BLIND CHANNEL IDENTIFICATION IN HIGH-DATA-RATE PULSE SHAPING OFDM/OQAM SYSTEMS

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Abstract—Pulse shaping OFDM systems based on offset QAM (OFDM/OQAM) are particularly attractive for wireless high-data-rate applications. Since OFDM/OQAM schemes do not employ a cyclic prefix, equalization has to be performed. In this paper, we introduce an algorithm for blind channel identification in pulse shaping OFDM/OQAM systems. Our approach assumes that the receiver knows the transmitter pulse shaping filter. Exploiting cyclostationarity of the received OFDM signal blind channel identification is accomplished using second-order statistics only. The proposed method exhibits low noise sensitivity, is computationally efficient, and does not require knowledge of the channel order. Finally, we provide simulation results demonstrating the performance of the new algorithm.

1. INTRODUCTION AND OUTLINE

Orthogonal frequency division multiplexing (OFDM) [1]-[6] has been adopted or proposed for several applications, such as satellite and terrestrial digital audio broadcasting (DAB), digital terrestrial TV broadcasting, asymmetric digital subscriber line (ADSL) for high-bit-rate digital subscriber services on twisted-pair channels, and broadband indoor wireless systems.

Recently, it has been pointed out in [7]-[9] that OFDM systems based on offset QAM (OFDM/OQAM) [10] bypass a major disadvantage of OFDM schemes based on ordinary QAM, namely the fact that time-frequency well-localized (and hence dispersion-robust) pulse shaping filters are prohibited in the case of critical time-frequency density where spectral efficiency is maximal. Pulse shaping OFDM/OQAM systems are therefore well-suited for wireless high-data-rate applications [9]. However, since no cyclic prefix (CP) [2] is employed, equalization has to be performed.

Usually, channel identification in OFDM systems is accomplished using training data resulting in a reduction of spectral efficiency [11]. In [12] an algorithm for the blind identification of time-dispersive channels in CP OFDM systems has been introduced. In this paper, based on the results reported in [12, 13], we develop a novel method for blind channel identification in pulse shaping OFDM/OQAM systems. Let us briefly summarize important features of our algorithm:

- it applies to *pulse shaping OFDM/OQAM* and biorthogonal frequency division multiplexing systems based on OQAM (BFDM/OQAM) [9] with *arbitrary pulse shapes.*
- it does *not require knowledge of the channel order.*
- *it exhibits low noise sensitivity.*
- *it does not need a CP.*
- it is *computationally efficient.*

The paper is organized as follows. Section 2 briefly reviews pulse shaping OFDM/OQAM systems and provides our assumptions and the problem statement. Section 3 shows that under quite general conditions pulse shaping OFDM/OQAM signals are cyclostationary. Section 4 introduces the novel channel identification method and discusses its properties. Finally, in Section 5 we present simulation results, and Section 6 concludes the paper.

2. PULSE SHAPING OFDM/OQAM SYSTEMS

OFDM/OQAM systems. In the following we consider pulse shaping OFDM/OQAM systems with the transmit signal given by $x[n] = \sum_{k=0}^{M-1} x_k[n]$, where

$$\begin{aligned} x_k[n] &= \sum_{l=-\infty}^{\infty} c_{k,l}^{\mathcal{R}} g[n-lM]\, e^{j\frac{2\pi}{M}k(n-\alpha/2)} \\ &+ \sum_{l=-\infty}^{\infty} j c_{k,l}^{\mathcal{I}} g[n+M/2-lM]\, e^{j\frac{2\pi}{M}k(n-\alpha/2)}. \end{aligned}$$

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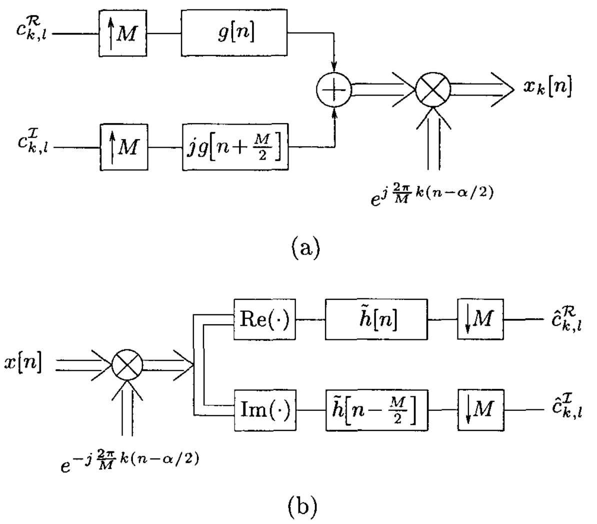

Fig. 1. Pulse shaping OFDM/OQAM system ($\tilde{h}[n] = h[-n]$): (a) k-th transmitter subchannel, (b) k-th receiver subchannel.

