ERROR FLOOR OF PULSE AMPLITUDE MODULATION WITH ADAPTIVE SAMPLING IN TIME-DISPERSIVE FADING CHANNELS

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ABSTRACT

We consider the error floor of coherently and differentially detected PAM (pulse amplitude modulation) in time-dispersive fading channels; PAM includes PSK (phase shift keying) and QAM as special cases. We introduce a new Zak-transform based method for computing the error floor. This method can be applied to arbitrary modulation formats and a very general class of time-dispersive channels. We prove that unfiltered BPSK exhibits no error floor when the sampling phase is chosen adaptively and (for coherent detection) the basis pulse satisfies a symmetry condition. For filtered BPSK there is an error floor, which depends on the filter bandwidth. For higher-order modulations, there is *always* an error floor; it can be attributed to I-Q crosstalk.

I. INTRODUCTION

The performance of unequalized wireless communication systems is often not limited by noise, but by intersymbol interference (ISI) caused by the time dispersion of the mobile radio channel; the bit error probability due to ISI is known as "error floor" [1]. Knowledge of the error floor is of particular importance in cordless telephones, e.g. PWT (Personal Wireless Telephone)[2], PHS (Personal Handyphone System)[3], and DECT (Digital Enhanced Cordless Telecommunications)[4], where equalizers are usually avoided because of cost reasons. The above mentioned systems are often also used in low-mobility PCS (personal communications systems), where the time dispersion of the mobile radio channel is larger than in "classical" cordless applications. Investigations of the error floor are thus of great practical as well as theoretical importance.

Most of the earlier investigations of the error floor assume a fixed sampling instant (a. k. a. "sampling phase"), i.e., the offset between the minimum excess delay of the channel and the time instant where the demodulator samples the received signal remains constant over time, see e.g. [5, 6, 7, 8, 9, 10]. In practice the sampling phase is often adjusted according to the instantaneous channel configuration ("adaptive sampling phase" or just "adaptive sampling" for short). This principle has first been suggested by Yoshida et al. [11, 12], who proposed to adjust the sampling phase according to the "eye pattern". Recently, this idea has been extended to training-sequence based adaptation of the sampling phase [13, 14]. In [13, 14], it has been demonstrated that the use of an adaptive sampling phase leads to a reduction of the error floor, with the achievable improvement depending on the modulation format and the filtering. Unfiltered BPSK and MSK have zero error floor, while ($\pi/4-$D)QPSK exhibits an error floor even in the case of adaptive sampling.

Hitherto, investigations of adaptive sampling were based on the assumption of a two-delay channel, which allows a comparatively simple computation of the error floor;[1] furthermore, only differential detection was considered. In this paper, we solve a problem that is in several respects more general: (i) we assume an N-delay Rayleigh or Rician fading channel; (ii) we assume general pulse amplitude modulation (PAM), including M-ary phase shift keying (PSK) and multi-level quadrature amplitude modulation (QAM); (iii) we consider both coherent and differential demodulation. Our approach is based on the so-called Zak-transform (ZT) [15, 16, 17], which will be seen to allow a simple and elegant treatment of the problem. Our new formulation furthermore allows physical insights into the reasons for the different behaviors of BPSK and QPSK.

The paper is organized as follows. In Sec. 2, we describe the considered system and give a brief description of the principle of adaptive sampling. Furthermore the most important properties of the ZT are reviewed, and the basic approach for the investigation of the error floor is outlined. In Secs. 3 and 4, we apply this method to coherent and differential detection, respectively. We furthermore provide physical interpretations of our findings. In Sec. 5, we summarize our results and give conclusions for the design of cordless communications systems.

II. GENERAL CONSIDERATIONS

A. The Basic System

When transmitting over an ideal channel, the received PAM signal can be written as

$$u(t) = \sum_k c_k g(t-k), \tag{1}$$

where the c_k are the complex-valued data symbols, $g(t)$ is the convolution of the transmitter pulse shape $b(t)$ and

[1] The analysis for differentially detected MSK in App. B of [13] assumes an $N-$delay channel, but exploits specific properties of MSK and thus does not allow a straightforward extension to PAM.

