Available at
www.ComputerScienceWeb.com
POWERED BY SCIENCE DIRECT

Signal Processing 83 (2003) 1379–1391

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Orthogonalization of OFDM/OQAM pulse shaping filters using the discrete Zak transform☆

Helmut Bölcskei[a,*], Pierre Duhamel[b], Rima Hleiss[b]

[a] Communication Technology Laboratory, ETH Zürich, ETF E122, Sternwartstrasse 7, 8092 Zürich, Switzerland
[b] Laboratoire des signaux et systemes, Supelec, 3 rue Joliot-Curie, F-91190 Gif-sur-Yvette, France

Received 1 August 2001; received in revised form 1 February 2003

This paper is dedicated to Prof. W.F.G. Mecklenbräuker on the occasion of his 65th birthday.

Abstract

We introduce a computationally efficient FFT-based algorithm for the design of OFDM/OQAM pulse shaping filters of arbitrary length and with arbitrary overlapping factors. The basic idea of our approach is to start from an arbitrary initial (nonorthogonal) filter and to perform an orthogonalization which yields an ISI- and ICI-free pulse shaping filter. The orthogonalization is performed in the discrete Zak transform domain and can be implemented using the FFT. We design filters for OFDM/OQAM systems with up to 1024 channels and overlapping factor 32. Finally, we present simulation examples assessing the performance of the designed pulse shaping filters in the presence of Doppler spread.
© 2003 Elsevier Science B.V. All rights reserved.

Keywords: Orthogonal frequency division multiplexing; Offset QAM; Pulse shaping; Filter design

1. Introduction and outline

Orthogonal frequency division multiplexing (OFDM) [2–4,10,12,16,17] is part of the European digital audio broadcasting (DAB) and digital video broadcasting (DVB) standards, and is employed for high-bit-rate digital subscriber services on twisted-pair channels such as in the asymmetric digital subscriber line (ADSL) standard. Important features of OFDM systems include high spectral efficiency and immunity to multipath fading and impulsive noise.

OFDM systems employing a cyclic prefix (CP) [12] offer robustness to time-dispersion (multipath fading) but do not provide protection against frequency-synchronization errors and frequency-dispersion caused by Doppler spread. Frequency-synchronization errors and frequency-dispersion cause a loss of orthogonality between the subcarriers and result in an error, which depends critically on the frequency-localization of the pulse shaping filter [13]. In the CP OFDM case, this pulse shaping filter is a rectangular function and hence exhibits poor frequency-decay which implies high sensitivity to frequency-synchronization errors and Doppler spread. This observation has motivated studies on the design of pulse shaping OFDM systems [4,7,9–11,13,15,19]. Another objective of pulse shaping is to reduce out-of-band emission and hence increase spectral efficiency. In this paper, we shall be concerned with

☆ This work was supported in part by FWF-grants J1629-TEC and J1868-TEC. Part of this paper was presented at IEEE ICC-99, Vancouver B.C., Canada, June 1999.
* Corresponding author. Fax: +41-1-6321209.
E-mail addresses: boelcskei@nari.ee.ethz.ch (H. Bölcskei), pierre.duhamel@lss.supelec.fr (P. Duhamel).

0165-1684/03/$ - see front matter © 2003 Elsevier Science B.V. All rights reserved.
doi:10.1016/S0165-1684(03)00087-2

the design of pulse shaping filters for OFDM systems based on offset QAM (OFDM/OQAM) [2,6,16].

1.1. Previous work

Let us first briefly review previous work on the design of pulse shaping filters for OFDM/OQAM systems. In [2,16] band-limited continuous-time OFDM/OQAM pulse shaping filters designed such that there is overlap in frequency only between neighboring subchannels have been proposed. In practice, the corresponding infinitely long pulses must be truncated, which inevitably leads to a loss of orthogonality. Furthermore, in order to keep the error due to the resulting nonorthogonality small, in general, the pulses must be rather long which gives rise to considerable reconstruction delay and increased computational complexity. In [10] a continuous-time OFDM/OQAM pulse shaping filter of twice the symbol length is proposed. A constrained optimization procedure for the design of continuous-time OFDM/OQAM pulse shaping filters of finite length is introduced in [19]. The method in [19] yields near-orthogonal pulse shaping filters, requires computationally expensive numerical optimization, and is restricted to integer overlapping factors.

1.2. Contributions

We introduce a computationally efficient FFT-based algorithm for the design of OFDM/OQAM pulse shaping filters of arbitrary length and with arbitrary overlapping factors. The basic idea of our algorithm is to start from an arbitrary (nonorthogonal) initial filter and to perform an orthogonalization such that the resulting filter is intersymbol interference (ISI)- and intercarrier interference (ICI)-free. Based on a new formulation of the OFDM/OQAM pulse shaping filter orthogonality conditions in terms of the discrete-time Zak transform (DZT) [1,5], we are able to perform the orthogonalization using the FFT. This yields a computationally highly efficient implementation and allows the design of filters with several thousands of taps. For such filter lengths the use of numerical optimization often becomes infeasible. We emphasize, however, that our design method does not involve an optimization which makes a judicious choice of the initial filter necessary in order to arrive at well-localized orthogonal filters. We provide guidelines on how to choose “good initial filters” and demonstrate our method by designing pulse shaping filters for OFDM/OQAM systems with up to 1024 channels and with filter length up to 8192.

1.3. Organization of the paper

The rest of this paper is organized as follows. In Section 2, we derive general orthogonality conditions for discrete-time OFDM/OQAM pulse shaping filters in terms of the DZT. Based on these orthogonality conditions, in Section 3, we provide an efficient FFT-based design algorithm. In Section 4, we present some design examples and we demonstrate the superiority of coded and uncoded pulse shaping OFDM/OQAM systems over OFDM systems without pulse shaping in the presence of Doppler spread. Finally, Section 5 concludes the paper.

