BLIND HIGH-RESOLUTION UPLINK SYNCHRONIZATION OF OFDM-BASED MULTIPLE ACCESS SCHEMES

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Abstract—One of the major problems in OFDM-based multiple access schemes is uplink synchronization. In this paper, we propose an algorithm for the *blind estimation* of multi-user OFDM synchronization parameters. Our method assumes that all users employ the same pulse shaping filter and uses second-order statistics only. The proposed approach provides high-resolution estimates in the sense of being able to resolve closely spaced time-frequency offsets. Finally, we provide simulation results demonstrating the performance of our algorithm.

1. MOTIVATION AND INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1]-[6] is part of the European digital audio broadcasting (DAB) standard. It is under investigation for digital video broadcasting (DVB) [7], and it is employed for high-bit-rate digital subscriber services on twisted-pair channels such as in the asymmetric digital subscriber line (ADSL) standard. To a lesser extent, OFDM has also been considered for mobile radio applications [3, 6, 8, 9, 10].

One of the major problems in OFDM-based multiple access schemes [6, 8, 9, 10] is synchronization on the system uplink (mobile to base station). On the downlink (base station to mobile) multi-user synchronization is easy to achieve, because the signals corresponding to the different mobiles all originate from the same source (base station). Methods for downlink synchronization have been proposed for example in [11]-[13]. If one of the mobiles is not synchronized on the downlink, the other users will not suffer from this failure. On the uplink, however, if one of the mobiles is not synchronized there will be interference between the users since the incoming signals are multiplexed in the base station. In [14] a random access protocol for uplink time-synchronization of multiple access OFDM has been proposed. A pilot symbol based approach for multi-user OFDM uplink synchronization has been provided in [8]. The use of pilot symbols or training

data, however, reduces the data rate.

In this paper, we introduce an algorithm for the *blind high-resolution* estimation of time-frequency offsets in the uplink of multiple access OFDM systems. Unlike the methods proposed in [14, 8], our approach does not make use of training sequences or pilot symbols. We assume that all users employ the same pulse shaping filter, which is known to the receiver (base station). The novel algorithm uses second-order statistics only, and provides high-resolution estimates in the sense of being able to resolve closely spaced time-frequency offsets.

The paper is organized as follows. Section 2 briefly describes the OFDM multiple access scheme and provides our assumptions and the problem statement. Section 3 presents the estimation algorithm. Section 4 provides simulation results, and Section 5 concludes the paper.

2. MULTIPLE ACCESS OFDM

The OFDM multiple access scheme considered in this paper has first been proposed in [8, 10]. We assume that the total available bandwidth is divided into N subcarriers (which are allowed to overlap in frequency) and each of the K users is assigned one or several of the subcarriers during a specified transmission time. After this transmission time the frequency assignment can be changed. In the following we make two simplifying assumptions:

- each user is assigned one subcarrier, i.e., $K = N$.

- the frequency assignment is kept constant over time.

The equivalent baseband signal of the i-th user is given by

$$s_i[n] = \sum_{l=-\infty}^{\infty} c_{i,l}\, g[n - lM] e^{j2\pi \frac{i}{N}(n-lM)}, \; i \in [0, N-1],$$

$$(1)$$

where $c_{i,l}$ denotes the i-th user's data symbols, $g[n]$ is the pulse shaping filter, and M is the symbol duration. Note that we are assuming that all users employ the same pulse shaping filter. For $M > N$, the OFDM system is said to employ a time-frequency guard region [15, 16]. The use of time-frequency guard regions reduces spectral efficiency [6]. However, the resulting advantages such as increased dispersion robustness [6] and the possibility to perform blind equalization [17] and blind synchronization [13] generally motivate their use. In the absence of distortions, orthogonality between the users holds if

$$A^{(g,g)}\left[lM, \frac{k}{N}\right) = \delta[l]\delta[k], \quad l \in \mathbb{Z}, \, k \in [0, N-1],$$

where $A^{(g,g)}[k,\theta] = \sum_{n=-\infty}^{\infty} g[n]g[n-k]e^{-j2\pi n\theta}$ denotes the auto-ambiguity function of $g[n]$ [18].

