Capacity Scaling Laws in MIMO Relay Networks

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***Abstract*— The use of multiple antennas at both ends of a wireless link, popularly known as multiple-input multiple-output (MIMO) wireless, has been shown to offer significant improvements in *spectral efficiency* and *link reliability* through *spatial multiplexing* and *space-time coding*, respectively. This paper demonstrates that similar performance gains can be obtained in wireless relay networks employing terminals with MIMO capability. We consider a setup where a designated source terminal communicates with a designated destination terminal, both equipped with M antennas, assisted by K single-antenna or multiple-antenna relay terminals using a *half-duplex* protocol. Assuming perfect channel state information (CSI) at the destination and the relay terminals and no CSI at the source, we show that the corresponding network capacity scales as $C = (M/2)\log(K) + O(1)$ for fixed M, arbitrary (but fixed) number of (transmit and receive) antennas N at each of the relay terminals, and $K \to \infty$. We propose a protocol that assigns each relay terminal to one of the multiplexed data streams forwarded in a "doubly coherent" fashion (through matched filtering) to the destination terminal. It is shown that this protocol achieves the cut-set upper bound on network capacity for fixed M and $K \to \infty$ (up to an $O(1)$-term) by employing independent stream decoding at the destination terminal. Our protocol performs *inter-stream interference cancellation* in a completely decentralized fashion, thereby orthogonalizing the effective MIMO channel between source and destination terminals. Finally, we discuss the case where the relay terminals do not have CSI and show that simple *amplify-and-forward relaying*, asymptotically in K, for fixed M and fixed $N \geq 1$, turns the relay network into a point-to-point MIMO link with high-SNR capacity $C = (M/2)\log(\text{SNR}) + O(1)$, demonstrating that the use of *relays as active scatterers* can recover spatial multiplexing gain in poor scattering environments.**

***Index Terms*— Relay channel, Adhoc network, distributed orthogonalization, MIMO, fading channels**

I. Introduction and Outline

THE use of multiple antennas at both ends of a wireless link, known as *multiple-input multiple-output* (MIMO) wireless yields significant improvements in *spectral efficiency* and *link reliability* through spatial multiplexing [1]–[5] and space-time coding [6]–[9], respectively. While the information-theoretic performance limits of point-to-point MIMO links are well understood [1]–[5] knowledge about the impact of MIMO terminals in a multiuser or network context is scarce. First results on MIMO broadcast and multiple-access channels have been reported in [10]–[18].

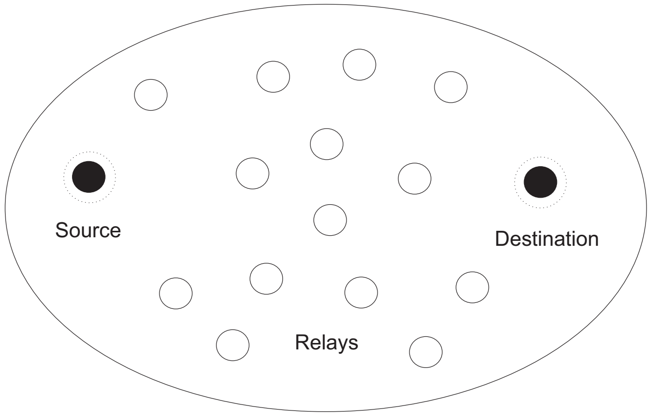

Fig. 1. Dense wireless relay network with dead-zones around source and destination terminals each equipped with M antennas. The relay terminals employ $N \geq 1$ (transmit and receive) antennas.

The aim of this paper is to quantify the impact of MIMO technology on large wireless relay networks, where a designated multiantenna source terminal communicates with a designated multiantenna destination terminal through a set of multiantenna or single-antenna relay terminals (see Fig. 1). We start by briefly reviewing MIMO gains in point-to-point links.