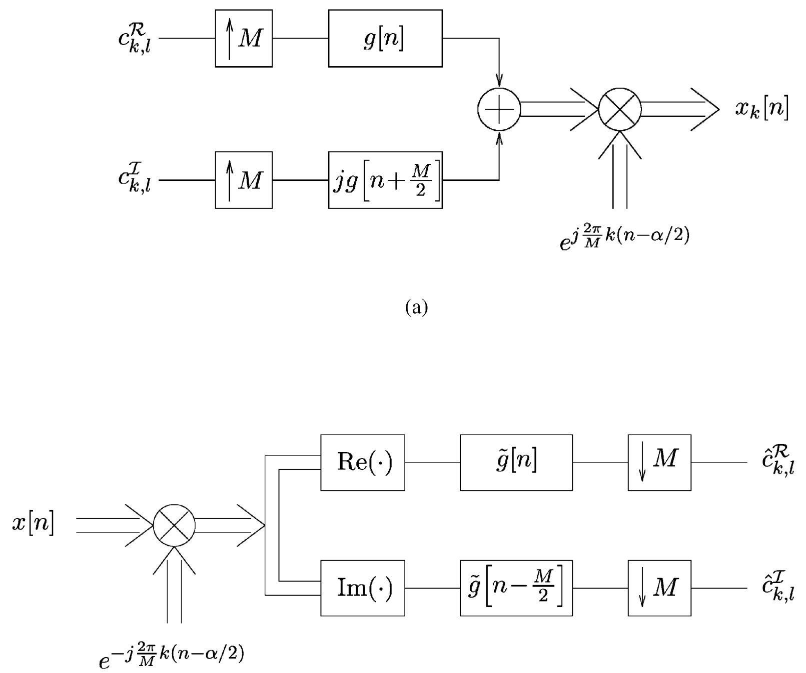
2. DZT-based orthogonality conditions

In the following, we consider the discrete-time complex baseband model of an M-channel OFDM/OQAM system where M is assumed to be even (Fig. 1). The transmit signal is given by

$$x[n]=\sum_{k=0}^{M-1}x_k[n]=\sum_{k=0}^{M-1}\Bigg[\sum_{l=-\infty}^{\infty}c_{k,l}^{\mathscr{R}}\,g[n-lM]\times e^{j(2\pi/M)k(n-\alpha/2)}+\sum_{l=-\infty}^{\infty}jc_{k,l}^{\mathscr{I}}\,g[n+M/2-lM]\,e^{j(2\pi/M)k(n-\alpha/2)}\Bigg],$$

where $c_{k,l}^{\mathscr{R}}=\mathrm{Re}\{c_{k,l}\}$ and $c_{k,l}^{\mathscr{I}}=\mathrm{Im}\{c_{k,l}\}$ denote the real and imaginary parts of the data symbols $c_{k,l}$, respectively, $g[n]$ is the real-valued pulse shaping filter and [1] $\alpha\in[0,M-1]$. Note that the $c_{k,l}^{\mathscr{I}}$ are modulated by $g[n+M/2]$. We emphasize that the proposed OFDM/OQAM system does not employ guard regions (in time or frequency) as is the case for CP OFDM systems (temporal guard region) and hence exhibits maximum spectral efficiency. The computational complexity of pulse shaping OFDM/OQAM systems is higher than that of CP OFDM systems. However,

[1] The significance of α will become clear later.



(a)

(b)

Fig. 1. Discrete-time multirate model of OFDM/OQAM system ($\tilde{g}[n] = g[-n]$): (a) kth transmitter subchannel, (b) kth receiver subchannel.

since the subchannel filters are obtained by complex modulation of a single filter efficient polyphase implementations are possible [20].

The approach used in the following to derive the orthogonality conditions up to Eq. (13) has first been suggested in a continuous-time setting in [19]. Note, however, that unlike [19] we included a general phase factor $e^{j(2\pi/M)k\alpha/2}$ in the modulator and the demodulator. The phase factor used in [19] in a continuous-time setting corresponds to $\alpha = M/2 - 1$ in the discrete-time case. This choice of α requires that the pulse shaping filter length is an integer multiple of the OFDM symbol length or equivalently the overlapping factor is an integer number. If pulse shaping filters with noninteger overlapping factors are desired α has to be chosen accordingly in order to ensure orthogonality. We note that our approach is more general than the one in [19] since we do not restrict the filter length to be an integer multiple of the OFDM symbol length. The specific choice of α for a given filter length will be discussed later in this section. Due to the additional phase factor $e^{j(2\pi/M)k\alpha/2}$ we get more general orthogonality conditions and also different symmetry conditions on $g[n]$ than in [19]. For this reason and because the exposition in [19] is for the continuous-time case, we decided to present the derivation of the orthogonality conditions in detail. The DZT-based formulation of the orthogonality conditions (19) constitutes the basis for our design algorithm and appears to be entirely new.

2.1. Orthogonality conditions

The pulse shaping filter $g[n]$ in Fig. 1 is said to be *orthogonal* if it satisfies perfect reconstruction, i.e., $\hat{c}_{k,l} = c_{k,l}$ for arbitrary $c_{k,l}$. Considering the equivalent path from the $(k+m)$th transmitter subchannel to the kth receiver subchannel (see Fig. 2), it follows that $g[n]$ is orthogonal if the following conditions are satisfied for $m \in [0, M-1], l \in \mathbb{Z}$:

$$[\mathrm{Re}\{g[n-lM]\, e^{j(2\pi/M)m(n-\alpha/2)}\} * \tilde{g}[n]]_{n=0} = \delta[l]\delta[m], \qquad (1)$$

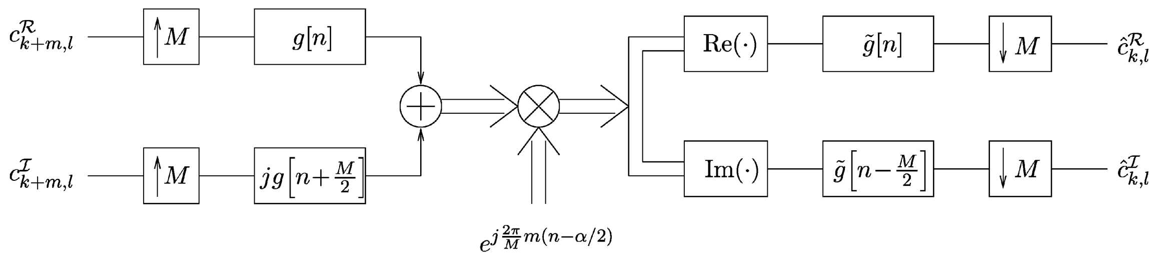


Fig. 2. Equivalent path from $(k+m)$th transmitter subchannel to kth receiver subchannel.