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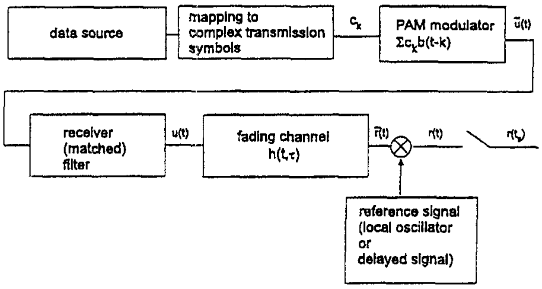

Figure 1: Block diagram of the considered system. Position of matched filter and fading channel are interchanged, see text.

the impulse response of the (matched) filter in the receiver. Without loss of generality, the symbol period satisfies $T = 1$. For unfiltered PAM, the "basis pulse" $g(t)$ is a triangular function. For BPSK, the data symbols satisfy $c_k \in \{+1, -1\}$; for QPSK, it is often convenient to write $c_k \in \{1+j, 1-j, -1+j, -1-j\}$.
For the mobile radio channel, we make the following assumptions: (i) the channel is Rayleigh-fading or Rician-fading [18] (ii) the channel does not change appreciably during one data burst. Practical coherence times are on the order of $10ms$ [13]. The length of a data burst in usual cordless systems, on the other hand, is less than $1ms$. The assumption of the quasi-stationary channel is thus well justified, which implies that the channel impulse response can be written as

$$h(t, \tau) = h(\tau) = \sum_{i=1}^{N} a_i e^{j\phi_i} \delta(\tau - \tau_i), \tag{2}$$

where the $a_i e^{j\phi_i}$ are complex-valued Gaussian random variables; if they are zero-mean, the amplitudes are Rayleigh-fading, otherwise, they are Rician-fading. The τ_i are assumed to be fixed, and τ_0 is set to 0 without loss of generality. We also assume that the time dispersion of the channel is small, i.e., the τ_i are smaller than 1. Since we consider the error floor, the noise added by the channel can be neglected. The receiver then samples the signal $r(t)$, i.e. the convolution of $u(t)$ with $h(t)$, at the time instants $k + t_s$ where t_s can vary with the channel configuration.

B. Adaptive Choice of the Sampling Phase

The training sequence-based adaptive determination of the sampling phase can be realized as follows [13]: during the training sequence, the received signal is oversampled (with respect to the symbol rate) by a factor N_{samp}, i.e., the sampling instants are given by

$$\begin{aligned} t_{k,n} &= \frac{n}{N_{samp}} + k, & (3) \\ n &= -N_{samp}, -N_{samp}+1 \ldots, 2N_{samp} \qquad k \in Z \end{aligned}$$

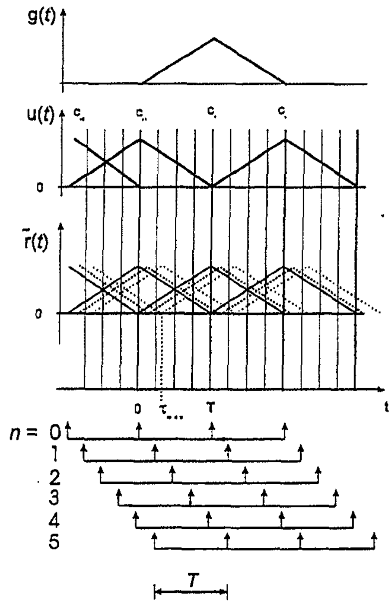

Figure 2: Principle of adaptive sampling with oversampling factor 4. $n = 0, 1$ fulfills the condition $0 < t_s < \tau_{\max}$. $n = 2, 3$ fulfills the condition $\tau_{\max} < t_s < 1$. $n = 4, 5$ is equal to $n = 0, 1$ with $s = 1$.