A. MIMO Gains in Point-to-Point Links

In point-to-point wireless links, MIMO systems improve *spectral efficiency* through *spatial multiplexing gain*, *link reliability* through *diversity gain* and *coverage* through *array gain*:

Spatial multiplexing in MIMO systems yields a *linear* (in the minimum of the number of transmit and receive antennas) increase in capacity for no additional power or bandwidth expenditure [1]–[5]. The corresponding gain is realized by simultaneously transmitting independent data streams in the same frequency band. Under conducive channel conditions (i.e., rich scattering), the receiver exploits differences in the spatial signatures of the multiplexed data streams to separate the different signals, thereby realizing a capacity gain.

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Diversity is a powerful technique to mitigate fading and increase robustness to interference [19]. Diversity gain is obtained by transmitting the data signal over multiple (ideally) *independently* fading paths (time/frequency/space) and performing proper combining at the receiver. Spatial (i.e., antenna) diversity is particularly attractive when compared with time/frequency diversity since it does not incur an expenditure in transmission time/bandwidth. Space-time coding [6]–[9] realizes spatial transmit diversity gain in point-to-point MIMO systems without requiring channel knowledge at the transmitter.

Array gain can be made available at the transmitter and the receiver, requires channel knowledge, and results in an increase in average receive signal-to-noise ratio (SNR) and hence improved coverage due to a coherent combining effect [19].

B. Coherent and Noncoherent MIMO Relay Networks

Throughout this paper, we focus on dense relay networks where a designated source terminal equipped with M antennas spatially multiplexes data (i.e., transmits statistically independent data streams from different antennas) to a designated destination terminal with M antennas through K relay terminals with $N \geq 1$ transmit/receive antennas each. Communication takes place over two time slots using a two-hop *half-duplex* protocol. We consider two scenarios:

- *Coherent MIMO relay network*: The source terminal has no channel state information (CSI), the destination terminal has perfect knowledge of all channels (asymptotically in K, knowledge of the composite $M \times M$ MIMO channel between source and destination terminals will suffice, cf. Theorem 2). Each relay terminal maintains perfect CSI for its $N \times M$ backward (i.e., channel between the source terminal and the relay terminal) and $M \times N$ forward (i.e., channel between the relay terminal and the destination terminal) channels.
- *Noncoherent MIMO relay network*: In this case, the assumptions on CSI in the source and destination terminals are the same as in the coherent case, the relay terminals have no CSI.

C. Contributions and Relation to Previous Work

We investigate the information-theoretic performance limits of large MIMO relay networks and quantify the corresponding multiplexing, diversity, and array gains. Throughout the paper the terminology *distributed array/diversity gain* will be used to designate array/diversity gain resulting from the use of relay terminals. Our framework is based on deriving asymptotic (in the number of relay terminals) network capacity, a concept pioneered for single-antenna systems and AWGN channels in [20], [21]. Results of the Gupta-Kumar [20], [22] and Gastpar-Vetterli [21] type have recently been reported for single antenna fading channels in [23], for the case of node mobility and large-scale fading in [24], for extended AWGN networks in [25], [26], and for multihop routing in [27].

Distributed space-time code design and information-theoretic performance limits for single-antenna fading relay channels (with a finite number of nodes) have recently been studied in [28]–[32]. Capacity results for MIMO relay channels with a finite number of relays can be found in [31], [33], [34].

The detailed contributions reported in this paper can be summarized as follows:

- Generalizing the main result in [21] to the fading MIMO case, we show that the coherent MIMO relay network capacity scales as $C = (M/2)\log(K) + O(1)$ for M fixed, N arbitrary (but fixed), and $K \to \infty$. In terms of traditional MIMO gains this result states that a spatial multiplexing gain of $M/2$, and a distributed per-stream array gain of K are obtained.
- We propose a simple (half-duplex) protocol that achieves the cut-set upper bound on coherent network capacity (up to an $O(1)$-term) in the large K limit, for fixed M and for arbitrary (but fixed) N by employing matched filtering at the relay terminals and independent stream decoding at the destination terminal. Our protocol assigns each relay terminal to one out of the M transmit-receive antenna pairs (relay partitioning) and requires knowledge (at the relay) only of the corresponding $N \times 1$ and $1 \times N$ backward and forward channels, respectively. We show that the proposed protocol effectively performs *inter-stream interference cancellation*, thereby orthogonalizing the MIMO channel between source and destination in a completely *decentralized* fashion.
- For noncoherent MIMO relay networks, we show that simple *amplify-and-forward (AF) relaying*, asymptotically in K, for fixed M and fixed $N \geq 1$, turns the relay network into a point-to-point MIMO link with high-SNR capacity $C = (M/2)\log(\text{SNR}) + O(1)$, demonstrating that a spatial multiplexing gain of $M/2$ and no distributed array gain are obtained. We find, however, that traditional receive array gain is realized. Our result also shows that using (even single-antenna) *relays as active scatterers* can recover spatial multiplexing gain in poor scattering environments.