$$[\mathrm{Re}\{\mathrm{j}g[n+M/2-lM]\,\mathrm{e}^{\mathrm{j}(2\pi/M)m(n-\alpha/2)}\} * \tilde{g}[n]]_{n=0} = 0, \quad (2)$$

$$[\mathrm{Im}\{g[n-lM]\,\mathrm{e}^{\mathrm{j}(2\pi/M)m(n-\alpha/2)}\} * \tilde{g}[n-M/2]]_{n=0} = 0, \quad (3)$$

$$[\mathrm{Im}\{\mathrm{j}g[n+M/2-lM]\,\mathrm{e}^{\mathrm{j}(2\pi/M)m(n-\alpha/2)}\} * \tilde{g}[n-M/2]]_{n=0} = \delta[l]\delta[m], \quad (4)$$

with $\tilde{g}[n]=g[-n]$. For $m \neq 0$, Eqs. (1)–(4) guarantee that ICI is perfectly cancelled, whereas for $m=0$, (1)–(4) reduce to the conditions for ISI cancellation. More specifically, for $m=0$, Eqs. (2) and (3) guarantee that there is no interference between the real and imaginary parts of $c_{k,l}$, whereas (1) and (4) guarantee zero ISI for $c^{\mathcal{R}}_{k,l}$ and $c^{\mathcal{I}}_{k,l}$, respectively. Rewriting (1)–(4) as

$$\sum_{n=-\infty}^{\infty} g[n-lM]g[n]\cos\left(\frac{2\pi}{M}m(n-\alpha/2)\right) = \delta[l]\delta[m], \quad (5)$$

$$\sum_{n=-\infty}^{\infty} g[n+M/2-lM]g[n]\sin\left(\frac{2\pi}{M}m(n-\alpha/2)\right) = 0, \quad (6)$$

$$\sum_{n=-\infty}^{\infty} g[n-lM]g[n+M/2]\sin\left(\frac{2\pi}{M}m(n-\alpha/2)\right) = 0, \quad (7)$$

$$\sum_{n=-\infty}^{\infty} g[n+M/2-lM]g[n+M/2] \times \cos\left(\frac{2\pi}{M}m(n-\alpha/2)\right) = \delta[l]\delta[m], \quad (8)$$

we shall next show that (8) is equivalent to (5) and that (7) is equivalent to (6). Setting $n \to n-M/2$ in (8), it follows that (8) is equivalent to (5). Setting $n \to n-M/2$ in (7), we can see that (7) is (6) with l replaced by $l+1$, which implies the equivalence of (7) and (6). We shall next demonstrate that (6) can be satisfied by imposing a simple symmetry constraint on the pulse shaping filter $g[n]$. Setting $f^{(l)}[n]=g[n+(M/2)-lM]g[n]$, Eq. (6) can be rewritten as

$$\frac{1}{2\mathrm{j}}\,\mathrm{e}^{-\mathrm{j}(\pi\alpha/M)m}F^{(l)}(\mathrm{e}^{-\mathrm{j}(2\pi/M)m}) - \frac{1}{2\mathrm{j}}\,\mathrm{e}^{\mathrm{j}(\pi\alpha/M)m}F^{(l)}(\mathrm{e}^{\mathrm{j}(2\pi/M)m}) = 0, \quad (9)$$

where $F^{(l)}(\mathrm{e}^{\mathrm{j}2\pi\theta}) = \sum_{n=-\infty}^{\infty} f^{(l)}[n]\,\mathrm{e}^{-\mathrm{j}2\pi n\theta}$ denotes the discrete-time Fourier transform of $f^{(l)}[n]$. In the following we take $g[n]$ to be an even function satisfying

$$g[n] = g[\alpha+(2r+1)M/2-n], \quad (10)$$

with $r \in \mathbb{Z}$ and $\alpha \in [0, M-1]$. The parameters r and α allow a flexible choice of the filter length L_g. We note that α has to be odd (even) for even (odd) L_g. Furthermore, from (10) it follows[2] that for a $g[n]$ supported in $[0, L_g-1]$ the parameter α has to be chosen as $\alpha=(L_g+(M/2)-1) \bmod M$. For $L_g=KM$ with $K \in \mathbb{Z}$ we get $\alpha=M/2-1$ which corresponds to the choice in [19]. Using (5) and (10) it can furthermore

[2] mod stands for the modulo operation.

be shown that $L_g \neq lM + 1$ with $l \in \mathbb{N}$. Since $f^{(l)}[n]$ satisfies

$$f^{(l)}[\alpha + (r+l)M - n] = f^{(l)}[n] \quad \text{or equivalently}$$
$$F^{(l)}(e^{-j2\pi\theta}) = e^{j2\pi((r+l)M+\alpha)\theta} F^{(l)}(e^{j2\pi\theta}), \tag{11}$$

it follows from (9) that (6) is satisfied. We are thus left with condition (5), which will next be shown to hold for odd m. Setting $f^{(l)}[n] = g[n-lM]g[n]$ and using $f^{(l)}[\alpha + (r+l)M + (M/2) - n] = f^{(l)}[n]$ or equivalently $F^{(l)}(e^{-j2\pi\theta}) = e^{j2\pi((r+l)M+\alpha+M/2)\theta} F^{(l)}(e^{j2\pi\theta})$, it follows that (5) can be rewritten as

$$\frac{1}{2} e^{j(\pi\alpha/M)m} F^{(l)}(e^{j(2\pi/M)m})[(-1)^m + 1] = \delta[l]\delta[m]. \tag{12}$$

For m odd (12) is obviously satisfied irrespectively of the specific $g[n]$ chosen. Note that so far we only made use of the symmetry of $g[n]$. The real design task lies in satisfying (5) for m even. Setting $m \to 2m$ in (5) we obtain

$$\frac{1}{2} \sum_{n=-\infty}^{\infty} g[n]g[n-lM] e^{j(2\pi/M/2)m(n-\alpha/2)} + \frac{1}{2} \sum_{n=-\infty}^{\infty} g[n]g[n-lM] e^{-j(2\pi/M/2)m(n-\alpha/2)} = \delta[l]\delta[m],$$

which upon defining $g_{l,m}[n] = g[n-lM]e^{j(2\pi/M/2)m(n-\alpha/2)}$ can be rewritten as

$$\frac{1}{2}\langle g, g_{l,-m}\rangle + \frac{1}{2}\langle g, g_{l,m}\rangle = \delta[l]\delta[m], \tag{13}$$

where $\langle a,b\rangle = \sum_{n=-\infty}^{\infty} a[n]b^*[n]$ denotes the inner product of the sequences $a[n]$ and $b[n]$. The continuous-time equivalent of (13) has been reported previously in [19]. From (13) it follows that a $g[n]$ satisfying the symmetry property (10) is orthogonal if the function set $\{g_{l,m}[n]\}_{m=0,1,\ldots,M-1,\, l\in\mathbb{Z}}$ constitutes an orthonormal basis, i.e., $\langle g_{l,m}, g_{l',m'}\rangle = \delta[l-l']\delta[m-m']$. We have thus reduced the design of the pulse shaping filter $g[n]$ to the design of an orthonormal basis consisting of time-frequency shifted versions of $g[n]$. Since orthonormality of the function set $\{g[n-lM]e^{j(2\pi/M/2)m(n-\alpha/2)}\}$ is equivalent to orthonormality of $\{g[n-lM]e^{j(2\pi/M/2)mn}\}$, we set $g_{l,m}[n] = g[n-lM]e^{j(2\pi/M/2)mn}$ in the following.