We then define $3N_{samp}+1$ symbol-rate sampled sequences q_n according to

$$\begin{array}{rlllll} q_1 = & r(t_{1,1}) & r(t_{2,1}) & r(t_{3,1}) & r(t_{4,1}) & .. \\ q_2 = & r(t_{1,2}) & r(t_{2,2}) & r(t_{3,2}) & r(t_{4,2}) & .. \\ .. & .. & .. & .. & .. & .. \\ q_n = & r(t_{1,n}) & r(t_{2,n}) & r(t_{3,n}) & r(t_{4,n}) & .. \\ .. & .. & .. & .. & .. & .. \end{array} \tag{4}$$

We then analyze the various sequences q_n and determine the one that shows the least deviation from the transmitted sequence. The corresponding n_{opt} provides the optimum sampling phase

$$t_{s,opt} = \frac{n_{opt}}{N_{samp}} \tag{5}$$

for detection of the unknown data in the burst.
Even though we have assumed that the maximum excess delay of the channel is smaller than one bit length, the channel can shift the optimum sampling phase over a range $-1 \le t_s \le 2$ [13]. Thus we are allowing n to vary in the range $-N_{samp} \le n \le 2N_{samp}$, which is larger than one bit length. We will incorporate this into our derivations by allowing the sampling phase to be $t_s + s$, where $t_s \in [0, 1)$, while the "word synchronization parameter" s can take on the values $-1, 0, 1$ independently of t_s (see Fig. 2).
We note that if there is filtering, the decision for a certain received bit will be influenced by m adjacent bits with

m depending on the filter bandwidth. When computing the BER, we thus have to compute the BER for each bit $m-$tuplet, and then average over the results.

C. The Zak Transform

The Zak transform [15, 16, 17] is a natural tool for studying the effects of different receiver sampling phases. In fact, it has been shown in [19] that the ZT is a useful tool for the study of the impacts of receiver sampling phase on the equalizability (symbol-spaced or fractionally spaced) of transmission channels.
The ZT of the continuous-time signal $x(t)$ reads [16]

$$Z_x(t,f) = \sum_{l=-\infty}^{\infty} x(t+l)\exp\left(-j2\pi l f\right). \tag{6}$$

$Z_x(t,f)$ is periodic in the frequency variable and quasi-periodic in the time variable, i.e.

$$\begin{aligned} Z_x(t+1,f) &= e^{j2\pi f} Z_x(t,f) && (7)\\ Z_x(t,f+1) &= Z_x(t,f). && (8) \end{aligned}$$

The signal $x(t)$ can be recovered from the ZT according to

$$x(t) = \int_0^1 Z_x(t,f)\,df. \tag{9}$$

The ZT of the shifted signal $x(t-\tau)$ is

$$Z_{x(t-\tau)} = Z_x(t-\tau,f). \tag{10}$$

The ZT of the product of two signals $x_p(t) = x_a(t)x_b(t)$ is given by

$$Z_{x_p}(t,f) = \int_0^1 Z_{x_a}(t,f-\nu)Z_{x_b}(t,\nu)\,d\nu. \tag{11}$$

From (6) it is obvious that $Z_x(t,f)$ is the discrete-time Fourier transform of the sampled version of the signal $x(t)$ shifted by an amount of t.

III. COHERENT DETECTION

A. Decision variable

In the following, we assume that $g(t)$ is supported (has finite values) in the interval $[-N_l, N_h)$. Hence,

$$Z_g(t,f) = \sum_{l=-N_l}^{N_h-1} g(t+l)e^{-j2\pi l f} \qquad \text{for} \quad t \in [0,1). \tag{12}$$

For t not in the fundamental interval, $Z_g(t,f)$ can be obtained from the above expression using the quasi-periodicity relation (7). In the special case of *unfiltered* PAM, $g(t)$ is a triangular pulse supported within $[0,2)$ with its peak at $t=1$. In that case, we obtain from (12)

$$Z_g(t,f) = t + (1-t)e^{-j2\pi f} \qquad \text{for} \quad t \in [0,1). \tag{13}$$

