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System Capacity of Wideband OFDM Communications over Fading Channels without Channel Knowledge

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Abstract — **Assuming PSK modulation, we derive an upper bound on the system capacity of pulse-shaped orthogonal frequency division multiplexing (OFDM) communications over time-frequency selective fading channels in the absence of channel state information at the transmitter and the receiver. We show that capacity tends to zero in the large bandwidth limit and quantify the impact of spread and shape of the scattering function on finite bandwidth capacity.**

I. System Model

Channel model. We consider a Rayleigh fading channel $\mathbb{H}$ with complex baseband input-output relation [1]

$$y(t) = \int_\tau \int_\nu S_\mathbb{H}(\tau,\nu)\, x(t-\tau)\, e^{j2\pi t\nu}\, d\tau\, d\nu + u(t)\,,$$

where $S_\mathbb{H}(\tau,\nu)$ denotes the spreading function [1] and $u(t)$ is zero-mean circularly symmetric white Gaussian noise with power spectral density N_0. We assume that $\mathbb{H}$ is wide-sense stationary with uncorrelated scattering (WSSUS) [1], i.e.[2], $\mathcal{E}\{S_\mathbb{H}(\tau,\nu)S^*_\mathbb{H}(\tau',\nu')\} = C_\mathbb{H}(\tau,\nu)\,\delta(\tau-\tau')\,\delta(\nu-\nu')$, where $C_\mathbb{H}(\tau,\nu)$ denotes the scattering function [1] supported in $[0,\tau_0]\times[-\nu_0/2,\nu_0/2]$. The channel is *underspread* [1], i.e., $\tau_0\nu_0 \ll 1$, with path loss $\sigma^2_\mathbb{H} = \int_\tau\int_\nu C_\mathbb{H}(\tau,\nu)\,d\tau\,d\nu$.

Signal model. We assume that the OFDM modulator and demodulator calculate $x(t) = \sum_{n=0}^{N-1}\sum_{k=0}^{K-1} X_{n,k}\, g_{n,k}(t)$ and $Y_{n,k} = \langle y, g_{n,k}\rangle \triangleq \int_{-\infty}^{\infty} y(t) g^*_{n,k}(t)\,dt$, respectively, with $g_{n,k}(t) \triangleq g(t-nT)\, e^{j2\pi kF(t-nT)}$, and $\langle g_{n,k}, g_{n',k'}\rangle = \delta[n-n']\delta[k-k']$. The data symbols $X_{n,k}$ are i.i.d. drawn from a PSK constellation ($|X_{n,k}|^2 = \sigma_x^2$), $x(t)$ has bandwidth $B = KF$, and $TF \geq 1$ [2]. The overall input-output relation can now be summarized as [2]

$$Y_{n,k} = H_{n,k}X_{n,k} + Z_{n,k}\,,\quad n\in[0,N-1],\ k\in[0,K-1], \quad (1)$$

where $Z_{n,k} = \langle u, g_{n,k}\rangle$ is circularly symmetric white Gaussian noise with variance $\sigma_z^2 = N_0$ and $H_{n,k} = L_\mathbb{H}(nT, kF)$ with the time-varying transfer function $L_\mathbb{H}(t,f) = \int_\tau\int_\nu S_\mathbb{H}(\tau,\nu)\, e^{-j2\pi(f\tau - t\nu)} d\tau d\nu$ [1]. Note that (1) neglects intersymbol and intercarrier interference which is well justified if the channel is underspread and $g(t)$ is adapted to $C_\mathbb{H}(\tau,\nu)$ [2].

II. System Capacity of OFDM Communications

Assuming that the channel $\mathbb{H}$ is unknown both at the transmitter and the receiver, the OFDM system capacity on the basis of the input-output relation (1) is given by

$$C \triangleq \lim_{N\to\infty} \frac{1}{NT}\, I(\mathbf{Y};\mathbf{X})\,,$$

where $\mathbf{Y} = \begin{bmatrix}\mathbf{Y}_0 \ldots \mathbf{Y}_{N-1}\end{bmatrix}$ with $\mathbf{Y}_n = [Y_{n,0} \ldots Y_{n,K-1}]$ (accordingly for $\mathbf{X}$), and $I(\mathbf{Y};\mathbf{X})$ denotes the mutual information between $\mathbf{Y}$ and $\mathbf{X}$.

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[2] Our notation is as follows: $\mathcal{E}$ denotes the expectation operator. Furthermore, $\delta[n] = 1$ for $n = 0$ and $\delta[n] = 0$ otherwise.

It can now be shown that $C \leq S$ with

$$S = \frac{B}{TF}\log_2\left(1 + \frac{TF}{B}\frac{P\sigma^2_\mathbb{H}}{N_0}\right) - B\int_\tau\int_\nu \log_2\left(1+\frac{P}{BN_0}\,C_\mathbb{H}(\tau,\nu)\right) d\tau\, d\nu\,, \quad (2)$$

where $P = \sigma_x^2 K/T$ denotes the transmit power. The first term in (2) equals the capacity of an AWGN channel with "effective bandwidth" $B_\text{eff} \triangleq B/(TF)$ operating at SNR $= P\sigma^2_\mathbb{H}/(B_\text{eff}N_0)$. The second term in (2) can be interpreted as capacity loss due to channel uncertainty [3]. It furthermore follows from (2) that $\lim_{B\to\infty} C = 0$ irrespectively of $C_\mathbb{H}(\tau,\nu)$, which is due to the fact that information is spread over all time-frequency slots resulting in signals that are not peaky in time-frequency. In the finite bandwidth case, S depends critically on spread and shape of $C_\mathbb{H}(\tau,\nu)$. More specifically, for fixed $\sigma^2_\mathbb{H}$ and fixed shape of the scattering function, S decreases for increasing spread $\tau_0\nu_0$. Fixing τ_0, ν_0 and $\sigma^2_\mathbb{H}$, the worst-case scattering function minimizing (2) is the "brick-shaped" scattering function $C_\mathbb{H}(\tau,\nu) = \sigma^2_\mathbb{H}/(\tau_0\nu_0)$ for $(\tau,\nu)\in[0,\tau_0]\times[-\nu_0/2,\nu_0/2]$ [3].

Numerical example. For $P = 1\,\text{mW}$, $TF = 1.25$, $\sigma^2_\mathbb{H} = 90\text{dB}$, $N_0 = 4.14\cdot 10^{-21}\,\text{W/Hz}$, two channels with "brick-shaped" scattering functions and spreads $\tau_0\nu_0 = 10^{-3}$ and 10^{-2}, respectively, Fig. 1 depicts S in (2) along with the AWGN channel capacity for the same receive SNR. S exhibits a maximum at the critical bandwidth B_crit. For $B < B_\text{crit}$, S is close to the AWGN capacity; for $B > B_\text{crit}$, S decreases and approaches zero asymptotically. Finally, we observe that larger spread $\tau_0\nu_0$ results in smaller B_crit and reduced S in the regime $B > B_\text{crit}$.

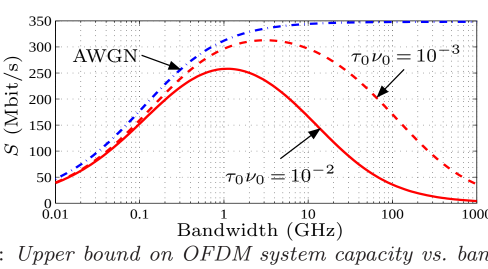

Fig. 1: *Upper bound on OFDM system capacity vs. bandwidth.*

References

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