2.2. DZT-based formulation

We shall next express the orthogonality condition (13) in terms of the DZT [1,5] of the function $g[n]$, which is defined as [3]

$$\mathcal{Z}_g(n,\theta) = \sum_{r=-\infty}^{\infty} g\left[n + r\frac{M}{2}\right] e^{-j2\pi r\theta} \tag{14}$$

with its inverse given by

$$g[n] = \int_0^1 \mathcal{Z}_g(n,\theta)\,d\theta. \tag{15}$$

The first step of this derivation consists of showing that $\{g_{l,m}[n]\}$ is orthogonal if and only if the following equivalent condition is satisfied:

$$\sum_{r=-\infty}^{\infty} g\left[n - r\frac{M}{2}\right] g\left[n - r\frac{M}{2} - lM\right] = \frac{2}{M}\delta[l]. \tag{16}$$

Applying the Poisson summation formula to (16), we obtain

$$\sum_{m=0}^{(M/2)-1} e^{j2\pi(m/M/2)n} \int_0^1 G(e^{j2\pi\theta}) G^*(e^{j2\pi(\theta-(m/M/2))}) \times e^{j2\pi\theta lM}\,d\theta = \delta[l],$$

which using the Parseval identity can be rewritten as

$$\sum_{m=0}^{(M/2)-1} e^{j2\pi(m/M/2)n} \langle g, g_{l,m}\rangle = \delta[l]. \tag{17}$$

Obviously (17) is satisfied if and only if $\langle g, g_{l,m}\rangle = \delta[l]\delta[m]$ which implies orthogonality of the set $\{g_{l,m}[n]\}$ and consequently orthogonality of the filter $g[n]$.

Next, inserting the DZT inversion formula (15) into (16) we get

$$\sum_{r=-\infty}^{\infty} \int_0^1 \mathcal{Z}_g\left(n - r\frac{M}{2}, \theta\right) d\theta \times \int_0^1 \mathcal{Z}_g\left(n - r\frac{M}{2} - lM, v\right) dv = \frac{2}{M}\delta[l]$$

[3] The DZT is the polyphase decomposition [20] evaluated on the unit circle. Throughout this paper, we shall use the term DZT since in our context it is rather a signal transformation.

which using the quasi-periodicity relation $\mathscr{Z}_g(n - r(M/2), \theta) = e^{-j2\pi r\theta}\mathscr{Z}_g(n, \theta)$ yields

$$\int_0^1 \mathscr{Z}_g(n,\theta)\mathscr{Z}_g(n,-\theta)\, e^{j2\pi 2l\theta}\, d\theta = \frac{2}{M}\delta[l]$$

or equivalently

$$\int_0^1 \mathscr{Z}_g(n,\theta)\mathscr{Z}_g(n,-\theta)e^{j2\pi l\theta}\, d\theta \underbrace{\sum_{s=-\infty}^{\infty}\delta[l-2s]}_{\frac{1}{2}\sum_{i=0}^{1} e^{-j2\pi(i/2)l}}$$

$$= \frac{2}{M}\delta[l]. \quad (18)$$

Since $g[n]=g^*[n]$ we have $\mathscr{Z}_g(n,-\theta)=\mathscr{Z}_g^*(n,\theta)$, and consequently (18) reduces to

$$\int_0^1 [|\mathscr{Z}_g(n,\theta)|^2 + |\mathscr{Z}_g(n,\theta-1/2)|^2]e^{j2\pi l\theta}\, d\theta$$

$$= \frac{4}{M}\delta[l],$$

which is satisfied if and only if

$$|\mathscr{Z}_g(n,\theta)|^2 + |\mathscr{Z}_g(n,\theta-1/2)|^2 = \frac{4}{M},$$

$$n = 0, 1, \ldots, \frac{M}{2} - 1, \theta \in [0, 1). \quad (19)$$

We have thus found a simple equivalent expression for the orthogonality condition (13) in terms of the DZT. Based on (19), in the next section, we shall derive our design method.

3. FFT-based pulse shaping filter design

In the previous section we have shown that the design of OFDM/OQAM pulse shaping filters reduces to the design of a symmetric function satisfying (19). The basic idea of our approach is to start from an arbitrary (nonorthogonal) symmetric filter $g[n]$ which is modified (orthogonalized) to obtain a symmetric pulse shaping filter satisfying (19). Typically the initial filter $g[n]$ will be a lowpass filter.

3.1. Orthogonalization

Starting from an arbitrary (lowpass) filter $g[n]$ satisfying the symmetry property

$$g\left[\alpha + (2r+1)\frac{M}{2} - n\right] = g[n]$$

with $r \in \mathbb{Z}$ and $\alpha \in [0, M-1]$, an orthogonal pulse shaping filter $g_o[n]$ can be obtained as

$$\mathscr{Z}_{g_o}(n,\theta) = \frac{2\mathscr{Z}_g(n,\theta)}{\sqrt{M|\mathscr{Z}_g(n,\theta)|^2 + M|\mathscr{Z}_g(n,\theta-\frac{1}{2})|^2}}. \quad (20)$$

Inserting (20) into (19) and using $\mathscr{Z}_g(n,\theta-1) = \mathscr{Z}_g(n,\theta)$, it is easily seen that

$$|\mathscr{Z}_{g_o}(n,\theta)|^2 + |\mathscr{Z}_{g_o}(n,\theta-1/2)|^2 = \frac{4}{M}$$

$$\text{for } n = 0, 1, \ldots, \frac{M}{2} - 1, \quad \theta \in [0, 1),$$

which proves the orthogonality of $g_o[n]$. It remains to show that $g_o[n]$ satisfies the same symmetry property as the initial filter $g[n]$. In the DZT domain (10) reads

$$\mathscr{Z}_g(\alpha - n, -\theta) = e^{j2\pi(2r+1)\theta}\mathscr{Z}_g(n,\theta), \quad (21)$$

which using (20) implies that

$$\mathscr{Z}_{g_o}(\alpha - n, -\theta) = e^{j2\pi(2r+1)\theta}\mathscr{Z}_{g_o}(n,\theta)$$

and consequently

$$g_o[n] = g_o\left[\alpha + (2r+1)\frac{M}{2} - n\right].$$

Note that although $g_o[n]$ satisfies the same symmetry property as the initial filter $g[n]$, it need not have the same length as $g[n]$. As we shall see later, in general it will be longer than $g[n]$. In the Appendix it is shown that $g_o[n]$ obtained from a compactly supported $g[n]$ will be compactly supported if and only if the initial filter satisfies

$$|\mathscr{Z}_g(n,\theta)|^2 + \left|\mathscr{Z}_g\left(n, \theta - \frac{1}{2}\right)\right|^2 = c_n,$$

$$0 \leqslant \theta < 1, \quad (22)$$

with arbitrary constants c_n. From (22) it furthermore follows that in this case

$$\mathscr{Z}_{g_o}(n,\theta) = \frac{2}{\sqrt{Mc_n}}\mathscr{Z}_g(n,\theta),$$

which implies that $g_o[n]$ will not only be compactly supported but also have the same support as $g[n]$. The class of compactly supported initial filters yielding compactly supported orthogonal filters is rather restricted. In practice, however, if the initial filter is sufficiently localized in time, in general the corresponding orthogonal filter will be well-localized in time as well and can therefore be truncated at the cost of a negligible reconstruction error.