Assumptions. Throughout this paper the influence of channel dispersions will be neglected. The uncertainty in the arrival times of the individual signals $s_i[n]$ ($i = 0, 1, ..., N-1$) will be modeled as a time-shift, and the unknown carrier frequency offsets are accounted for by frequency shifts. The signal received at the base station is therefore given by

$$r[n] = \sum_{i=0}^{N-1} \sqrt{P_i}e^{j2\pi\theta_i n}s_i[n-n_i] + \rho[n] \qquad (2)$$

with the power P_i, the time-delays $n_i \in \mathbb{Z}$, and the frequency offsets θ_i, where $|\theta_i| < \frac{1}{2N}$ ($i = 0, 1, ..., N - 1$). Furthermore, $\rho[n]$ is wide-sense-stationary noise which is assumed to be independent of the data symbols. The correlation function of the noise process is given by[1] $c_\rho[\tau] = \mathcal{E}\{\rho[n]\rho^*[n-\tau]\}$. The data symbols are taken from a finite complex alphabet and satisfy $\mathcal{E}\{c_{i,l}c^*_{i',l'}\} = \sigma_c^2\delta[i - i']\delta[l - l']$. We furthermore assume that the base station knows the pulse shaping filter $g[n]$ and the variance σ_c^2. This knowledge constitutes the basis for the estimation algorithms discussed in the paper. Finally, we assume that time-frequency guard regions are employed, i.e., $M > N$.

Problem statement. In order to synchronize the N different mobiles it is necessary to estimate the parameters n_i and θ_i from the received signal $r[n]$. In the following, we are aiming at computationally efficient estimates that do not require knowledge of the distributions of $x[n]$ and $\rho[n]$, and apply to the blind or nondata-aided scenario.

3. BLIND ESTIMATION OF SYNCHRONIZATION PARAMETERS

The correlation function of a nonstationary stochastic process is defined as $c_r[n,\tau] = \mathcal{E}\{r[n]r^*[n-\tau]\}$ with τ being an integer lag parameter[2]. The signal $r[n]$ is

said to be second-order cyclostationary (CS) with period M if $c_r[n + M, \tau] = c_r[n, \tau]$ [19]. Using (1) and (2) it can be shown that

$$c_r[n,\tau] = \sigma_c^2 \sum_{i=0}^{N-1} P_i\,e^{j2\pi\epsilon_i\tau} \sum_{l=-\infty}^{\infty} g[n - n_i - lM]$$
$$g[n - n_i - lM - \tau] + c_\rho[\tau], \qquad (3)$$

where $\epsilon_i = \theta_i + \frac{i}{N}$. From (3) it follows that $c_r[n + M, \tau] = c_r[n, \tau]$ for every τ, which implies cyclostationarity of $r[n]$. There are several possibilities of evoking cyclostationarity in a multi-user OFDM signal, for example by using time-frequency guard regions (i.e. $M > N$) or by employing pulse shaping.

For a fixed lag τ, the correlation function $c_r[n, \tau]$ can be expanded into a Fourier series with respect to n with the Fourier series coefficients $C_r[k, \tau] = \frac{1}{M}\sum_{n=0}^{M-1} c_r[n, \tau]e^{-j\frac{2\pi}{M}kn}$ given by

$$C_r[k,\tau] = \frac{\sigma_c^2}{M}A^{(g,g)}\left[\tau, \frac{k}{M}\right)$$
$$\sum_{i=0}^{N-1} P_i\,e^{j2\pi\epsilon_i\tau}e^{-j\frac{2\pi}{M}kn_i} + c_\rho[\tau]\delta_M[k],$$

where $\delta_M[k] = \sum_{s=-\infty}^{\infty}\delta[k - sM]$. Since $g[n]$ and σ_c^2 are known to the base station their influence can be eliminated by defining

$$C[k,\tau] = \begin{cases} \dfrac{C_r[k,\tau]}{\frac{\sigma_c^2}{M}A^{(g,g)}[\tau, \frac{k}{M})}, & [k,\tau] \in \mathcal{I} \\ 0, & \text{else,} \end{cases} \qquad (4)$$

where $\mathcal{I} := \{[k,\tau]\,|\,A^{(g,g)}[\tau, \frac{k}{M}) \neq 0\}$. We thus have

$$C[k,\tau] = \sum_{i=0}^{N-1} P_i\,e^{j2\pi\epsilon_i\tau}e^{-j\frac{2\pi}{M}kn_i} + \frac{c_\rho[\tau]}{\frac{\sigma_c^2}{M}A^{(g,g)}[\tau, \frac{k}{M})}\delta_M[k]$$
$$(5)$$

for $[k, \tau] \in \mathcal{I}$. Since $C[k, \tau]$ is known only on $\mathcal{I}$, the pulse shaping filter $g[n]$ should be designed such that $A^{(g,g)}[\tau, \frac{k}{M})$ does not vanish on the points of interest.