The ZT of the transmitted signal (including receiver filtering) is

$$Z_u(t,f) = Z_g(t,f)C(f), \tag{14}$$

where $C(f)$ denotes the discrete-time Fourier transform of the data sequence c_k, i.e., $C(f) = \sum_k c_k e^{-j2\pi k f}$. Employing the channel model discussed in Sec. 2, the received signal is given by $\tilde{r}(t) = \Sigma_i a_i \exp(j\phi_i)u(t-\tau_i)$. Using (10), the ZT of the received signal reads

$$Z_{\tilde{r}}(t,f) = \sum_i a_i e^{j\phi_i} Z_u(t-\tau_i,f). \tag{15}$$

The quasi-periodicity of the ZT, Eq. (7), allows to take the word synchronization into account by simply multiplying $Z_{\tilde{r}}(t,f)$ by $\exp(j2\pi f s)$, where s is the word synch parameter.
For $-1 \le t_s - \tau_i < 0$, it follows from (7) that

$$Z_u(t_s-\tau_i,f) = Z_u(t_s+1-\tau_i,f)e^{-j2\pi f} \tag{16}$$

This guarantees that the first argument of Z_u and consequently that of Z_g lies within the fundamental interval $t \in [0,1)$. This is necessary because (12), which will be used extensively in the following, is valid for $t \in [0,1)$.
We now have to distinguish two different cases:

- The receiver samples in the range $\tau_{\max} \le t_s < 1$, i.e., $0 \le t_s - \tau_i < 1$ (note that due to our definitions of $g(t)$, the sampling time in a distortionless channel is $t_s = 1$. In order to fall into the fundamental inverval, we have to use $t_s = 0$ and $s = 1$ in this case). Applying the inversion equation (9) and taking into account the effect of word synchronization, the received signal is given by:

$$\tilde{r}(t_s) = \int_0^1 e^{j2\pi f s} \sum_i a_i e^{j\phi_i} Z_g(t_s-\tau_i,f)C(f)df \tag{17}$$

which yields

$$\tilde{r}(t_s) = \sum_i a_i e^{j\phi_i} \sum_k c_k \sum_{l=-N_l}^{N_h-1} g(t_s+l-\tau_i)\delta_{s-k-l} \tag{18}$$

with $\delta_k = \delta_{k,0}$ denoting the Kronecker delta. The actual decision variable is obtained by multiplying the received signal with a local oscillator (LO). If the LO uses a narrow-band filtered version of the received signal, then multiplication with the LO and application of gain control can be written as division by $\Sigma a_{i'}e^{j\phi_{i'}}$ so that the decision variable is obtained as

$$r(t_s) = \sum_k c_k \sum_{l=-N_l}^{N_h-1} y_l(t_s)\delta_{s-k-l}, \tag{19}$$

where

$$y_l(t_s) = \frac{\sum_i a_i e^{j\phi_i} g(t_s+l-\tau_i)}{\sum_{i'} a_{i'} e^{j\phi_{i'}}} \tag{20}$$

is a random variable obtained by dividing two correlated Gaussian random variables. The corresponding correlation coefficient depends on the value of t_s. To simplify notation, the dependence of y_l on t_s will henceforth be written explicitly only when required.

- The receiver samples in the range $0 \leq t_s \leq \tau_{\max}$. In the following, N_a denotes the number of advanced echoes, i.e., echoes whith delays smaller than t_s, and N_d the number of delayed echoes. Note that N_a and N_d depend on the sampling time t_s. Advanced echoes satisfy $0 \leq t_s - \tau_i < 1$, whereas for delayed echoes, we have $-1 \leq t_s - \tau_i < 0$. Now using (16) and defining

$$y_{a,l}(t_s) = \frac{\sum_{i=1}^{N_a(t_s)} a_i e^{j\phi_i} g(t_s + l - \tau_i)}{\sum_{i'=1}^{N} a_{i'} e^{j\phi_{i'}}} \tag{21}$$

$$y_{d,l}(t_s) = \frac{\sum_{i=N_a(t_s)+1}^{N} a_i e^{j\phi_i} g(t_s + l - \tau_i)}{\sum_{i'=1}^{N} a_{i'} e^{j\phi_{i'}}}$$

the decision variable is obtained as

$$r(t_s) = \sum_k c_k \sum_{l=-N_l}^{N_h-1} y_{a,l}\delta_{s-k-l} + \sum_k c_k \sum_{l=-N_l}^{N_h-1} y_{d,l}\,\delta_{s-k-l-1} \tag{22}$$

It is important to recognize that although the random variables $y_{d,l}$, and $y_{a,l}$ are correlated, they can in principle take on every complex value.