3.2. Implementation of the algorithm

We shall now turn to the implementation of the proposed orthogonalization procedure. As already mentioned above, in general the resulting orthogonal filter $g_o[n]$ will be longer than the initial filter. Therefore, $g[n]$ has to be sufficiently zero-padded before applying the orthogonalization. The DZT has to be evaluated on a discrete time-frequency grid according to [1]

$$\mathscr{Z}_g[n,k] = \sum_{r=0}^{K-1} g\left[n + r\frac{M}{2}\right] \mathrm{e}^{-\mathrm{j}2\pi(k/K)r},$$

$$n = 0,1,\ldots,\frac{M}{2}-1, \quad k = 0,1,\ldots,K-1$$

with the length of the zero-padded filter satisfying $L_g = (M/2)K$. It is easily seen that the computation of $\mathscr{Z}_g[n,k]$ reduces to the column-wise FFT of the $K \times M/2$ matrix

$$\mathbf{G} = \begin{pmatrix} g[0] & g[1] & \cdots & g[M/2-1] \\ g[M/2] & g[M/2+1] & \cdots & g[M-1] \\ g[M] & g[M+1] & \cdots & g[3M/2-1] \\ \vdots & \vdots & & \vdots \\ g[L_g - M/2] & g[L_g - M/2+1] & \cdots & g[L_g - 1] \end{pmatrix}.$$

The orthogonalization procedure can finally be summarized as follows:

- Design an initial filter $g[n]$ satisfying the symmetry condition (10).
- Perform zero-padding of $g[n]$ to obtain a filter of length L_g.
- Compute the DZT of the orthogonal filter $g_o[n]$ according to [4]

$$\mathscr{Z}_{g_o}[n,k] = \frac{2\mathscr{Z}_g[n,k]}{\sqrt{M|\mathscr{Z}_g[n,k]|^2 + M|\mathscr{Z}_g[n,k-(K/2)]|^2}}. \quad (23)$$

- Compute the inverse DZT to obtain the orthogonal pulse shaping filter $g_o[n]$, i.e., compute the inverse FFT of the columns of the $K \times M/2$ matrix $\mathscr{Z}_{g_o}[n,k]$.

We note that this orthogonalization procedure does not automatically guarantee that $g_o[n]$ is well-localized in frequency. In practice, however, starting from a lowpass filter with bandwidth approximately equal to $1/M$ yields well-localized orthogonal filters (see the design examples in Section 4). We emphasize that the above orthogonalization procedure is *computationally very cheap*, since it consists of FFTs (forward and inverse DZT) and divisions in the DZT domain (cf. (23)). More specifically, the design of an orthogonal pulse shaping filter of length L_g requires $M/2$ FFTs of length K, L_g squaring operations, L_g additions, L_g square root operations, and $M/2$ IFFTs of length K. For $M = 256$ and $L_g = 1024$ the total flop count in MATLAB was 14,356.

The OFDM/IOTA approach proposed in [10] and further elaborated in [14,18] is based on ideas similar to those underlying our orthogonalization method. However, in the case of OFDM/IOTA the initial filter has to be the gaussian function and an orthogonalization is performed in the time-domain and in the frequency-domain separately leading to what is called extended gaussian functions in [14,18]. Recently, it has been shown in [8] that the orthogonalization method proposed in [10,14,18] can only be applied to a very restricted class of initial filters. Since our approach can be applied to arbitrary initial filters, it is more general than the OFDM/IOTA approach. We finally note that an orthogonalization similar to the one employed in this paper has been used in [21,1] to design so-called tight Weyl–Heisenberg frames.

4. Design examples

In this section, we shall demonstrate how the proposed orthogonalization procedure can be used to design frequency well-localized pulse shaping filters for OFDM/OQAM systems. We shall see that the basic philosophy of our orthogonalization method is to start from an initial lowpass filter with bandwidth approximately equal to $1/M$. This will in general yield orthogonal filters which are again well-localized.

[4] For the sake of simplicity we assume that K is even.