D. Notation

The superscripts ^{T}, ^{H}, and * stand for transposition, conjugate transpose, and element-wise conjugation, respectively. $\mathcal{E}$ denotes the expectation operator. $\det(\mathbf{A})$ stands for the determinant of the matrix $\mathbf{A}$. $\|\mathbf{a}\|$ denotes the Euclidean norm of the vector $\mathbf{a}$. $\mathbf{I}_m$ stands for the $m \times m$ identity matrix. For an $m \times n$ matrix $\mathbf{A} = [\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_n]$, we define the $mn \times 1$ vector $\text{vec}(\mathbf{A}) = [\mathbf{a}_1^T\ \mathbf{a}_2^T\ \ldots\ \mathbf{a}_n^T]^T$. $\mathbf{A} \otimes \mathbf{B}$ stands for the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$. $|\mathcal{X}|$ is the cardinality of the set $\mathcal{X}$. The notation $u(x) = O(v(x))$ denotes that $|u(x)/v(x)|$ remains bounded as $x \to \infty$. A circularly symmetric complex Gaussian random variable (RV) is a RV $Z = X + jY \sim \mathcal{CN}(0, \sigma^2)$, where X and Y are i.i.d. $\mathcal{N}(0, \sigma^2/2)$. $\text{VAR}(X)$ stands for the variance of the RV X. $\xrightarrow{\text{w.p. } 1}$ denotes convergence with probability 1. Throughout the paper all logarithms, unless specified otherwise, are to the base 2.

TABLE I

HALF-DUPLEX TWO-HOP PROTOCOL. COMMUNICATION TAKES PLACE OVER TWO TIME SLOTS. ($\mathcal{A} \rightarrow \mathcal{B}$ SIGNIFIES COMMUNICATION FROM TERMINAL $\mathcal{A}$ TO TERMINAL $\mathcal{B}$).

Time slot 1	Time slot 2
$\mathcal{S} \rightarrow \mathcal{R}_k,\ \ k=1,2,\ldots,K$	$\mathcal{R}_k \rightarrow \mathcal{D},\ \ k=1,2,\ldots,K$

E. Organization of the Paper

The rest of this paper is organized as follows. Section II describes the channel and signal model. In Section III, we derive an upper bound on the capacity of the coherent MIMO relay network. Section IV introduces a simple protocol that achieves this upper bound asymptotically in K (up to an $O(1)$-term) and hence establishes (asymptotic) network capacity. Section V contains an analysis of a simple AF-based protocol for noncoherent MIMO relay networks and derives the corresponding asymptotic network capacity. We conclude in Section VI.

II. CHANNEL AND SIGNAL MODEL

A. General Assumptions

The discussion in the remainder of this section applies to both coherent and noncoherent MIMO relay networks as defined in Section I-B. We consider a wireless network consisting of $K+2$ terminals, with a designated source-destination terminal pair and K relay terminals located randomly and independently in a domain of fixed area (see Fig. 1). The source and destination terminals, equipped with M antennas each, are denoted by $\mathcal{S}$ and $\mathcal{D}$, respectively. The kth relay terminal employs $N \geq 1$ transmit/receive antennas and is denoted by $\mathcal{R}_k$ $(k=1,2,\ldots,K)$. Throughout the paper, we assume that M and N are finite and consider the $K \rightarrow \infty$ capacity behavior of the network. We furthermore assume a "dead-zone" of non-zero radius around $\mathcal{S}$ and $\mathcal{D}$ [21] that is free of relay terminals, no direct link between $\mathcal{S}$ and $\mathcal{D}$ (e.g., due to large distance between $\mathcal{S}$ and $\mathcal{D}$ or due to $\mathcal{D}$ being located deep inside a building and $\mathcal{S}$ outside the building), and transmission in spatial multiplexing mode (i.e., the source terminal transmits independent data streams from different antennas) over two time slots using two hops in half-duplex mode (i.e., the terminals cannot transmit and receive simultaneously). In the first time slot the relay terminals receive the signal transmitted by the source terminal. After processing the received signals, the relay terminals simultaneously transmit the processed data to the destination terminal during the second time slot while the source terminal is silent (see Table I).

