

# Transmit Optimization for Spatial Multiplexing in the Presence of Spatial Fading Correlation

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**Abstract**— Multiple-input multiple-output (MIMO) wireless systems employ spatial multiplexing to increase data rate. The performance of spatial multiplexing is highly dependent on channel statistics which in turn depend on antenna spacing and richness of scattering. It has been shown in [10] that the presence of transmit correlation can have a detrimental effect on the performance of multi-antenna signaling techniques. In this paper, we present a novel scheme to (partly) mitigate the performance loss of spatial multiplexing in the presence of highly correlated fading at the transmitter. The adaptation to be performed at the transmitter is a form of power allocation and/or relative phase adjustment between the different symbol streams to be multiplexed. We consider the cases of dual-polarized as well as uni-polarized antennas and derive estimates of the uncoded average symbol error rate as a function of channel statistics, power allocation and phase adjustment. We then optimize power allocation and phase adjustment and demonstrate that this form of preprocessing can yield SNR gains of up to 4 dB over the case where no precoding is employed.

## I. INTRODUCTION

Multiple antennas at the transmitter and receiver in a wireless system can increase capacity dramatically by the use of a technique called spatial multiplexing [1]-[5]. However, the resulting gain is heavily dependent on channel statistics such as transmit and receive antenna correlation which are in turn related to antenna spacing and richness of scattering. In general, antenna spacings of several wavelengths are required to achieve sufficient decorrelation. In [10] it has been shown that the presence of transmit correlation due to lack of scattering and/or insufficient transmit antenna spacing can have a detrimental impact on multi-antenna signaling techniques. Depending on the channel statistics and the symbol constellation used there are data vectors which tend to “excite” the null space of the channel resulting

in large error rates.

**Contributions.** In this paper, we propose a novel technique to alleviate the impact of transmit correlation. Our approach comprises of power allocation (subject to a total power constraint) and relative phase adjustment between the symbol streams to be multiplexed at the transmitter. The optimal settings for power allocation and phase adjustment are based on *knowledge of the channel statistics and symbol constellation* and can be determined numerically. Due to limitations on the dynamic range of the power amplifiers at the transmitter, it may not be possible in practice to allocate unequal amounts of power to the symbol streams. In such a situation, we show that improved performance can be achieved by simply adjusting the relative phase between the symbol streams. Throughout the paper we assume that the channel statistics are changing slowly, which is certainly a valid assumption in the fixed wireless case. Space to support multiple antennas at the base-station or subscriber unit is expensive or possibly unavailable. Recently, the use of dual-polarized antennas [6], [7] has been proposed for spatial multiplexing as a cost-effective way of realizing significant multiplexing gain, where two spatially separated antennas are replaced by a single antenna structure with orthogonally polarized elements [8], [9]. In this paper, we consider the general case of spatial multiplexing systems employing uni-polarized or dual-polarized antennas.

Using an estimate of the average symbol error rate as a function of power allocation and phase setting, we perform a numerical search to find the optimal (in the sense of minimizing the average symbol error rate estimate) power allocation and phase setting. We then show that the adaptation using the resulting settings can yield up to an order of magnitude improvement in uncoded average symbol error rate or for a fixed average symbol error rate an SNR gain of up to 4 dB. These gains will be demonstrated by simulations.

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**Organization of the paper.** The organization of this paper is as follows. Section II introduces the channel model. In Section III we present our technique for average symbol error rate reduction. Section IV provides simulation results, and Section V contains our conclusions.

## II. THE CHANNEL MODEL

We consider a system with 2 transmit and 2 receive antennas or 1 dual-polarized transmit antenna and 1 dual-polarized receive antenna with the signals being launched and received on orthogonal polarizations. We assume flat fading over the frequency band of interest. In both cases the underlying channel is a  $2 \times 2$  link with the relationship between the transmitted signal and the received signal given by

$$\mathbf{r} = \sqrt{E_s} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (1)$$

where<sup>1</sup>

- $\mathbf{x} = [x_0 \ x_1]^T$  is the  $2 \times 1$  transmitted signal vector (also called codevector) whose elements are drawn from a finite (possibly complex) scalar constellation such that the average energy of the constellation is 1
- $\mathbf{H} = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix}$  is the  $2 \times 2$  channel matrix or the polarization matrix in the case of dual-polarized antennas
- $\mathbf{n}$  is the  $2 \times 1$  complex-valued Gaussian noise vector with  $\mathcal{E}\{\mathbf{n}\mathbf{n}^\dagger\} = \sigma_n^2 \mathbf{I}_2$  and  $\mathcal{E}\{\mathbf{n}\} = \mathbf{0}$
- $E_s$  is the energy available at each of the transmit antennas if equal power is allocated to the two symbol streams
- $\mathbf{r} = [r_0 \ r_1]^T$  is the  $2 \times 1$  received signal vector.