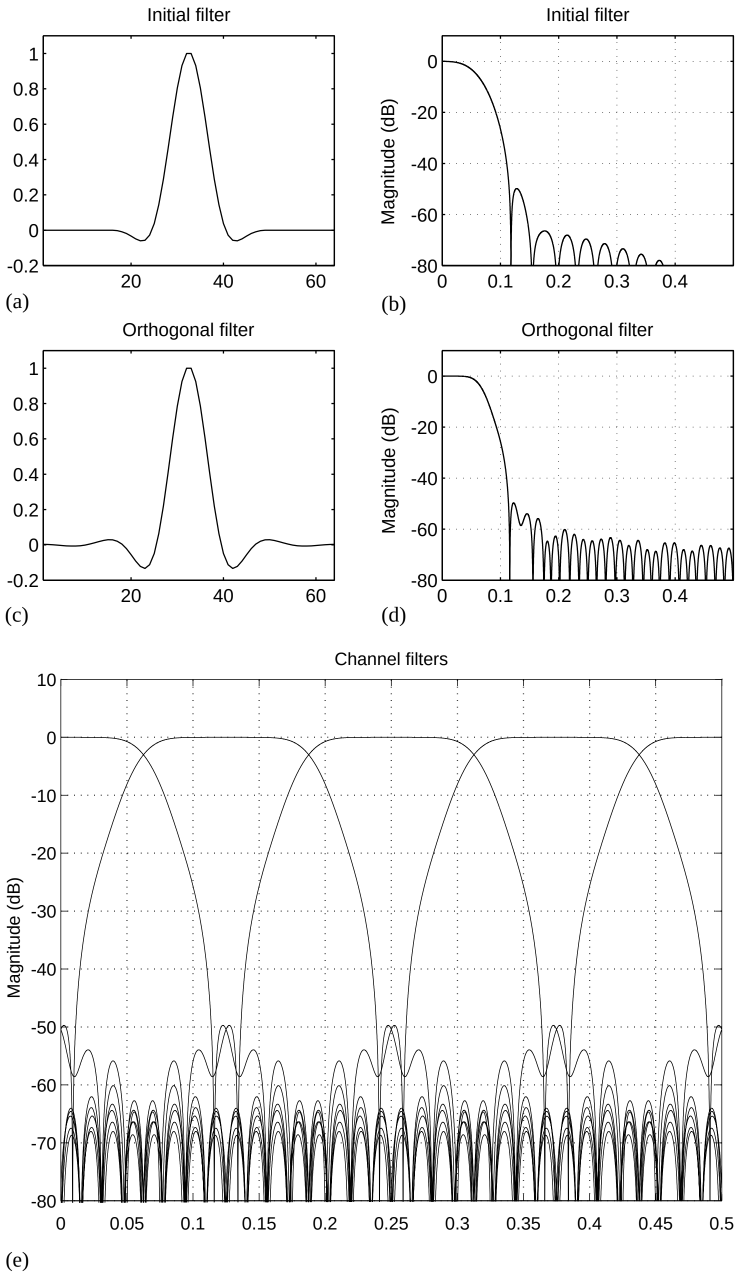


Fig. 3. 8-channel OFDM/OQAM pulse shaping filter of length 64: (a, b) initial filter, (c, d) orthogonal filter, (e) corresponding channel filters.

Design Example 1. In the first example, we designed an 8-channel OFDM/OQAM system. The initial filter is a 32 tap lowpass filter with bandwidth $1/M = 0.125$ (designed using the MATLAB function FIR1) zero-padded to obtain a filter of overall length 64. Figs. 3a and b show the initial filter and its transfer function, respectively. Figs. 3c and d show the resulting orthogonal filter's impulse response and transfer

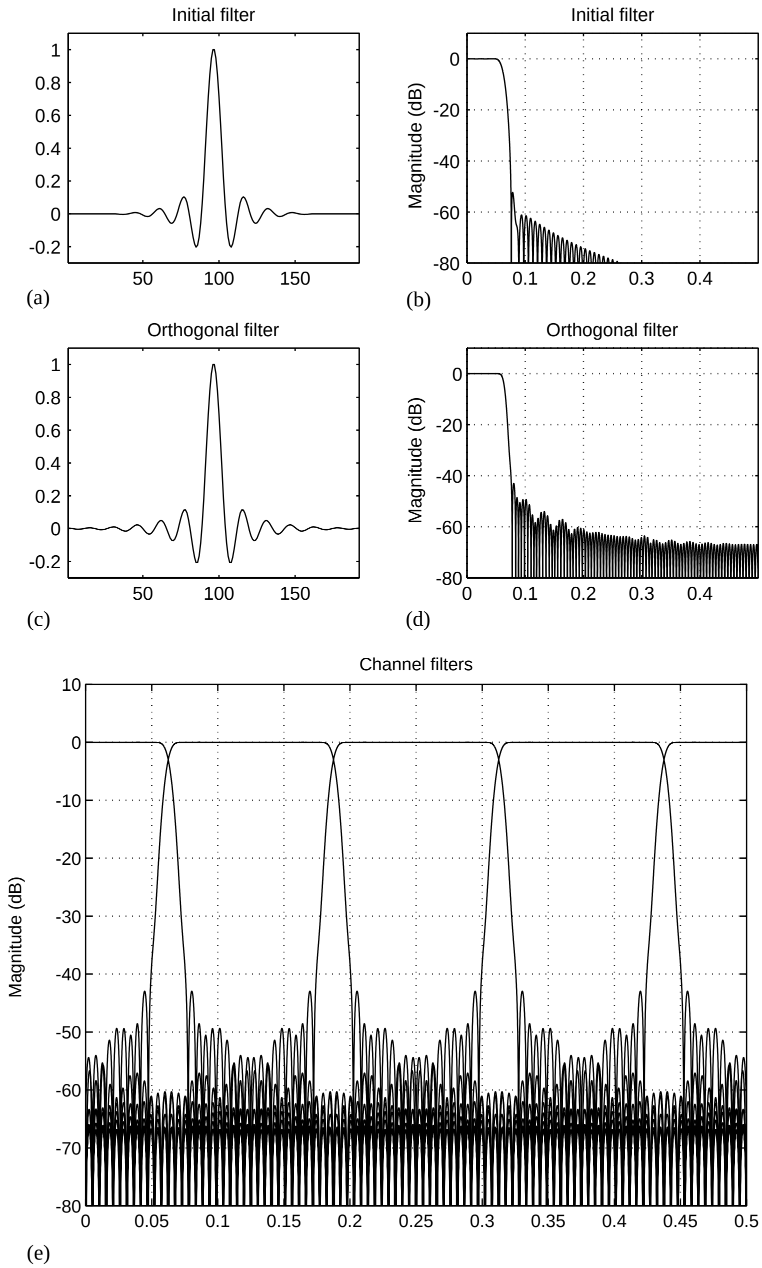


Fig. 4. 8-channel OFDM/OQAM pulse shaping filter of length 192: (a, b) initial filter, (c, d) orthogonal filter, (e) corresponding channel filters.

function, respectively. The corresponding subchannel filters are depicted in Fig. 3e. We can see that the orthogonal filter has good localization in both time and frequency.

Design Example 2. In the second design example, we demonstrate how the frequency-localization of the pulse shaping filter can be improved by increasing the filter length. We designed an 8-channel