B. Error floor of BPSK

Based on the general equations for the decision variable, Eqs. (19), (22), we shall next analyze the error floor of unfiltered BPSK. Let us assume that the sampling time satisfies $\tau_{\max} < t_s \leq 1$, and that $g(t)$ is supported in $[0,2)$. For BPSK, the possible symbols are $c_k = \pm 1$, and the decision is based on the real part of the output variable, so that we have

$$\mathrm{Re}\{r(t_s)\} = c_s \widehat{y}_0 + c_{s-1}\widehat{y}_1 \tag{23}$$

where $\widehat{y}_l = \mathrm{Re}\{y_l\}$. For the case $c_s = c_{s-1}$, we require that

$$\max_t \{\widehat{y}_0(t) + \widehat{y}_1(t)\} > 0 \qquad \text{for} \quad t \in [\tau_{\max}, 1) \tag{24}$$

in order to achieve a correct decision. The possibility of changing the word synch parameter s does not help in that case, because $c_s = c_{s-1}$, so that using a different word synch parameter does not alter the decision. In principle, the random variables $\widehat{y}_0$ and $\widehat{y}_1$ can take on arbitrary values. However, if we choose the basis pulse $g(t)$ such that

$$g(t) + g(t+1) = 1 \qquad \text{for} \quad t \in [0,1), \tag{25}$$

then $\widehat{y}_0 + \widehat{y}_1 = 1$ for all t. In other words, satisfying Eq. (25) guarantees correct transmission when $c_s = c_{s-1}$. The implications of this condition will be discussed more extensively below.

If $c_s = -c_{s-1}$, we get $r(t_s) = c_s[2\widehat{y}_0 - 1]$. One certain channel constellation corresponds to a certain value of $\widehat{y}_0$. Depending on the channel constellation, $\widehat{y}_0$ can take on all possible values. If the term in brackets is larger than zero then[2] $\mathrm{Re}\{r(t_s)\} = c_s$; otherwise, $\mathrm{Re}\{r(t_s)\} = -c_s$, and thus $\mathrm{Re}\{r(t_s)\} = c_{s-1}$.

The aim of all detection and synchronization procedures is to have the detected bit agree with the transmitted c_k. If we use for some channel constellations ($\widehat{y}_0 < \frac{1}{2}$) the word synch shift $s = 0$, and for the other channel constellations the shift $s = +1$, then we always detect correctly. The training sequence allows us to find out the current channel constellation, and thus the correct word synch shift. The exact value of t_s does not even play a role for the error floor, just as long as we sample within the correct interval. Of course, t_s becomes important when noise is also taken into account.

The above considerations relied on the fact that only two bits can influence the decision. This requires on the one hand that $\tau_{\max} \leq t_s < 1$, and on the other hand that the support of the basis pulse is restricted to the range $[0,2)$. Otherwise, several bits, and consequently several $\widehat{y}_l$ will influence the decision at the receiver. It is thus not possible to *always* find a value for s that guarantees an error-free transmission for *all* bit combinations.

When we apply filtering, e.g., to make the spectrum of the transmitted signal narrower, and thus increase the spectral efficiency, the basis pulse extends over a range that is larger than $[0,2)$. The smaller the filter bandwidth, the larger the number of relevant $\widehat{y}_l$, and also the smaller the correlation between them. This proves the (intuitively clear) conjecture that the smaller the bandwidth, the larger is the error floor for adaptive sampling.