B. Channel and Signal Model

Throughout the paper, we assume that all channels are frequency-flat block fading with the same block length, independent realizations across blocks and the block length taken to be an integer multiple of the duration of a time slot. Transmission/reception between the terminals is perfectly synchronized. The input-output relation for the $\mathcal{S} \rightarrow \mathcal{R}_k$ link[1]

[1] $\mathcal{A} \rightarrow \mathcal{B}$ signifies communication from terminal $\mathcal{A}$ to terminal $\mathcal{B}$.

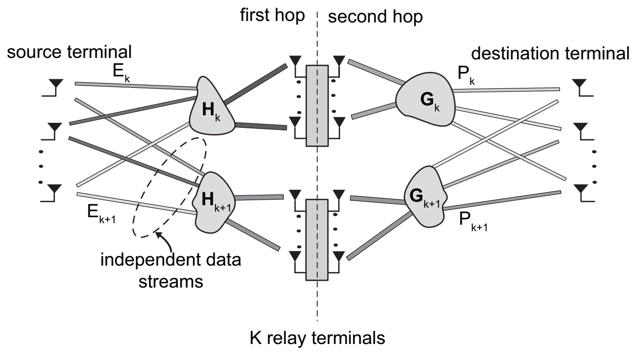

Fig. 2. MIMO wireless relay network setup. The relay terminals employ $N \geq 1$ antennas.

is given by

$$\mathbf{r}_k = \sqrt{\frac{E_k}{M}}\,\mathbf{H}_k\mathbf{s} + \mathbf{n}_k, \quad k=1,2,\ldots,K \tag{1}$$

where $\mathbf{r}_k$ denotes the $N \times 1$ received vector signal, E_k is the average energy received at the kth relay terminal over one symbol period through the $\mathcal{S} \rightarrow \mathcal{R}_k$ link (having accounted for path loss and shadowing), $\mathbf{H}_k = [\mathbf{h}_{k,1}\ \mathbf{h}_{k,2}\ \ldots\ \mathbf{h}_{k,M}]$ is the $N \times M$ random channel matrix corresponding to the $\mathcal{S} \rightarrow \mathcal{R}_k$ link, independent across relay terminals (i.e., independent across k) and consisting of i.i.d. $\mathcal{CN}(0,1)$ entries, $\mathbf{s} = [s_1\, s_2\, \ldots\, s_M]^T$ is the $M \times 1$ circularly symmetric complex Gaussian transmit signal vector satisfying $\mathcal{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_M$ (recall that the signaling mode is spatial multiplexing), and $\mathbf{n}_k$ is an $N \times 1$ spatio-temporally white circularly symmetric complex Gaussian noise vector sequence, independent across k, with covariance matrix $\mathcal{E}\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_n^2\mathbf{I}_N$ $(k=1,2,\ldots,K)$.