Note that in the dual-polarized case, the transmitter launches the signals  $x_0$  and  $x_1$  on two orthogonal polarizations and the receiver captures the signals  $r_0$  and  $r_1$  on the corresponding polarizations.

We assume that the channel is purely Rayleigh fading, i.e., the matrix  $\mathbf{H}$  consists of (in general) correlated zero-mean complex Gaussian random variables. The correlations between the elements of the matrix and the variances of the elements depend on the propagation conditions and the choice of polarizations, respectively.

Throughout the paper we assume that

$$\mathcal{E}\{|h_{0,0}|^2\} = \mathcal{E}\{|h_{1,1}|^2\} = 1 \quad (2)$$

$$\mathcal{E}\{|h_{0,1}|^2\} = \mathcal{E}\{|h_{1,0}|^2\} = \alpha, \quad (3)$$

<sup>1</sup>The superscripts  $T$  and  $\dagger$  stand for transpose and conjugate transpose, respectively.  $\mathcal{E}$  stands for the expectation operator.  $\mathbf{I}_m$  is the  $m \times m$  identity matrix.

where  $\alpha = 1$  in the case of 2 uni-polarized transmit and 2 uni-polarized receive antennas. In the case of a  $1 \times 1$  link with dual-polarized antennas,  $\alpha$  depends on the cross-polarization discrimination (XPD), i.e., the antenna's ability to separate orthogonal polarizations [11]. Good XPD yields small  $\alpha$  and vice versa. Furthermore, we define the following correlation coefficients<sup>2</sup>

$$t = \frac{\mathcal{E}\{h_{0,0}h_{0,1}^*\}}{\sqrt{\alpha}} = \frac{\mathcal{E}\{h_{1,0}h_{1,1}^*\}}{\sqrt{\alpha}} \quad (4)$$

$$r = \frac{\mathcal{E}\{h_{0,0}h_{1,0}^*\}}{\sqrt{\alpha}} = \frac{\mathcal{E}\{h_{0,1}h_{1,1}^*\}}{\sqrt{\alpha}}, \quad (5)$$

where  $t$  is the transmit correlation and  $r$  is the receive correlation. The assumption that the transmit correlation is the same for both polarizations or antennas at the receiver and vice versa is a simplifying assumption. It is particularly true in the dual-polarized case if we launch and receive on orthogonal polarizations at  $\pm 45^\circ$  with respect to the earth. For the sake of simplicity, throughout the paper we assume that  $\mathcal{E}\{h_{0,0}h_{1,1}^*\} = \mathcal{E}\{h_{1,0}h_{0,1}^*\} = 0$ .

## III. NOVEL SCHEME FOR TRANSMIT OPTIMIZATION

Throughout the paper, we assume that the statistics of the channel are known, the instantaneous channel realization is unknown at the transmitter and perfectly known at the receiver, and maximum-likelihood (ML) decoding is performed. The receiver computes the estimate of the transmitted signal vector according to

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{r} - \sqrt{E_s} \mathbf{H} \mathbf{x}\|^2, \quad (6)$$

where the minimization is performed over the set of all possible  $2 \times 1$  codevectors  $\mathbf{x}$ .

Our scheme for transmit preprocessing can be written as the premultiplication of the symbol vector to be transmitted by a diagonal matrix according to

$$\mathbf{x}' = \begin{bmatrix} \sqrt{2\gamma}e^{j\theta} & 0 \\ 0 & \sqrt{2(1-\gamma)} \end{bmatrix} \mathbf{x}, \quad (7)$$

where

- $\mathbf{x}$  is the codevector to be transmitted
- $\gamma$  and  $1-\gamma$  represent fractions of the total power allotted to the individual scalar symbol streams,  $0 < \gamma < 1$
- $\theta$  is the phase shift applied to one of the streams
- $\mathbf{x}'$  is the preprocessed transmitted signal vector.