Let us now consider the implications of Eq. (25). We require a certain symmetry from the basis pulse. For unfiltered PAM, i.e., when $g(t)$ is a triangular pulse, the condition is obviously satisfied. In the case of fractional-bit detection, where the integrator in the receiver integrates not from 0 to 1, but only from T_f to 1, we have

$$g(t) = \begin{cases} \frac{t}{1-T_f} & 0 < t < 1 - T_f \\ 1 & 1 - T_f < t < 1 \\ 1 - \frac{t-1}{1-T_f} & 1 < t < 2 - T_f \\ 0 & \textit{otherwise} \end{cases} \tag{26}$$

and hence the symmetry condition is satisfied.

[2]strictly speaking, of course $\mathrm{Re}\{r(t_s)\} = \varkappa c_s$ where $\varkappa$ is a positive constant. However, this has no influence on the decision; so to simplify notation, we do not explicitely write this positive constant in the following.

We stress that for zero error floor, *both* the symmetry condition *and* the support condition (i.e. $g(t)$ must be zero outside the interval $[0, 2)$) must be fulfilled. Nyquist pulses, for example, do not allow to achieve zero error floor - they fulfill neither condition.

C. Error floor of QPSK

Also for QPSK, we start with the assumption that the sampling instant lies between $\tau_{\max}$ and 1. The output variable is then

$$r(t_s) = c_s y_0 + c_{s-1} y_1 \tag{27}$$

The transmit symbols satisfy $c_k \in \{1+j, 1-j, -1+j, -1-j\}$; this allows to make the decisions for the received symbols separately for real and imaginary part. We write the real part of the output signal as

$$\begin{aligned} \mathrm{Re}\{r(t_s)\} \;=\; & \mathrm{Re}\{c_s\}\,\mathrm{Re}\{y_0\} + \mathrm{Re}\{c_{s-1}\}\,\mathrm{Re}\{y_1\} \\ & - \mathrm{Im}\{c_s\}\,\mathrm{Im}\{y_0\} - \mathrm{Im}\{c_{s-1}\}\mathrm{Im}\{y_1\} \end{aligned} \tag{28}$$

If the second line in Eq. (28) would not be present, the error floor could be eliminated by appropriate choice of the word synch parameter s, as we proved for the BPSK case. The complications arising in the QPSK case stem from the fact that the real and imaginary parts of the symbols c_s are independent of each other. If $\mathrm{Im}\{c_s\} = \mathrm{Im}\{c_{s-1}\}$, then the second line in Eq. (28) is zero (due to our symmetry requirement, $y_0 + y_1 = 1$), and we have to choose the word synch parameter just as in the case of BPSK. This means that s is fixed, and we have no more "degrees of freedom". For those symbol combinations where $\mathrm{Im}\{c_s\} = -\mathrm{Im}\{c_{s-1}\}$, the second line can take on either a large positive value, or a large negative value, according to the *quadrature* component of the symbols. This second line thus acts as interference, and can cause erroneous decisions. Obviously, an error-free transmission is not possible when the sampling time lies between 0 and $\tau_{\max}$, because in that case, even more bits influence the decision. Similar conclusions hold when the modulation format is not QPSK, but a more complex QAM modulation format. As long as there is an independent quadrature component, it can lead to uncontrollable interference (crosstalk).

This derivation corroborates Yoshida's [11] conjecture that the "quadrature component interference" (I-Q crosstalk) is a major reason for the error floor in systems with adaptive sampling phase. We also note, however, that this is not true for MSK. While in MSK the transmit symbols also can take on 4 values in the constellation diagram $(+1, +j, -1, -j)$, in-phase and quadrature phase are not independent of each other: if $\mathrm{Im}(c) \neq 0$, then $\mathrm{Re}\{c\} = 0$ and vice versa. This explains why MSK can be transmitted with zero error floor (see [13]).