Note that $C[k, \tau]$ can be interpreted as the 2-D correlation function of a superposition of N 2-D statistically independent complex sinusoids (CISOIDS) in noise. The problem of estimating the OFDM multi-user synchronization parameters n_i and θ_i has therefore been reduced to the estimation of the 2-D CISOID frequencies $\frac{n_i}{M}$ and ϵ_i. Now, the estimation of the synchronization parameter pairs (n_i, θ_i) can be decoupled into two independent 1-D problems. The resulting two decoupled parameter sets have to be combined to correct parameter pairs [20], which may be achieved by minimizing an appropriate cost function [21]. Employing 2-D Unitary ESPRIT [22, 23] (a closed-form 2-D angle estimation algorithm) automatic pairing is achieved. In the following we will not elaborate on the pairing

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problem. Instead, we shall describe 1-D algorithms for estimating the time-delays n_i and the frequeny offsets θ_i, separately. The resulting parameter sets can be paired using one of the methods suggested in [21, 23].

Estimation of the time-delays. For $\tau = 0$, the 2-D correlation function $\mathcal{C}[k, \tau]$ reduces to the 1-D correlation function of a noisy superposition of N statistically independent 1-D CISOIDS with frequencies $\frac{n_i}{M}$ and amplitudes $\sqrt{P_i}$, i.e.,

$$\mathcal{C}[k, 0] = \sum_{i=0}^{N-1} P_i\, e^{-j\frac{2\pi}{M}kn_i} + \frac{c_\rho[0]}{\frac{\sigma_c^2}{M} A^{(g,g)}\left[0, \frac{k}{M}\right]}\delta_M[k].$$

Note that here the 1-D correlation function $\mathcal{C}[k, 0]$ is M-periodic in k. Since we assumed that our system employs time-frequency guard regions, we have $M > N$. Therefore, high-resolution algorithms such as MUSIC or ESPRIT can be employed to estimate the frequencies $\frac{n_i}{M}$ and hence the time-delays n_i.

Estimation of frequency offsets. For $k = 0$, the 2-D correlation function $\mathcal{C}[k, \tau]$ can be interpreted as the 1-D correlation function of a noisy superposition of N statistically independent 1-D CISOIDS with frequencies ϵ_i and amplitudes $\sqrt{P_i}$, i.e.,

$$\mathcal{C}[0, \tau] = \sum_{i=0}^{N-1} P_i\, e^{j2\pi\epsilon_i\tau} + \frac{c_\rho[\tau]}{\frac{\sigma_c^2}{M} A^{(g,g)}[\tau, 0]}.$$

Now, from $\mathcal{C}[0, \tau]$ with $0 \leq \tau \leq \tau_{max}$, where $\tau_{max} \geq N$, the frequency offsets $\theta_i = \epsilon_i - \frac{i}{N}$ ($i = 0, 1, ..., N-1$) can be estimated using a one-dimensional high-resolution frequency estimation algorithm.

If paired estimates of the synchronization parameters n_i and θ_i are available, the power levels P_i can be estimated by solving a linear system of equations.

Estimation of the cyclic statistics. The cyclic statistics $\mathcal{C}_r[k, \tau]$ can be estimated from a finite data record $\{r[n]\}_{n=0}^{L-1}$ of length L according to

$$\hat{\mathcal{C}}_r[k, \tau] = \frac{1}{L}\sum_{n=0}^{L-1} r[n]r^*[n - \tau]e^{-j\frac{2\pi}{M}kn}.$$

An estimate of $\mathcal{C}[k, \tau]$ is then obtained as

$$\hat{\mathcal{C}}[k, \tau] = \frac{\hat{\mathcal{C}}_r[k, \tau]}{\frac{\sigma_c^2}{M} A^{(g,g)}\left[\tau, \frac{k}{M}\right]}.$$

4. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the performance of the proposed estimators. We simulated an OFDM multiple access scheme with $K = N = 8$ users, symbol length $M = 16$, and pulse shaping filter length 96. The data symbols were i.i.d. 4-PSK symbols with $\sigma_c^2 = 4$. The signal-to-noise-ratio