The absence of precoding is equivalent to setting  $\gamma = 0.5$  and  $\theta = 0$  with the precoding matrix in place. To

<sup>2</sup>The superscript  $*$  stands for complex conjugate.

decode the transmitted signal, the receiver must know precisely the precoding matrix applied at the transmitter. ML decoding at the receiver can then be performed as before by simply considering the channel matrix to be the product of the original channel matrix  $\mathbf{H}$  and the diagonal precoding matrix.

Given that codevector  $\mathbf{f}$  was transmitted, we shall next calculate an upper bound on the probability (averaged over the channel) that the receiver confuses  $\mathbf{f}$  for another codevector, say  $\mathbf{g}$ , following the derivation in [8]. The upper bound on the probability of error between two codewords is a function of the codevector difference (also known as an error event),  $\mathbf{f} - \mathbf{g} = \mathbf{e} = [e_0 \ e_1]^T$ , and the channel statistics. We denote it by

$$\Lambda_{\gamma,\theta}(\mathbf{e}) = \frac{1}{1 + \lambda_{1,\gamma,\theta}(\mathbf{e}) \frac{E_s}{4\sigma_n^2}} \frac{1}{1 + \lambda_{2,\gamma,\theta}(\mathbf{e}) \frac{E_s}{4\sigma_n^2}},$$

where  $\lambda_{1,\gamma,\theta}(\mathbf{e})$  and  $\lambda_{2,\gamma,\theta}(\mathbf{e})$  are given by

$$\lambda_{1,2,\gamma,\theta}(\mathbf{e}) = \frac{a_{\gamma,\theta} + d_{\gamma,\theta}}{2} \pm \frac{\sqrt{(a_{\gamma,\theta} - d_{\gamma,\theta})^2 + 4b_{\gamma,\theta}c_{\gamma,\theta}}}{2}, \quad (8)$$

with<sup>3</sup>

$$a_{\gamma,\theta} = 2(\gamma|e_0|^2 + \alpha(1-\gamma)|e_1|^2) \quad (9)$$

$$+ 2\Re\{t\sqrt{\alpha\gamma(1-\gamma)}e_0e_1^*e^{j\theta}\}$$

$$b_{\gamma,\theta} = 2r\sqrt{\alpha}(\gamma|e_0|^2 + (1-\gamma)|e_1|^2) \quad (10)$$

$$c_{\gamma,\theta} = 2r^*\sqrt{\alpha}(\gamma|e_0|^2 + (1-\gamma)|e_1|^2) \quad (11)$$

$$d_{\gamma,\theta} = 2(\alpha\gamma|e_0|^2 + (1-\gamma)|e_1|^2) \quad (12)$$

$$+ 2\Re\{t\sqrt{\alpha\gamma(1-\gamma)}e_0e_1^*e^{j\theta}\}.$$

For the remainder of the paper, we will assume that the elements of the codevector to be transmitted are drawn from a 4-QAM constellation. In this case  $e_0$  and  $e_1$  can take on any one of the values from the set  $\{0, \pm d_{min}, \pm d_{min}j, \pm d_{min}(1+j), \pm d_{min}(1-j)\}$ , where  $d_{min}$  is the minimum distance of separation of the underlying scalar constellation. Note that  $e_0$  and  $e_1$  cannot be identically zero.

Enumerating all possible pairs of (dissimilar) codevectors  $\mathbf{f}$  and  $\mathbf{g}$  (there are 240 in total), we observe that certain error events  $\mathbf{e}$  are more frequent than others. We define the relative frequency of an error event as

$$\psi(\mathbf{e}) = \frac{w(e_0)w(e_1)}{240}, \quad (13)$$

<sup>3</sup> $\Re\{x\}$  stands for the real part of  $x$ .

where

$$w(x) = \begin{cases} 4, & x = 0 \\ 2, & x = \pm d_{min}, \pm d_{min}j \\ 1, & x = \pm d_{min}(1+j), \pm d_{min}(1-j). \end{cases} \quad (14)$$

Additionally, different error events cause a different number of transmitted symbols to be in error. If both  $e_0$  and  $e_1$  are non-zero, then both  $x_0$  and  $x_1$  are in error. On the other hand, if either  $e_0$  or  $e_1$  is zero then this corresponds to only one symbol error. We define a new function to reflect this

$$s(\mathbf{e}) = \begin{cases} 2, & e_0 \neq 0, e_1 \neq 0 \\ 1, & e_0 = 0 \text{ or } e_1 = 0. \end{cases} \quad (15)$$

