponding array gain is given by M^α . We note that $K \geq M^2$ was a standing assumption in [3]. The second part of Theorem 2 above shows that $K \propto M^2$ is indeed fundamental in the sense that it is the minimum rate of growth required to realize strictly positive per source-destination terminal pair capacity in the large- M limit. Finally, we note that, just like in P1, no knowledge of the small-scale fading coefficients $h_{k,l}$ and $f_{l,k}$ is needed at the destination terminals.

C. Relation Between P1 and P2

Summarizing the results in Theorems 1 and 2, we can conclude that both protocols P1 and P2 perform distributed orthogonalization of the effective MIMO channel between the \mathcal{S}_l and the \mathcal{D}_l . However, the minimum rate of growth required to realize a strictly positive per source-destination terminal pair capacity in the large- M limit is $K \propto M^3$ in P1 whereas it is (only) $K \propto M^2$ in P2. This order of magnitude reduction comes, however, at the cost of each relay having to know all M backward and M forward channels. We can therefore conclude that P1 and P2 trade off the number of relay terminals for channel knowledge at the relays.

D. Numerical Results

For $K = M^3$ in P1 and $K = M^2$ in P2, Fig. 2 shows the lower bounds on per source-destination terminal pair capacity (6) and (9), respectively, along with the upper bound obtained (through Monte Carlo methods) by assuming that each of the destination terminals \mathcal{D}_l knows the small-scale fading coefficients. We can see that for increasing M the bounds become increasingly tight. Fig. 3 shows the same bounds for $K = M^{2.5}$ and $K = M^{3.5}$ in P1 and correspondingly $K = M^{1.5}$ and $K = M^{2.5}$ in P2. We observe that the bounds are again increasingly tight for increasing M . Moreover, we can see that for $K = M^{3.5}$ in P1 and $K = M^{2.5}$ in P2, the per source-destination terminal pair capacity increases with increasing M , which can be attributed to distributed array gain. For $K = M^{2.5}$ in P1 and $K = M^{1.5}$ in P2, the per source-destination terminal pair capacity approaches zero in the large- M limit which agrees with the converse parts of Theorems 1 and 2.

V. CONCLUSION

We established the fundamental rates of growth of the number of relays K as a function of the number of source-destination terminal pairs M for the two-hop half-duplex relaying protocols introduced in [1], [2] and [3]. The protocol

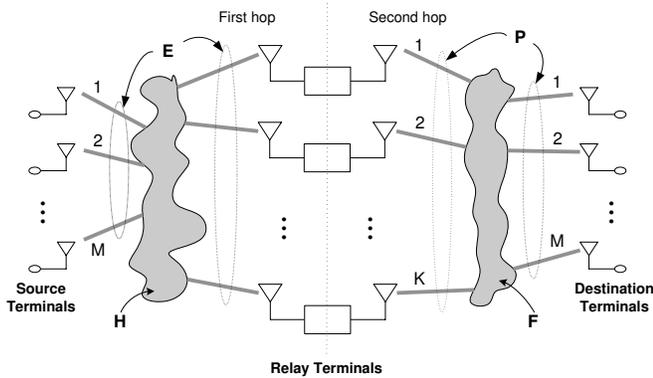


Fig. 1. Interference relay network.

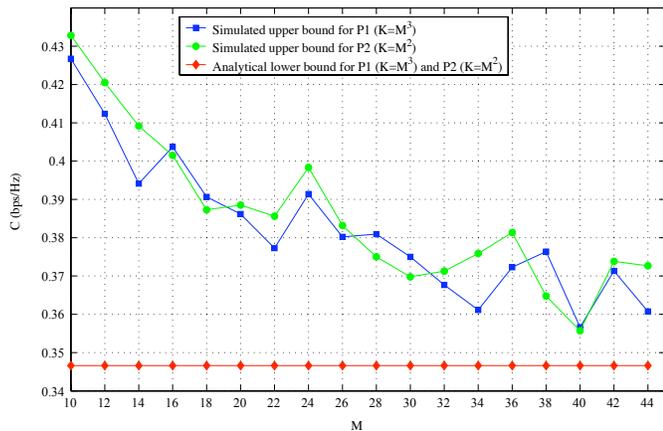


Fig. 2. Upper and lower bounds on C_{P1} and C_{P2} for $K = M^3$ in P1 and $K = M^2$ in P2.

in [1], [2] requires relay partitioning and the knowledge of one backward and one forward fading coefficient per relay; the corresponding minimum rate of growth is $K \propto M^3$ for the per source-destination terminal pair capacity to be strictly positive in the large- M limit. The protocol in [3] requires the knowledge of all M backward and M forward fading coefficients at each of the relay terminals; the corresponding minimum rate of growth was shown to be $K \propto M^2$.

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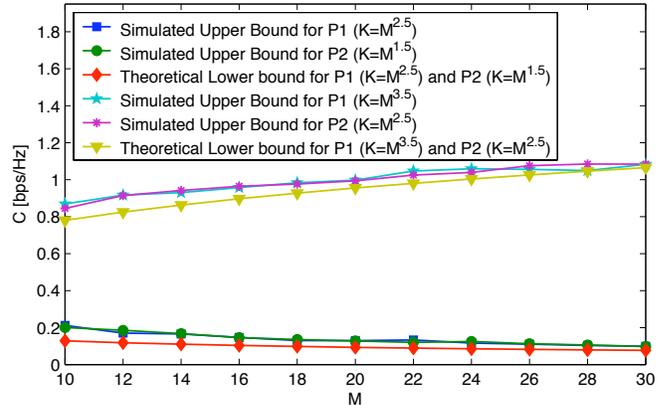


Fig. 3. Upper and lower bounds on C_{P1} and C_{P2} for $K = M^{2.5}$ and $K = M^{3.5}$ in P1 and $K = M^{1.5}$ and $K = M^{2.5}$ in P2.

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