Here, $c^{\mathcal{R}}_{k,l} = \mathrm{Re}\{c_{k,l}\}$ and $c^{\mathcal{I}}_{k,l} = \mathrm{Im}\{c_{k,l}\}$ denote the real and imaginary parts of the data symbols $c_{k,l}$, respectively, $g[n]$ is the real-valued transmitter pulse shaping filter of length L_g, and[1] $\alpha = (L_g + \frac{M}{2} - 1) \bmod M$. Throughout this paper we assume that the number of subcarriers M is even. The receiver pulse shaping filter is denoted as $h[n]$ (see Fig. 1). In the orthogonal case (OFDM/OQAM) we have $h[n] = g[n]$.

Assumptions. We assume perfect synchronization, i.e., the received OFDM signal is given by

$$r[n] = (x * d)[n] + \rho[n], \qquad (1)$$

where $d[n]$ denotes the (complex-valued) time-dispersive channel of length L_d, and $\rho[n]$ is a wide-sense-stationary noise process, independent of the data symbols $c_{k,l}$. The correlation function of the noise process is given by[2] $c_\rho[\tau] = \mathcal{E}\{\rho[n]\rho^*[n-\tau]\}$. The symbols $c_{k,l}$ are drawn from a finite-alphabet complex constellation and satisfy $\mathcal{E}\{c^{\mathcal{R}}_{k,l}c^{\mathcal{R}}_{k',l'}\} = \sigma^2_{c,R}\delta[k-k']\delta[l-l'], \mathcal{E}\{c^{\mathcal{I}}_{k,l}c^{\mathcal{I}}_{k',l'}\} = \sigma^2_{c,I}\delta[k-k']\delta[l-l']$, and $\mathcal{E}\{c^{\mathcal{R}}_{k,l}c^{\mathcal{I}}_{k',l'}\} = 0$. We furthermore assume that the receiver knows the transmitter pulse shaping filter $g[n]$, and the variances $\sigma^2_{c,R}$ and $\sigma^2_{c,I}$. This knowledge constitutes the basis for the blind identification algorithms discussed in the paper.

Problem statement. We want to derive estimates of the channel impulse response $d[n]$ from second-order statistics of the received signal $r[n]$, aiming at estimators that do not require knowledge of the distributions of $x[n]$ and $\rho[n]$, and apply to the blind (nondata-aided) scenario.

[1] mod stands for the modulo operation.
[2] $\mathcal{E}$ denotes the expectation operator.

3. CYCLOSTATIONARITY IN PULSE SHAPING OFDM/OQAM SYSTEMS

The key to our blind estimation algorithm is cyclostationarity of the received signal. We shall first show that under quite general conditions the OFDM/OQAM transmit signal $x[n]$ is cyclostationary (CS) [14], which using (1) implies cyclostationarity of the received signal (see Sec. 4). The correlation function of a (nonstationary) stochastic process is defined as $c_x[n,\tau] = \mathcal{E}\{x[n]x^*[n-\tau]\}$, where τ is an integer lag parameter.[3] The signal $x[n]$ is said to be second-order CS with period M if $c_x[n,\tau] = c_x[n+M,\tau]$ [14].

We shall next establish cyclostationarity of pulse shaping OFDM/OQAM transmit signals. It can be shown that the correlation function of $x[n]$ is given by [15]

$$c_x[n,\tau] = \delta_M[\tau]\left[\sigma^2_{c,R}a^{(\tau)}_M[n] + \sigma^2_{c,I}a^{(\tau)}_M\left[n + \frac{M}{2}\right]\right], \qquad (2)$$

where $\delta_M[\tau] = M\sum_{s=-\infty}^{\infty}\delta[\tau - sM]$, and $a^{(\tau)}_M[n] = \sum_{l=-\infty}^{\infty} g[n-lM]g[n-lM-\tau]$. From $a^{(\tau)}_M[n+M] = a^{(\tau)}_M[n]$, it follows that $c_x[n+M,\tau] = c_x[n,\tau]$. Thus, the signal $x[n]$ is indeed CS with period M. If no pulse shaping is employed, i.e., $g[n] = \frac{1}{\sqrt{M}}$ for $n \in [0, M-1]$ and 0 else, we obtain

$$c_x[n,\tau] = (\sigma^2_{c,R} + \sigma^2_{c,I})\,\delta[\tau].$$

This implies that the signal $x[n]$ is stationary and therefore blind identification of arbitrary time-dispersive channels using second-order statistics is not possible [16].

