Space-Frequency Codes for Broadband Fading Channels

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Abstract — **Recently, the use of space-frequency coding in Orthogonal Frequency Division Multiplexing (OFDM)-based broadband multi-antenna systems has been proposed. The design criteria for space-frequency codes derived in [1] are vastly different from the well-known design criteria for space-time codes. In this paper, we provide an explicit construction of a class of space-frequency codes which achieve full spatial and frequency diversity.**

I. Summary

Most of the previous work on space-time coding [2] has been restricted to narrowband channels where spatial diversity only is available. Recently, the use of multiple antennas in broadband fading channels in combination with *Orthogonal Frequency Division Multiplexing* (OFDM) has been proposed [3, 4, 1]. Considering a strategy which consists of coding across antennas and OFDM tones and is therefore called *space-frequency coding*, [1] derives the design criteria for space-frequency codes and shows that these criteria are vastly different from the well-known design criteria for space-time codes in narrowband slow or fast fading channels. Employing known space-time codes as space-frequency codes (by coding across space and frequency rather than across space and time) in general provides spatial diversity but fails to exploit the additionally available frequency diversity [1]. Therefore, the OFDM-broadband case calls for new code designs.

Design Criteria. We assume a multi-antenna OFDM system with M_T transmit and M_R receive antennas and N OFDM tones, and organize the transmitted data symbols (taken from a finite complex constellation) into frequency vectors $\mathbf{c}_k = [c_k^{(0)}\ c_k^{(1)}\ \ldots\ c_k^{(M_T-1)}]^T$ $(k = 0, 1, \ldots, N-1)$ with $c_k^{(i)}$ denoting the data symbol transmitted from the i-th antenna on the k-th tone. Let $\mathbf{C} = [\mathbf{c}_0\ \mathbf{c}_1\ \ldots\ \mathbf{c}_{N-1}]$ and $\mathbf{E} = [\mathbf{e}_0\ \mathbf{e}_1\ \ldots\ \mathbf{e}_{N-1}]$ be two different space-frequency codewords of size $M_T \times N$ and assume that $\mathbf{C}$ was transmitted. For a Rayleigh fading channel with uniform power delay profile, L uncorrelated equi-spaced taps and uncorrelated spatial fading, an upper bound on the expected pairwise error probability (averaged over all channel realizations) was derived in [1] as[2]

$$P(\mathbf{C} \to \mathbf{E}) \leq \prod_{i=0}^{rank(\mathbf{S})-1} \left(1 + \lambda_i(\mathbf{S})\frac{\rho}{4}\right)^{-M_R}, \qquad (1)$$

where ρ stands for the signal-to-noise ratio, $\mathbf{S} = \mathbf{G}(\mathbf{C},\mathbf{E})\mathbf{G}^H(\mathbf{C},\mathbf{E})$ with the $N \times M_T L$ matrix

$$\mathbf{G}(\mathbf{C},\mathbf{E}) = \left[(\mathbf{C}-\mathbf{E})^T \quad \mathbf{D}(\mathbf{C}-\mathbf{E})^T \ \ldots\ \mathbf{D}^{L-1}(\mathbf{C}-\mathbf{E})^T\right]$$

and $\mathbf{D} = \mathrm{diag}\{e^{-j\frac{2\pi}{N}k}\}_{k=0}^{N-1}$.

Code Design. In order to simplify the presentation, we consider the case $M_T = 2, M_R = 1$ and assume a two-tap channel (i.e. $L = 2$). The general case is treated in [5]. The maximum achievable diversity order in the present example is 4. In the following, let $\mathbf{F} = [\mathbf{f}_0\ \mathbf{f}_1\ \ldots\ \mathbf{f}_{N-1}]$ denote the $N \times N$ FFT-matrix with entries $[\mathbf{F}]_{m,n} = \frac{1}{\sqrt{N}}e^{-j2\pi\frac{mn}{N}}$ $(m, n = 0, 1, \ldots, N-1)$. Note that $\mathbf{F}$ is unitary, i.e., $\mathbf{F}\mathbf{F}^H = \mathbf{F}^H\mathbf{F} = \mathbf{I}_N$ and hence its columns are orthogonal to each other. Note furthermore that left-multiplying $\mathbf{F}$ by the diagonal matrix $\mathbf{D}$ yields a cyclic shift of the columns of $\mathbf{F}$ by one position to the right. We construct the $N \times 2$ space-frequency codewords $\mathbf{C}^T$ according to

$$\mathbf{C}^T = [\mathbf{F}_1\mathbf{c} \quad \mathbf{F}_2\mathbf{c}], \qquad (2)$$

where $\mathbf{c}$ is a $K \times 1$ vector of data symbols taken from a finite (complex) constellation, and $\mathbf{F}_1$ and $\mathbf{F}_2$ are $N \times K$ matrices containing columns of the FFT-matrix $\mathbf{F}$. These columns have to be chosen such that the following *design criteria* are satisfied:

- $\mathbf{F}_1^H\mathbf{F}_1 = \mathbf{F}_2^H\mathbf{F}_2 = \mathbf{I}_K$. This condition can be met by choosing the columns in $\mathbf{F}_1$ and $\mathbf{F}_2$ such that no two columns in $\mathbf{F}_1$ and no two columns in $\mathbf{F}_2$ are equal.
- $\mathbf{F}_2^H\mathbf{F}_1 = \mathbf{0}_K$. This condition can be satisfied by choosing different columns for $\mathbf{F}_1$ and $\mathbf{F}_2$.
- $\mathbf{F}_1^H\mathbf{D}^H\mathbf{F}_1 = \mathbf{F}_2^H\mathbf{D}^H\mathbf{F}_2 = \mathbf{0}_K$. This condition can be met by choosing the columns in $\mathbf{F}_1$ and $\mathbf{F}_2$ such that no two neighboring[3] columns in $\mathbf{F}$ are assigned to $\mathbf{F}_1$ or $\mathbf{F}_2$.
- $\mathbf{F}_1^H\mathbf{D}^H\mathbf{F}_2 = \mathbf{F}_2^H\mathbf{D}^H\mathbf{F}_1 = \mathbf{0}_K$. This condition can be satisfied by choosing the columns in $\mathbf{F}_1$ and $\mathbf{F}_2$ such that no two neighboring columns in $\mathbf{F}$ are assigned to $\mathbf{F}_1$ and $\mathbf{F}_2$.

An example for $N = 8$ and $K = 2$ is given by $\mathbf{F}_1 = [\mathbf{f}_0\ \mathbf{f}_2]$ and $\mathbf{F}_2 = [\mathbf{f}_4\ \mathbf{f}_6]$, respectively. It can be shown that if the conditions above are satisfied, we have

$$\mathbf{G}^H(\mathbf{C},\mathbf{E})\mathbf{G}(\mathbf{C},\mathbf{E}) = \|\mathbf{c}-\mathbf{e}\|^2\mathbf{I}_4, \qquad (3)$$

which implies that the rank of $\mathbf{S}$ is 4 and the nonzero eigenvalues of $\mathbf{S}$ satisfy $\lambda_i(\mathbf{S}) = \|\mathbf{c}-\mathbf{e}\|^2$ $(i = 0, 1, 2, 3)$. This proves that our space-frequency code achieves full (in this case 4-th order) diversity. The four design criteria above impose limits on the code rate the details of which are discussed in [5].

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[2] The superscripts ^{T} and ^{H} stand for transpose and conjugate transpose, respectively.

[3] Note that $\mathbf{f}_0$ and $\mathbf{f}_{N-1}$ are considered neighboring columns.

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