IV. DIFFERENTIAL DETECTION

A. Decision variable

For differential detection, the form of the output signal is somewhat more complicated. The decision variable is $\widetilde{r}(t_s)\widetilde{r}^*(t_s-1)$. Using (9) and (11), the decision variable (including word synchronization) can be written as

$$\begin{aligned} r(t_s) \;=\; & \int_0^1 \int_0^1 e^{j2\pi s f} Z_{\widetilde{r}}(t_s, f-\nu) \\ & Z^*_{\widetilde{r}}(t_s, -\nu) e^{-j2\pi\nu}\, d\nu df \end{aligned} \tag{29}$$

where we have used the fact that $Z_{x^*(t-1)} = Z^*_x(t, -f)\exp(-j2\pi f)$. If we have only advanced echoes, i.e. if the sampling time lies between $\tau_{\max}$ and 1, the decision variable $r(t_s)$ is given by

$$\begin{aligned} & \int_0^1 \int_0^1 e^{j2\pi s f} e^{-j2\pi\nu} \cdot \sum_i a_i e^{j\phi_i} \sum_k c_k e^{-j2\pi k(f-\nu)} \\ & \left[\sum_l g(t+l-\tau_i) e^{-j2\pi k(f-\nu)}\right] \cdot \\ & \sum_{i'} a^*_{i'} e^{-j\phi_{i'}} \sum_{k'} c^*_{k'} e^{-j2\pi k'\nu} \\ & \left[\sum_{l'} g^*(t+l'-\tau_i) e^{-j2\pi k'\nu}\right] d\nu df \end{aligned}$$

If the support of $g(t)$ is limited to $[0, 2)$ and $\tau_{\max} < t_s \leq 1$, we get

$$\frac{r(t_s)}{|p|^2} = c_s c^*_{s-1} + c_s c^*_{s-2} v^* + c_{s-1} c^*_{s-1} v + c_{s-1} c^*_{s-2} v v^* \tag{30}$$

where we have introduced the random variables

$$\begin{aligned} p &= \sum_i a_i \exp(j\phi_i) g(t_s - \tau_i) \\ q &= \sum_i a_i \exp(j\phi_i) g(t_s + 1 - \tau_i) \\ v &= \frac{q}{p} \end{aligned} \tag{31}$$

B. BPSK

For the computation of the error floor of BPSK, let us assume that sampling occurs at $\tau_{\max} \leq t_s \leq 1$. Furthermore, the symbols are ± 1, so that the complex conjugation of the symbols has no effect; we also replace v by $\widetilde{v} = \mathrm{Re}\{v\} = \mathrm{Re}\{v^*\}$. The variable vv^* is real, and is correlated with $\widetilde{v}$, but can take on all possible values. The positive factor $|p|^2$ has no influence on the decision and will therefore be omitted in the following. The decision variable can then be written as

$$\begin{aligned} \mathrm{Re}\{r(t_s)\} \;=\; & vv^*\,[c_{s-1} c_{s-2}] + \\ & \widetilde{v}\,[c_s c_{s-2} + c_{s-1} c_{s-1}] + c_s c_{s-1} \end{aligned} \tag{32}$$

Considering the various possible bit combinations (analogously to the case of coherent detection), we find that we have to distinguish the following channel configurations[3]

- $|v|^2 > 1$: in that case, $s = 0$
- $|v|^2 < 1$: in that case, $s = 1$.

It is noteworthy that for differential detection, we need no symmetry of the basis pulse. The symmetry inherent in the differential detection procedure is sufficient to obtain zero error floor if the support condition is fulfilled.

C. QPSK

As an example for QPSK, we consider $\pi/4$—shifted DQPSK, since this has the greatest practical importance. Abbreviating $c_s c_{s-1}^*$ as d_s, we know that all d_s satisfy $d_s \in \{1+j, 1-j, -1+j, -1-j\}$. The output signal can then be written as