(SNR) was defined as $\mathrm{SNR} = 10\log_{10}\left(\sigma_c^2/\sigma_q^2\right)$, where σ_q^2 is the variance of the white noise process $\rho[n]$. All results were obtained by averaging over $I = 200$ independent Monte Carlo trials. Each realization consisted of 128 data symbols per user. The estimator performance was measured in terms of average bias and mean square error (MSE) defined as $\frac{1}{IN}\sum_{l=0}^{N-1}\left|\sum_{i=0}^{I-1}\left[\frac{\hat{n}_l^{(i)}}{M} - \frac{n_l}{M}\right]\right|$ and $\frac{1}{INM^2}\sum_{i=0}^{I-1}\left\|\hat{\mathbf{n}}^{(i)} - \mathbf{n}\right\|^2$, respectively, for the time-delay estimator, and $\frac{1}{IN}\sum_{l=0}^{N-1}\left|\sum_{i=0}^{I-1}\left[\hat{\theta}_l^{(i)} - \theta_l\right]\right|$ and $\frac{1}{IN}\sum_{i=0}^{I-1}\left\|\hat{\boldsymbol{\theta}}^{(i)} - \boldsymbol{\theta}\right\|^2$, respectively, for the frequency offset estimator. The time-frequency offsets and the user power levels were chosen as

$$\mathbf{n} = [0\ 4\ 6\ 7\ 8\ 10\ 11\ 15]$$
$$\boldsymbol{\theta} = [0.04\ -0.03\ 0.06\ -0.04\ 0.01\ 0\ -0.04\ 0.02]$$
$$\mathbf{p} = [1.77\ 2.25\ 1.00\ 1.44\ 4.00\ 1.21\ 1.00\ 3.24].$$

Furthermore, in both simulation examples the 1-D ESPRIT algorithm [24] has been used for estimating the synchronization parameters.

Simulation Example 1. In the first simulation example we consider the estimation of frequency offsets. The estimates of the cyclic statistics $\hat{\mathcal{C}}_r[k, \tau]$ were obtained using the entire data record. Fig. 1 shows the average bias and the MSE of the frequency offset estimator as a function of the SNR. The results were obtained by applying the ESPRIT algorithm to $\mathcal{C}[0, \tau]$ with $0 \leq \tau \leq 11$. We can observe that the estimator performance is more or less independent of the SNR.
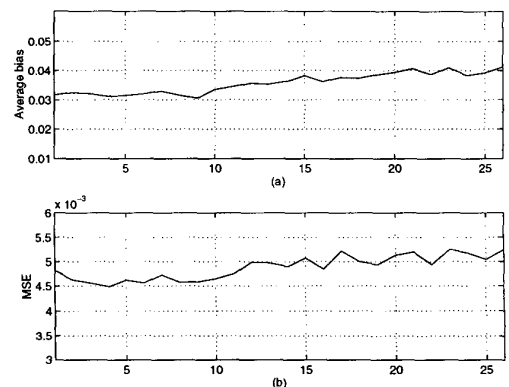

Fig. 1: (a) Average Bias and (b) MSE of the frequency offset estimator as a function of the SNR in dB.

Simulation Example 2. In the second simulation example, we investigate the effect of the length L of the data record used for estimating the cyclic statistics $\hat{\mathcal{C}}_r[k, \tau]$ on the performance of the time-delay estimator. For $\mathrm{SNR} = 9\mathrm{dB}$, Fig. 2 shows the average bias

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and the MSE of the time-delay estimator as a function of the data record length. (Note that in Fig. 2 the length of the data record has been specified in OFDM symbols. The actual length of the data record is therefore obtained by multiplying the number of symbols by 16). The estimates were obtained from $C[k, 0]$ with $0 \leq k \leq 15$. We can see that the performance of the estimator improves with increasing data record length.
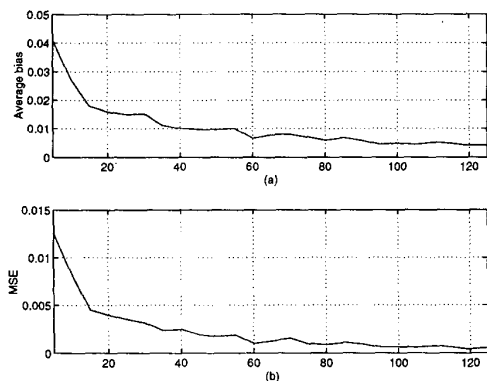

Fig. 2: (a) Average bias and (b) MSE of the time-delay estimator as a function of the data record length (specified in OFDM symbols).

5. CONCLUSION

We introduced a high-resolution method for the blind estimation of OFDM multi-user synchronization parameters. Our approach assumes that all users employ the same pulse shaping filter and relies on second-order statistics of the signal received at the base station. We provided algorithms for the separate estimation of time-delays and frequency offsets. The novel method is computationally efficient and exhibits low noise sensitivity. Simulation results were provided to demonstrate the performance of our algorithm.