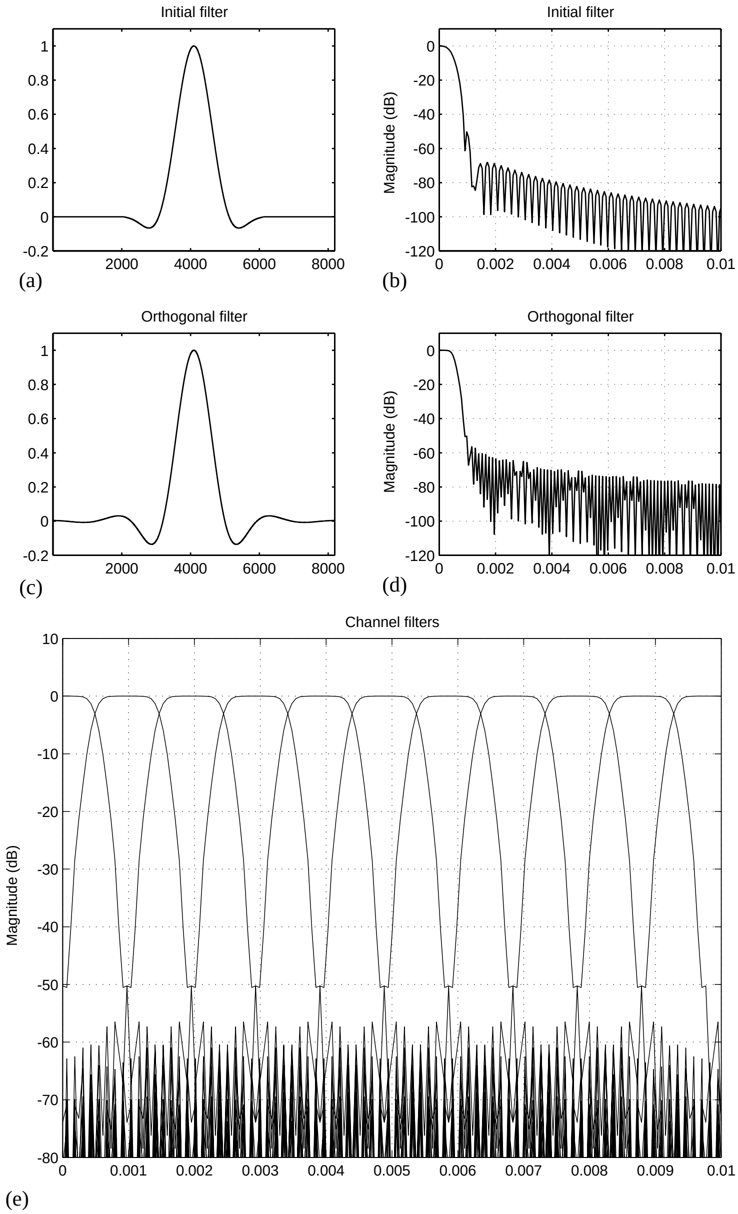
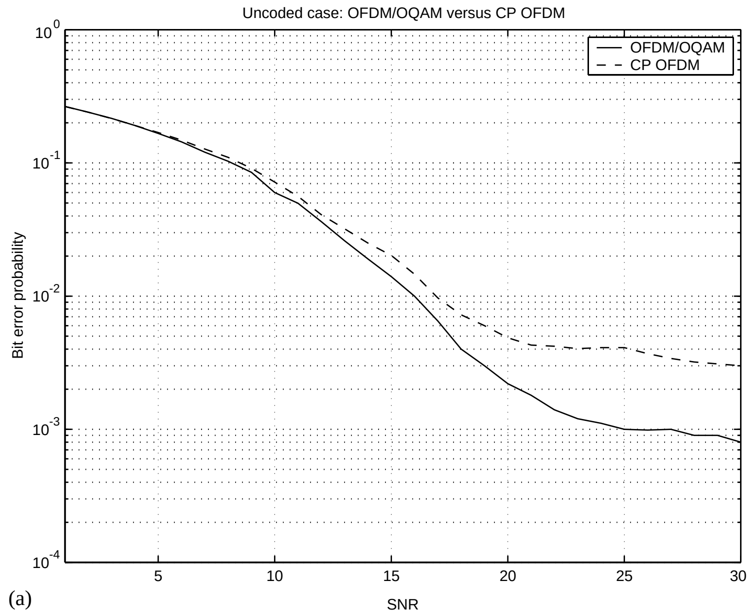


Fig. 5. 1024-channel OFDM/OQAM pulse shaping filter of length 8192: (a, b) initial filter, (c, d) orthogonal filter, (e) corresponding channel filters.

OFDM/OQAM system using a pulse shaping filter of length 192. The initial filter is shown in Figs. 4a and b. (It was designed using the MATLAB function FIR1 with nominal bandwidth 0.125). Figs. 4c and d show the resulting orthogonal filter and its transfer function, respectively. The corresponding channel filters are depicted in Fig. 4e. Comparing Figs. 3e and 4e, we can see that the pulse shaping filter in this



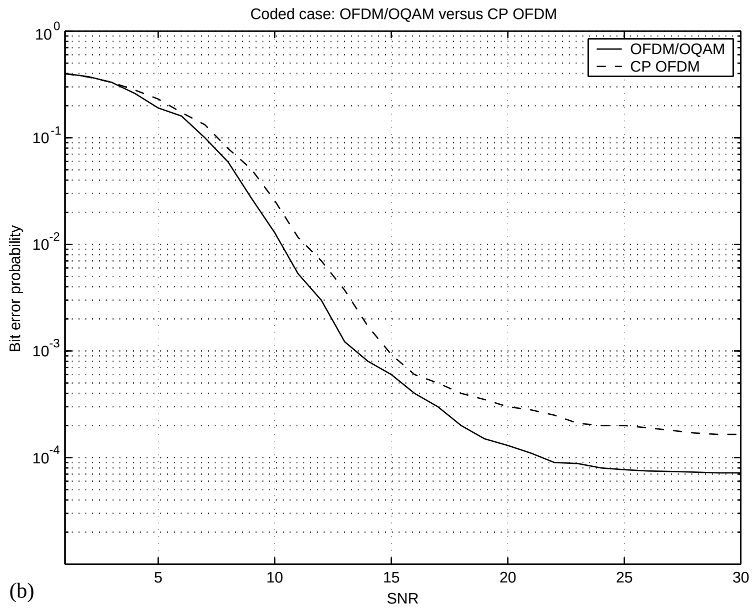


Fig. 6. Comparison between pulse shaping OFDM/OQAM and CP OFDM system. Estimated bit error probability for the (a) uncoded case, and (b) coded case.

example has much better frequency selectivity than that in the first design example. Note, however, that this improved frequency-selectivity comes at the cost of increased filter length.

Design Example 3. In the last design example, we demonstrate that our approach allows the design of OFDM/OQAM systems with a large number of channels and long pulse shaping filters. For $M = 1024$

channels, we designed a pulse shaping filter of length 8192 (i.e., the overlapping factor is 8). The initial filter shown in Figs. 5a and b is a zero-padded lowpass filter (again designed using the MATLAB function FIR1) with nominal bandwidth 1/1024. Figs. 5c and d show the resulting orthogonal filter and its transfer function, respectively. Finally, Fig. 5e shows the corresponding channel filters.

Simulation Example. We shall finally present simulation results giving quantitative support for the use of pulse shaping OFDM/OQAM systems in frequency-dispersive environments. We simulated a stochastic wireless channel with Doppler spread and negligible delay spread. The rms Doppler bandwidth was chosen to be 5% of the subcarrier spacing, which can be seen as a worst case scenario for many applications. We compared an OFDM system with no pulse shaping (i.e. a CP OFDM system) with 16 subcarriers and CP length 4 to a pulse shaping OFDM/OQAM system with 16 subcarriers and pulse shaping filter length 128. All results have been obtained by averaging over 500 independent channel realizations with a packet size of 512 OFDM symbols. We estimated the bit error rate in the uncoded and in the coded case (convolutional code of rate 1/2 and constraint length 3) using 16-QAM in both cases. Coding was performed across tones only and no interleaving was used. Figs. 6a and b show the bit error rate as a function of SNR $= 10\log(\sigma_c^2/\sigma_\rho^2)$ for the uncoded and the coded case, respectively. Here,[5] $\sigma_c^2 = \mathscr{E}|c_{k,l}|^2$, and σ_ρ^2 denotes the variance of the additive white channel noise. It can be seen that in the low-SNR regime (noise dominated) there is no difference between the pulse shaping OFDM/OQAM system and the CP OFDM system. In the high-SNR regime where ICI dominates over channel noise the curves flaten out and pulse shaping OFDM/OQAM outperforms CP OFDM significantly, which is due to the fact that there is much less ICI in the pulse shaping OFDM/OQAM case.