$$r(t_s) = d_s + d_s d_{s-1} v^* + v + d_{s-1} v v^* \qquad (33)$$

Closer inspection shows (analogously to the coherently detected QPSK) that the word synch parameter s cannot be chosen in such a way that the error floor becomes zero: for $\mathrm{Re}\{r(t_s)\}$ there are always contributions involving the quadrature components, $\mathrm{Im}\{d_s\}$, $\mathrm{Im}\{d_{s-1}\}$. Since the sign of these cannot be controlled (they are independent of the real parts), there is always the possibility that a large (if v is large) quantity with a sign that can be both positive and negative (depending on $\mathrm{Im}\{d_s\}$) is added. This prevents an error-free decision. The situation for $\mathrm{Im}\{r(t_s)\}$ is completely analogous.
Let us consider an example: $\mathrm{Re}\{d_{s-1}\} = -1$, $\mathrm{Re}\{d_s\} = 1$, $\mathrm{Re}\{v\} = 0$, $\mathrm{Im}\{v\} = 1.2$. If $\mathrm{Im}\{d_{s-1}\} = \mathrm{Im}\{d_s\}$, then $\mathrm{Re}\{r(1)\} = -0.44$, i.e. the word synch parameter s must be chosen to be $s = 1$. If, however, $\mathrm{Im}\{d_s\} = -1$, $\mathrm{Im}\{d_{s-1}\} = 1$, then $\mathrm{Re}\{r(1)\} = +1.96$, and s must be chosen to be 0. Since s cannot simultaneously have two different values, error-free transmission is not possible.

V. SUMMARY AND CONCLUSIONS

We computed the error floor for PAM modulation with adaptive choice of the sampling phase. Our derivations were based on the Zak transform, a tool that lends itself naturally to the problem of adaptive sampling. The receiver output (decision variables) was computed by exploiting elementary and well-documented properties of the Zak transform. The applied method is thus simple and applicable to a large class of modulation formats.
With this approach, we have shown that for BPSK, the error floor can be reduced to zero just as long as the maximum excess delay of the channel is smaller than the symbol length, and the basis pulse is zero outside the interval $[0, 2)$. For QPSK, such a reduction is not possible because of the quadrature-component interference. The same is true for any higher-order QAM modulation format. These results are valid both for coherent and for

[3] The details are omitted because of space restrictions

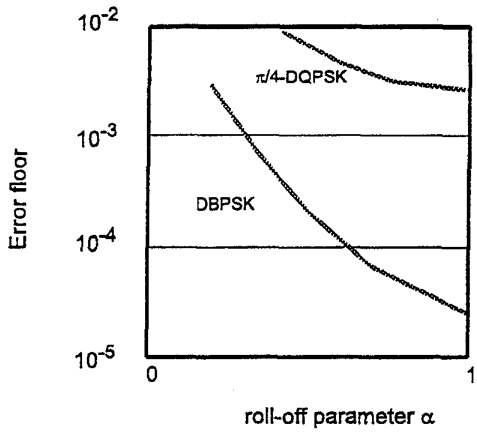

Figure 3: Error floor for DBPSK and $\pi/4$-DQPSK for $S/T = 0.2$. Nyquist filters with roll-off parameter α.

differential detection of the received signal. If we apply filtering, so that the length of the basis pulse is longer than 2, several bits influence the decision at the receiver. It is thus not possible to *always* find a value for s that guarantees an error-free transmission for *all* bit combinations.
For the design of cordless systems, the above considerations have important consequences.

- BPSK is superior to QPSK with respect to the error floor. This is in contrast to the widespread opinion that BPSK is inferior in time-dispersive environments because the symbol duration is shorter (for identical bit rate). We showed, however, that the advantages of adaptive sampling with BPSK more than compensate this disadvantage. For cordless phones in highly time-dispersive channels, use of the "simple" BPSK is thus an attractive approach.
- The filter bandwidth has a very strong influence on the error floor of BPSK, while it has less influence on QPSK. This effect can also be seen from Fig. 3, where the error floor for BPSK with Nyquist pulses is shown as a function of the roll-off parameter (the numerical values are obtained by computer experiments).
- We showed that the quadrature-component interference prevents zero error floor for QPSK. We furthermore showed that this must also hold for more complex modulation format, such as M-ary PSK and combined amplitude- and phase shift keying, if the real and imaginary parts of the transmission symbols are uncorrelated.
- Not only the delay spread, but also the maximum excess delay plays an important role in describing the

cordless telephone channel; it determines whether in principle zero error floor is possible or not.

- For coherent detection of BPSK, we require that the basis pulse fulfills a symmetry condition in addition to the requirement of limited support of the pulse (limited to $[0, 2)$). For differential detection, no such symmetry requirement exists, because there is an inherent symmetry in the detection method.

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