We conclude this section by noting that it is remarkable that pulse shaping OFDM/OQAM signals are CS although no guard interval (like a CP for example) is employed and consequently the transmit signal contains no redundancy.

4. BLIND IDENTIFICATION ALGORITHM

In this section, we shall introduce a novel method for the blind identification of time-dispersive channels from second-order statistics of the received noisy signal $r[n]$. The correlation function of $r[n]$ is given by

$$c_r[n,\tau] = \sum_{s=0}^{L_d-1} d[s] \sum_{r=-\infty}^{\infty} c_x[n-s,r]\,d^*[s-\tau+r] + c_\rho[\tau],$$

which using (2) shows that $c_r[n+M,\tau] = c_r[n,\tau]$. Hence, $r[n]$ is CS with period M. The Fourier series coefficients $C_r[k,\tau] = \frac{1}{M}\sum_{n=0}^{M-1} c_r[n,\tau]\,e^{-j\frac{2\pi}{M}kn}$ of $c_r[n,\tau]$ with respect to n are given by

$$C_r[k,\tau] = \sum_{s=0}^{L_d-1} d[s]\,e^{-j\frac{2\pi}{M}ks} \sum_{r=-\infty}^{\infty} d^*[s-\tau+r]\,C_x[k,r] + c_\rho[\tau]\delta[k], \quad k = 0,1,...,M-1, \qquad (3)$$

[3] For stationary processes the correlation function $c_x[n,\tau]$ is a function of the lag parameter τ only.

with the Fourier series coefficients

$$\mathcal{C}_x[k,\tau] = \frac{1}{M}\,\delta_M[\tau]A^{(g,g)}\left[\tau, \frac{k}{M}\right)\,[\sigma^2_{c,R} + (-1)^k\sigma^2_{c,I}] \quad (4)$$

of $c_x[n,\tau]$ with respect to n. Here,

$$A^{(g,g)}[\tau,\theta) = \sum_{n=-\infty}^{\infty} g[n]g[n-\tau]\,e^{-j2\pi n\theta}$$

denotes the auto-ambiguity function of $g[n]$ [17]. In the following, we shall need the cyclic spectrum of $r[n]$ defined by [14] $\mathcal{S}_r[k,z) = \sum_{\tau=-\infty}^{\infty} \mathcal{C}_r[k,\tau]\,z^{-\tau}$. Using (3), we get[4]

$$\mathcal{S}_r[k,z) = D\left(z\,e^{j\frac{2\pi}{M}k}\right)\mathcal{S}_x[k,z)\tilde{D}(z) + S_\rho(z)\delta[k],$$

where $\mathcal{S}_x[k,z)$ denotes the cyclic spectrum of $x[n]$, and $S_\rho(z) = \sum_{\tau=-\infty}^{\infty} c_\rho[\tau]z^{-\tau}$. Now, following a procedure first suggested in [13] and later used in [18, 12] we can take two different cycles $k_1 \in [1, M-1]$ and $k_2 \in [1, M-1]$ to find

$$\begin{aligned} & D\left(z\,e^{j\frac{2\pi}{M}k_2}\right)\mathcal{S}_r[k_1,z)\mathcal{S}_x[k_2,z) \\ = \; & D\left(z\,e^{j\frac{2\pi}{M}k_1}\right)\mathcal{S}_r[k_2,z)\mathcal{S}_x[k_1,z). \end{aligned} \quad (5)$$

Rewriting (5) in the time-domain yields

$$\sum_{l=0}^{L_d-1}\left[a_{r,x}^{(k_1,k_2)}[n-l]\,e^{-j\frac{2\pi}{M}k_2 l} - a_{r,x}^{(k_2,k_1)}[n-l]\,e^{-j\frac{2\pi}{M}k_1 l}\right] d[l] = 0 \quad (6)$$

with

$$a_{r,x}^{(k,l)}[n] = \sum_{s=-\infty}^{\infty} \mathcal{C}_r[k,s]\,\mathcal{C}_x[l,n-s]. \quad (7)$$