5. Conclusion

We provided DZT-based orthogonality conditions for OFDM/OQAM pulse shaping filters with arbitrary length and arbitrary overlapping factors. Based on these orthogonality conditions we proposed a computationally highly efficient FFT-based design method which basically consists of orthogonalizing an initial (nonorthogonal) lowpass filter. Our algorithm makes the design of pulse shaping filters of lengths up to several thousands of taps possible. We emphasize that the new algorithm does not involve optimization and therefore requires a judicious choice of the initial filter. We found that choosing the initial filter as a lowpass filter with bandwidth $1/M$ where M denotes the number of channels in the OFDM system yields time-frequency well-localized orthogonal filters. Finally, we provided design examples and we demonstrated the superiority of coded and uncoded pulse shaping OFDM/OQAM over OFDM systems without pulse shaping (CP OFDM) in the presence of Doppler spread.

Appendix

We show that a compactly supported initial filter $g[n]$ yields a compactly supported orthogonal filter $g_o[n]$ if and only if

$$|\mathscr{Z}_g(n,\theta)|^2 + \left|\mathscr{Z}_g\left(n,\theta-\frac{1}{2}\right)\right|^2 = c_n \tag{24}$$

with arbitrary constants $c_n \in \mathbb{R}$. The proof is based on the observation that the DZT of a compactly supported $g[n]$ is a polynomial in $e^{j2\pi\theta}$ for all n. Assume that $g[n]$ is compactly supported and satisfies (24). It then follows from (20) that $\mathscr{Z}_{g_o}(n,\theta) = (2/\sqrt{Mc_n})\mathscr{Z}_g(n,\theta)$ which is again polynomial and hence $g_o[n]$ is compactly supported. We shall next prove necessity. Assume that both $g[n]$ and $g_o[n]$ are compactly supported, i.e., both $\mathscr{Z}_g(n,\theta)$ and $\mathscr{Z}_{g_o}(n,\theta)$ are polynomials in $e^{j2\pi\theta}$. From (20) it therefore follows that

$$\sqrt{c_n(\theta)} = \sqrt{|\mathscr{Z}_g(n,\theta)|^2 + \left|\mathscr{Z}_g\left(n,\theta-\frac{1}{2}\right)\right|^2}$$

has to be a monomial in $e^{j2\pi\theta}$, which implies that $c_n(\theta)$ has to be a monomial in $e^{j2\pi\theta}$. Since $c_n(\theta) = c_n^*(\theta)$ for all n, we must have $c_n(\theta) = c_n$ with some constants $c_n \in \mathbb{R}$.

[5] $\mathscr{E}$ denotes the expectation operator.

References

[1] H. Bölcskei, F. Hlawatsch, Discrete Zak transforms, polyphase transforms, and applications, IEEE Trans. Signal Process. 45 (April 1997) 851–866.

[2] R.W. Chang, Synthesis of band-limited orthogonal signals for multi-channel data transmission, Bell System Tech. J. 45 (December 1966) 1775–1796.

[3] L.J. Cimini, Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing, IEEE Trans. Comm. 33 (July 1985) 665–675.

[4] R. Haas, Application des transmissions à porteuses multiples aux communications radio mobiles, Ph.D. Thesis, Ecole Nationale Supérieure des Télécommunications Paris, Paris, France, January 1996.

[5] C. Heil, A discrete Zak transform, Technical Report MTR-89W000128, The MITRE Corporation, December 1989.

[6] B. Hirosaki, An orthogonally multiplexed QAM system using the discrete Fourier transform, IEEE Trans. Comm. 29 (July 1981) 982–989.

[7] B. Hirosaki, S. Hasegawa, A. Sabato, Advanced groupband data modem using orthogonally multiplexed QAM technique, IEEE Trans. Comm. 34 (June 1986) 587–592.

[8] A.J.E.M. Janssen, H. Bölcskei, Equivalence of two methods for constructing tight Gabor frames, IEEE Signal Process. Lett. 7 (April 2000) 79–82.

[9] W. Kozek, A.F. Molisch, Robust and efficient multicarrier communication by nonorthogonal Weyl–Heisenberg systems, IEEE J. Sel. Areas Comm. 16 (October 1998) 1579–1589.

[10] B. LeFloch, M. Alard, C. Berrou, Coded orthogonal frequency division multiplex, Proc. IEEE 83 (June 1995) 982–996.

[11] R. Li, G. Stette, Time-limited orthogonal multicarrier modulation schemes, IEEE Trans. Comm. 43 (February 1995) 1269–1272.

[12] A. Peled, A. Ruiz, Frequency domain data transmission using reduced computational complexity algorithms, in: Proceedings of the IEEE ICASSP-80, Denver, CO, 1980, pp. 964–967.

[13] P.K. Remvik, N. Holte, Carrier frequency offset robustness for OFDM systems with different pulse shaping filters, in: Proceedings of the IEEE GLOBECOM-97, Phoenix, AZ, 1997, pp. 11–15.

[14] C. Roche, P. Siohan, Bancs de filtres modulés de type IOTA/EGF: Le cas orthogonal, Technical Report 5225, CNET, February 1998.

[15] M. Sablatash, Transmission of all-digital advanced television: state of the art and future directions, IEEE Trans. Broadcast. 40 (June 1994) 102–121.

[16] B.R. Saltzberg, Performance of an efficient parallel data transmission system, IEEE Trans. Comm. Technol. 15 (December 1967) 805–811.

[17] M. Sandell, Design and analysis of estimators for multicarrier modulation and ultrasonic imaging, Ph.D. Thesis, Lulea University of Technology, Lulea, Sweden, 1996.

[18] P. Siohan, C. Roche, Analytical design for a family of cosine modulated filter banks, in: Proceedings of the IEEE ISCAS-98, Monterey, CA, May 1998.

[19] A. Vahlin, N. Holte, Optimal finite duration pulses for OFDM, IEEE Trans. Comm. 4 (January 1996) 10–14.

[20] P.P. Vaidyanathan, Multirate Systems and Filter Banks, Prentice-Hall, Englewood Cliffs, NJ, 1993.

[21] M. Zibulski, Y.Y. Zeevi, Oversampling in the Gabor scheme, IEEE Trans. Signal Process. 41 (August 1993) 2679–2687.