An estimate of the average symbol error rate,  $P_e(\gamma, \theta)$ , is now obtained as in [8]

$$P_e(\gamma, \theta) \approx \sum_{\mathbf{e}} \Lambda_{\gamma,\theta}(\mathbf{e}) \psi(\mathbf{e}) s(\mathbf{e}). \quad (16)$$

We note that  $P_e(\gamma, \theta)$  is in general not an upper bound on the symbol error rate. This technique for estimating the average symbol error rate has first been proposed in [8] for spatial multiplexing without precoding using dual-polarized antennas. The scheme we propose seeks to change the contributions of the various error events so that the new contributions in conjunction with the respective relative frequencies and the number of scalar symbols in error result in a lower overall average symbol error rate. The optimal (in the sense of minimizing the symbol error rate estimate) settings for the power allocation ( $\gamma_{opt}$ ) and the phase shift ( $\theta_{opt}$ ) to minimize the uncoded symbol error rate can be found by minimizing the probability of error expression above with respect to the variables  $\gamma$  and  $\theta$

$$(\gamma_{opt}, \theta_{opt}) = \arg \min_{(\gamma, \theta)} P_e(\gamma, \theta). \quad (17)$$

The function  $P_e(\gamma, \theta)$  is not necessarily convex in the arguments  $\gamma$  and  $\theta$ . We therefore resort to a numerical search over  $\gamma$  and  $\theta$  to determine the optimal settings. It suffices to restrict our search to the domain  $0 < \gamma \leq 0.5$  and  $0 \leq \theta \leq \frac{\pi}{2}$ . Unequal power allocation between the streams to be multiplexed requires the use of more expensive power amplifiers, to cope with the increased dynamic range. If cost is an issue, the symbol error rate can still be reduced by simply optimizing over the phase shift which is relatively simple and inexpensive to implement. For this situation

$$\theta_{opt} = \arg \min_{\theta} P_e(\gamma = 0.5, \theta). \quad (18)$$

A simple, intuitive explanation for the use of power allocation and relative phase adjustment between the streams to be multiplexed can be found by examining the channel geometry induced by the channel statistics relative to the geometry of the signals being transmitted. For example, consider a situation where there is poor XPD or equivalently uni-polarized antennas,  $\alpha \approx 1$ , and very high transmit correlation,  $t \approx 1$ . Under such propagation conditions the realizations of  $\mathbf{H}$  will in general have parallel (almost identical) columns and hence the error event  $[d_{min} \ -d_{min}]^T$  and its scalar multiples have large components lying in the null-space of the channel which causes a large symbol error rate. By using power allocation and/or phase shifting we seek to avoid these error vectors. Arbitrary settings of power allocation or phase shift can increase the magnitudes of the projections of other error events onto the null-space (or other eigendirections with low gain) of the channel, possibly increasing the symbol error rate. The optimal settings of power allocation and/or phase shifting strike a balance by taking into account the relative frequency of occurrence of the error events in addition to the associated probability of error which is directly related to the projection of the error events onto the null-space (or eigendirections with low gain) of the channel.

#### IV. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the effectiveness of the technique described in the previous section. In our simulations, we use 4-QAM and ML decoding. The signal-to-noise ratio (SNR) was defined as  $10\log\left(\frac{2E_s}{\sigma_n^2}\right)$  (dB).

We assume that the transmitter is able to perform both unequal power allocation and phase shifting. To find the optimal power allocation between symbol streams and optimal phase shift, we perform a numerical search over  $0 < \gamma \leq 0.5$  and  $0 \leq \theta \leq \frac{\pi}{2}$  as described in (17). The optimal power allocation and phase shift settings are functions of the channel statistics and SNR. For high SNR ( $> 12$  dB) however, the optimal power and phase appear to be constant for given channel statistics.

##### A. The i.i.d channel model

We first study the widely used i.i.d channel model which can be framed in the context of our channel model by setting  $r = 0$ ,  $t = 0$  and  $\alpha = 1$ . Observing the symmetry in the i.i.d channel model and the fact that all directions are equally good, it makes intuitive sense to allocate equal power to both symbol streams. Fig. 1 below is a plot of  $P_e(\gamma, \theta)$  as a function of  $\gamma$  and  $\theta$  (from (16)) for the i.i.d channel at an SNR of 20 dB.