From (4) it follows that $\mathcal{C}_x[k,\tau] = 0$ for $|\tau| \geq L_g$. Furthermore, (3) implies that $\mathcal{C}_r[k,\tau] = 0$ for $k \in [1, M-1]$ and $|\tau| \geq L_g + L_d - 1$. We therefore have $a_{r,x}^{(k,l)}[n] = 0$ for $|n| \geq 2L_g + L_d - 2$. Note furthermore, that the influence of stationary additive noise $\rho[n]$ can be eliminated by considering nonzero cycles only. Eq. (6) can be rewritten in vector-matrix form as

$$\underbrace{[\mathbf{T}_{r,x}^{(k_1,k_2)}\,\mathbf{D}^{k_2} - \mathbf{T}_{r,x}^{(k_2,k_1)}\,\mathbf{D}^{k_1}]}_{\mathbf{S}_{r,x}^{(k_1,k_2)}}\mathbf{d} = \mathbf{0} \quad (8)$$

with the $(4L_g + 3L_d - 6) \times L_d$ Toeplitz matrices $\mathbf{T}_{r,x}^{(k,l)}$ with first row $[a_{r,x}^{(k,l)}[-2L_g - L_d + 3]\ 0\ \ldots\ 0]$ and first column $[a_{r,x}^{(k,l)}[-2L_g - L_d + 3]\ \ldots\ a_{r,x}^{(k,l)}[2L_g + L_d - 3]\ 0\ \ldots\ 0]$, and the $L_d \times L_d$ diagonal matrix $\mathbf{D} = \text{diag}\{e^{-j\frac{2\pi}{M}l}\}_{l=0}^{L_d-1}$. Furthermore, $\mathbf{d} = [d[0]\ d[1]\ \ldots\ d[L_d-1]]^T$ and $\mathbf{0}$ denotes the $(4L_g + 3L_d - 6) \times 1$ zero vector. From (8) it follows that the channel $d[n]$ can be uniquely recovered up to a complex scalar from (6) if the matrix $\mathbf{S}_{r,x}^{(k_1,k_2)}$ has nullity one. Using Theorem 1 in [18] it can be shown that the channel is uniquely identifiable within a complex scalar if and only if there is no $l \in [1, L_d-1]$ such that $e^{-j\frac{2\pi}{M}k_1 l} = e^{-j\frac{2\pi}{M}k_2 l}$. In this case, if the true correlations $\mathcal{C}_r[k,\tau]$ are known the channel can be found as the unique null eigenvector of[5] $\mathbf{S}_{r,x}^{(k_1,k_2)^H}\mathbf{S}_{r,x}^{(k_1,k_2)}$. In [15] it is shown that if the true correlations $\mathcal{C}_r[k,\tau]$ are available, the channel order can be estimated from $\mathbf{S}_{r,x}^{(k_1,k_2)}$. In practice, however, $\mathcal{C}_r[k,\tau]$ has to be estimated from a finite data record $\{r[n]\}_{n=0}^{L-1}$ of length L according to [14, 19]

$$\hat{\mathcal{C}}_r[k,\tau] = \frac{1}{L}\sum_{n=0}^{L-1} r[n]r^*[n-\tau]e^{-j\frac{2\pi}{M}kn}. \quad (9)$$

Based on these estimates of $\mathcal{C}_r[k,\tau]$ a channel estimate can be obtained by solving the following optimization problem:

$$\begin{aligned} \hat{D}(z) \;=\; & \arg\min_{D(z)}\left\|D\left(z\,e^{j\frac{2\pi}{M}k_2}\right)\hat{\mathcal{S}}_r[k_1,z)\mathcal{S}_x[k_2,z)\right. \\ & \left. -\; D\left(z\,e^{j\frac{2\pi}{M}k_1}\right)\hat{\mathcal{S}}_r[k_2,z)\mathcal{S}_x[k_1,z)\right\|^2, \end{aligned}$$

where $\hat{\mathcal{S}}_r[k,z) = \sum_{\tau=-\infty}^{\infty} \hat{\mathcal{C}}_r[k,\tau]\,z^{-\tau}$. The solution of this problem $\hat{D}(z) = \sum_{l=0}^{L_d-1}\hat{d}[l]\,z^{-l}$ is obtained from (8) as

$$\hat{\mathbf{d}} = \arg\min_{\mathbf{d}\neq\mathbf{0}}\left\|\hat{\mathbf{S}}_{r,x}^{(k_1,k_2)}\mathbf{d}\right\|^2, \quad (10)$$

where $\hat{\mathbf{S}}_{r,x}^{(k_1,k_2)}$ is an estimate of $\mathbf{S}_{r,x}^{(k_1,k_2)}$ obtained by replacing $\mathcal{C}_r[k,\tau]$ in (7) by $\hat{\mathcal{C}}_r[k,\tau]$. Since $g[n]$, $\sigma^2_{c,R}$, and $\sigma^2_{c,I}$ are known to the receiver, the cyclic spectra $\mathcal{S}_x[k,z)$ can be computed from (4). The solution of (10) is the eigenvector of $\hat{\mathbf{S}}_{r,x}^{(k_1,k_2)^H}\hat{\mathbf{S}}_{r,x}^{(k_1,k_2)}$ associated to the smallest eigenvalue. The influence of the choice of the cycles k_1 and k_2 on the performance of the algorithm is currently under investigation.

5. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the performance of the proposed identification algorithm. We used an OFDM/OQAM system with $M = 8$ channels and pulse shaping filter length 16. The data symbols were i.i.d. 64-QAM symbols with $\frac{\sigma^2_{c,R}}{\sigma^2_{c,I}} = 4$. The signal-to-noise-ratio (SNR) was defined as $\text{SNR} = 10\log_{10}\left(\frac{\sigma^2_{c,R}+\sigma^2_{c,I}}{\sigma^2_\rho}\right)$, where σ^2_ρ is the variance of the white noise process $\rho[n]$. All results were obtained by averaging over $I = 200$ independent Monte Carlo trials. Each realization consisted of 1024 data symbols (i.e. 128 OFDM symbols). Furthermore, in both simulation examples we used the cycles $k_1 = 1$ and $k_2 = 7$ to estimate the 5-tap time-dispersive channel $\mathbf{d} = \frac{1}{\sqrt{8}}[1\ 2\ -1\ 1\ -1]$. The estimator performance was measured in terms of the average bias [18] given by

[4]Here, $\tilde{D}(z) = D^*\left(\frac{1}{z^*}\right)$ is the paraconjugate of $D(z)$.

[5]The superscript ^{H} stands for conjugate transposition.

$\frac{1}{IL_d}\sum_{l=0}^{L_d-1}\left|\sum_{i=0}^{I-1}\left[\hat{d}^{(i)}[l]-d[l]\right]\right|$ and the mean square error (MSE) $\frac{1}{IL_d\|d\|^2}\sum_{i=0}^{I-1}\|\hat{\mathbf{d}}^{(i)}-\mathbf{d}\|^2$.

Simulation Example 1. In the first simulation example we computed the average bias and the MSE of the channel estimator (see Figs. 2 (a) and (b)) as a function of the SNR in dB. We can see that the performance of the estimator improves with increasing SNR.

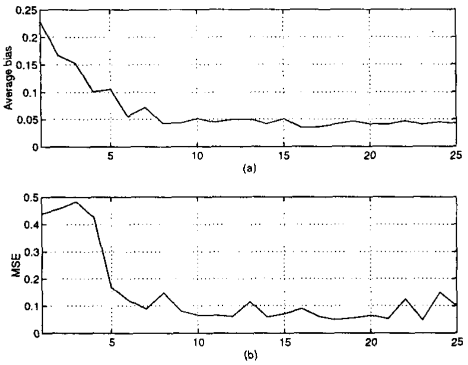

Fig. 2. (a) Average bias and (b) MSE of the channel estimator as a function of the SNR in dB.

Simulation Example 2. In the second simulation example, we investigate the effect of the length L of the data record used for estimating the cyclic statistics $\hat{C}_r[k,\tau]$ on the performance of the channel estimator. For SNR = 15dB, Figs. 3 (a) and (b) show the average bias and the MSE, respectively, of the channel estimator as a function of L. (Note that in Fig. 3 the length of the data record has been specified in OFDM symbols. The actual length of the data record is therefore obtained by multiplying the number of symbols by 8). We can observe that the performance of the estimator improves with increasing data record length.

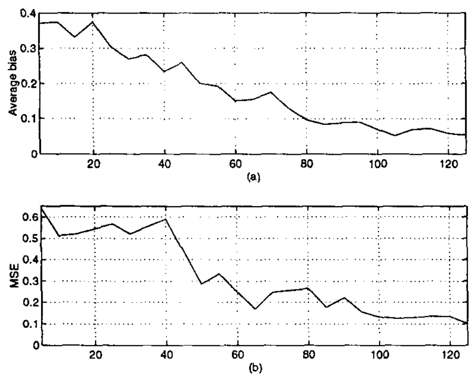

Fig. 3. (a) Average bias and (b) MSE of the channel estimator as a function of the data record length (specified in OFDM symbols).

6. CONCLUSION

We introduced an algorithm for blind channel identification in pulse shaping OFDM/OQAM systems. Our approach is based on the assumption that the receiver knows the transmitter pulse shaping filter. The proposed algorithm exploits cyclostationarity of the received signal, does not require knowledge of the channel order, and exhibits low noise sensitivity.

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