6. REFERENCES

[1] R. W. Chang, "Synthesis of band-limited orthogonal signals for multi-channel data transmission," *Bell Syst. Tech. J.*, vol. 45, pp. 1775–1796, Dec. 1966.

[2] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proc. IEEE ICASSP-80*, (Denver, CO), pp. 964–967, 1980.

[3] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Comm.*, vol. 33, pp. 665–675, July 1985.

[4] J. S. Chow, J. C. Tu, and J. M. Cioffi, "A discrete multitone transceiver system for HDSL applications," *IEEE J. Sel. Areas Comm.*, vol. 9, pp. 895–908, Aug. 1991.

[5] B. LeFloch, M. Alard, and C. Berrou, "Coded orthogonal frequency division multiplex," *Proc. of IEEE*, vol. 83, pp. 982–996, June 1995.

[6] R. Haas, *Application des transmissions à porteuses multiples aux communications radio mobiles.* PhD thesis, Ecole Nationale Supérieure des Télécommunications Paris, Paris, France, Jan. 1996.

[7] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," *IEEE Communications Magazine*, pp. 100–109, Feb. 1995.

[8] L. Wei and C. Schlegel, "Synchronization requirements for multi-user OFDM on satellite mobile and two-path Rayleigh fading channels," *IEEE Trans. Comm.*, vol. 43, pp. 887–895, Feb. 1995.

[9] E. F. Casas and C. Leung, "OFDM for data communication over mobile radio FM channels – Part I: Analysis and experimental results," *IEEE Trans. Comm.*, vol. 39, pp. 783–793, May 1991.

[10] M. Wahlqvist, C. Östberg, J. J. van de Beek, O. Edfors, and P. O. Börjesson, "A conceptual study of OFDM-based multiple access schemes: Part 1 – Air interface requirements," Tech. Rep. 117/96, ETSI STC SMG2 meeting no. 18, Helsinki, Finland, May 1996.

[11] P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Trans. Comm.*, vol. 42, pp. 2908–2914, Oct. 1994.

[12] J.-J. van de Beek, M. Sandell, and P.-O. Börjesson, "ML estimation of timing and frequency offset in OFDM systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 1800–1805, July 1997.

[13] H. Bölcskei, "Blind estimation of symbol timing and carrier frequency offset in pulse shaping OFDM systems," in *Proc. IEEE ICASSP-99*, (Phoenix, Arizona), March 1999. to appear.

[14] M. Wahlqvist, R. Larsson, and C. Östberg, "Time synchronization in the uplink of an OFDM system," in *IEEE VTC96*, (Atlanta (GA)), pp. 1569–1573, Feb. 1996.

[15] R. Haas and J. C. Belfiore, "A time-frequency well-localized pulse for multiple carrier transmission," *Wireless Personal Communications*, vol. 5, pp. 1–18, 1997.

[16] W. Kozek and A. F. Molisch, "Robust and efficient multi-carrier communication by nonorthogonal Weyl-Heisenberg systems," *IEEE J. Sel. Areas Comm.*, vol. 16, pp. 1579–1589, Oct. 1998.

[17] R. W. Heath and G. B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems," *IEEE Trans. Sig. Proc.*, Aug. 1997. submitted.

[18] R. Tolimieri and M. An, *Time-Frequency Representations.* Boston: Birkhäuser, 1998.

[19] W. A. Gardner, ed., *Cyclostationarity in communications and signal processing.* Piscataway (NJ): IEEE Press, 1995.

[20] M. D. Zoltowski and D. Stavrinides, "Sensor array signal processing via a Procrustes rotations based eigenanalysis of the ESPRIT data pencil," *IEEE Trans. Signal Processing*, vol. 37, pp. 832–861, June 1989.

[21] C. R. Rao, L. C. Zhao, and B. Zhou, "A novel algorithm for 2-dimensional frequency estimation," in *Asilomar Conf. Signals, Systems, Computers*, (Pacific Grove, CA), pp. 199–202, Nov. 1993.

[22] M. D. Zoltowski, M. Haardt, and C. P. Mathews, "Closed-form 2D angle estimation with rectangular arrays in element space or beamspace via Unitary ESPRIT," *IEEE Trans. Signal Processing*, vol. 44, pp. 316–328, Feb. 1996.

[23] M. Haardt, *Efficient one-, two-, and multidimensional high-resolution array signal processing.* PhD thesis, Technische Universität München, Munich, Germany, Nov. 1996.

[24] R. Roy, A. Paulraj, and T. Kailath, "ESPRIT - A subspace rotation approach to estimation of parameters of cisoids in noise," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 1340–1342, Oct. 1986.