From Fig. 1 it is clear that the minimum symbol error

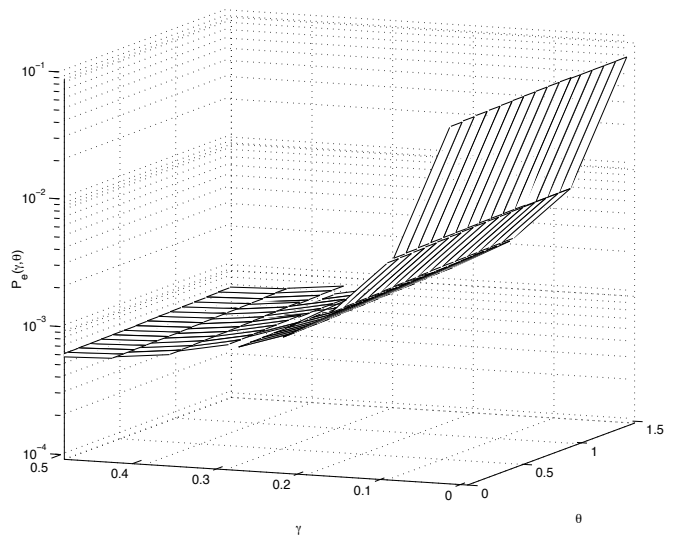


Fig. 1.  $P_e(\gamma, \theta)$  vs  $(\gamma, \theta)$  for  $r = 0$ ,  $t = 0$ ,  $\alpha = 1$  and SNR = 20 dB

rate occurs when  $\gamma = 0.5$ . Furthermore, the phase shift is irrelevant and makes no difference to the symbol error rate for a fixed  $\gamma$ . Thus our intuition is confirmed. Equal power allocation between the individual symbol streams is optimal for the i.i.d channel.

##### B. Correlated fading with poor XPD

We now consider the case of highly correlated fading with poor XPD for the  $1 \times 1$  dual-polarized antenna system. For our simulation we take  $r = 0$ ,  $t = 0.95$  and  $\alpha = 0.8$ . A plot of  $P_e(\gamma, \theta)$  as a function of  $\gamma$  and  $\theta$  is shown in Fig. 2 for an SNR of 20 dB.

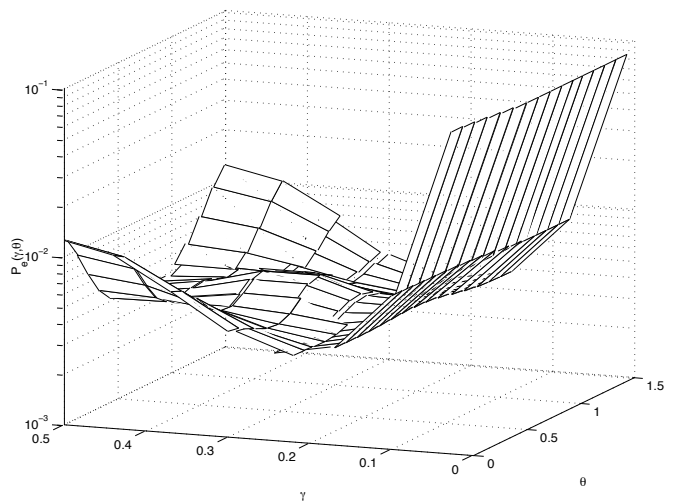


Fig. 2.  $P_e(\gamma, \theta)$  vs  $(\gamma, \theta)$  for  $r = 0$ ,  $t = 0.95$ ,  $\alpha = 0.8$  and SNR = 20 dB

From Fig. 2 it is clear that equal power allocation between symbol streams is not optimal and that introducing a phase shift can have a profound effect on the average symbol error rate. By setting  $\gamma = 0.22$  and  $\theta = 0.2$  we expect an order of magnitude improvement in the symbol error rate. In the event that the transmitter is not able to allocate unequal amounts of power to the symbol streams we observe that phase shifting alone can decrease the symbol error rate for the channel under consideration. For  $\gamma = 0.5$ , the minimum symbol error rate occurs when  $\theta = 0.8$ . To verify the improvements in performance we simulate the actual system with and without the optimal values of power allocation and/or phase shifting at the transmitter. The results are shown in Fig. 3.

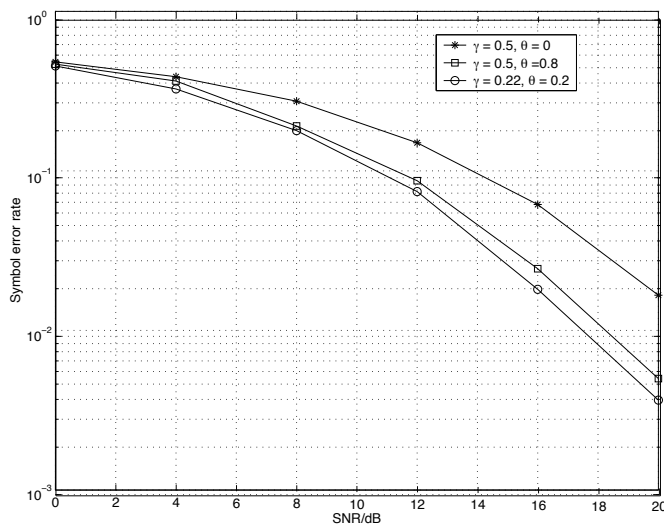


Fig. 3. Comparison of system performance with and without power allocation and/or phase shifting

From the average symbol error rate curves presented in Fig. 3, it is clear that the optimal values of power allocation and/or phase shifting yield a gain in terms of SNR of up to 4 dB. Also, for the channel statistics under consideration, phase shifting alone performs almost as well as power allocation and phase shifting. We note, however, that this is generally not the case.

## V. CONCLUSIONS

We studied the effects of power allocation and relative phase adjustment between symbol streams for spatial multiplexing with and without polarization diversity given knowledge of the channel statistics. We provided an expression for calculating an estimate of the uncoded average symbol error rate with power allocation and phase shifting. Using this expression it is pos-

sible to determine numerically the optimal (in the sense of minimizing the average symbol error rate estimate) settings for power allocation and phase shifting. The optimal settings of these parameters can result in significant improvements in system performance. The idea of power allocation and phase shifting between the symbol streams can be thought of as precoding with the objective of increasing the multiplexing gain. We demonstrated the performance gains of the new technique by means of simulation results.

## REFERENCES

- [1] A. J. Paulraj and T. Kailath, "Increasing capacity in wireless broadcast systems using distributed transmission/directional reception," *U.S. Patent*, No. 5,345,599, 1994.
- [2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," Tech. Rep. #BL0112170-950615-07TM, AT&T Bell Laboratories, 1995.
- [3] G. J. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol.6, no.3, pp. 311-335, March 1998.
- [4] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, pp. 41-59, Autumn 1998.
- [5] H. Bölcskei, D. Gesbert, and A. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Trans. Comm.*, in press.
- [6] J. J. A. Lempiäinen and J. K. Laiho-Steffens, "The performance of polarization diversity schemes at a base station in small/micro cells at 1800MHz," *IEEE Trans. Veh. Tech.*, vol. 47, pp. 1087-1092, Aug. 1999.
- [7] R. G. Vaughan, "Polarization diversity in mobile communications," *IEEE Trans. Veh. Tech.*, vol.39, pp. 177-186, Aug. 1990.
- [8] H. Bölcskei, R. Nabar, V. Erceg, D. Gesbert, and A. Paulraj, "Performance of spatial multiplexing in the presence of polarization diversity," *Proc. IEEE ICASSP 2001, Salt Lake City (UT)*, May 2001.
- [9] R. Nabar, H. Bölcskei, V. Erceg, D. Gesbert, and A. Paulraj, "Performance of multi-antenna signaling strategies in the presence of polarization diversity," *IEEE Trans. Sig. Proc.*, to be submitted, Aug. 2001.
- [10] H. Bölcskei and A. J. Paulraj, "Performance analysis of space-time codes in correlated Rayleigh fading environments," *Asilomar Conf. on Signals, Systems, and Computers, Pacific Grove (CA)*, Oct./Nov. 2000.
- [11] D. Baum, D. Gore, R. Nabar, S. Panchanathan, K. V. S. Hari, V. Erceg, and A. J. Paulraj, "Measurement and characterization of broadband MIMO fixed wireless channels at 2.5 GHz," *IEEE Int. Conf. on Personal Wireless Comm.*, pp. 203-206, Dec. 2000.