Each relay terminal processes its received vector signal $\mathbf{r}_k$ to produce the $N \times 1$ vector signal $\mathbf{t}_k$, which is then transmitted to the destination terminal over the second time slot. The $M \times 1$ vector signal received at the destination terminal is consequently given by

$$\mathbf{y} = \sum_{k=1}^{K} \sqrt{\frac{P_k}{N}}\,\mathbf{G}_k\mathbf{t}_k + \mathbf{z} \tag{2}$$

where P_k denotes the average signal energy over one symbol period contributed by the kth relay terminal (having accounted for path loss and shadowing in the $\mathcal{R}_k \rightarrow \mathcal{D}$ link), $\mathbf{G}_k = [\mathbf{g}_{k,1}\ \mathbf{g}_{k,2}\ \ldots\ \mathbf{g}_{k,M}]^T$ is the corresponding $M \times N$ channel matrix with i.i.d. $\mathcal{CN}(0,1)$ entries, independent across k, and $\mathbf{z} = [z_1\ z_2\ \ldots\ z_M]^T$ denotes an $M \times 1$ spatio-temporally white circularly symmetric complex Gaussian noise vector sequence satisfying $\mathcal{E}\{\mathbf{z}\mathbf{z}^H\} = \sigma_n^2\mathbf{I}_M$. The transmit signal vectors $\mathbf{t}_k$ will in general depend both on the forward and backward channels, $\mathbf{G}_k$ and $\mathbf{H}_k$, respectively, and are chosen to satisfy $\mathcal{E}\{\|\mathbf{t}_k\|^2\} \leq N$ thus imposing a per-relay average power constraint. We emphasize that *assuming a total power constraint across relays and uniform (across relays) power allocation does not change the main results of the paper* given by (17) and Theorem 3. All noise signals throughout the network are assumed independent of the transmit signals. The entire setup described above is summarized schematically in Fig. 2.

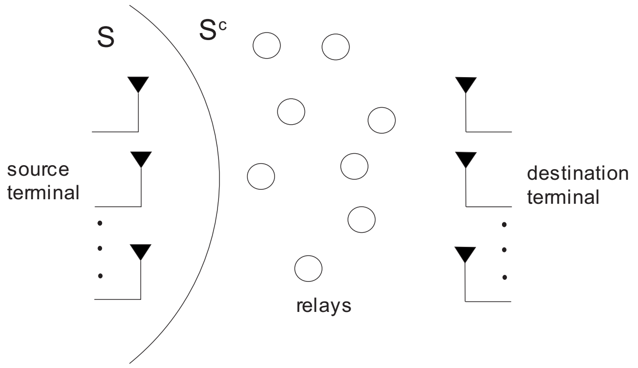

Fig. 3. Applying a broadcast cut to the coherent relay network.

As already mentioned above, the path-loss and shadowing effects are described by $\{E_k\}_{k=1}^K$ for the first hop and $\{P_k\}_{k=1}^K$ for the second hop. We assume that these parameters are independent RVs, strictly positive, bounded, and remain constant over the entire time period of interest. The randomness of the E_k and P_k reflects the fact that the relay terminal locations are chosen randomly, strict positivity is a consequence of the assumption that the domain under consideration is of fixed area, and the dead-zone assumption implies that the parameters E_k and P_k are bounded. Even though we do not specify the exact relay locations, as is done in geometrical network models such as those used in [26], the network topology and the propagation conditions are accounted for through the statistics of the E_k and the P_k. If the relay terminals are located randomly and *uniformly* within the domain of interest both the E_k and the P_k can be assumed to be independent. If the relay terminals are confined to certain geographic areas (e.g., only at cell edge or only within a certain range from source or destination terminals) this can be accounted for through different means in the E_k and the P_k. The entire discussion in the paper applies to general independent, positive and bounded E_k and P_k. However, we shall see that the i.i.d. assumption on the E_k and the P_k often leads to significantly stronger results (cf. Theorems 2 and 3).

III. Upper Bound on Coherent MIMO Relay Network Capacity

In this section, we derive an upper bound on the capacity of coherent relay networks (as defined in Section I-B) in conjunction with two-hop relaying as described in Section II. In Section IV, we introduce a simple protocol for coherent MIMO relay networks that leads to a corresponding lower bound on network capacity. This lower bound is then shown to asymptotically (up to an $O(1)$-term) approach the upper bound presented in Theorem 1 below, which ultimately establishes the asymptotic network capacity scaling behavior.

Theorem 1: The capacity of the coherent MIMO relay network under two-hop relaying satisfies

$$C \le C_{\text{upper}} = \frac{M}{2}\log\left(1 + \frac{N}{M\sigma_n^2}\sum_{k=1}^{K} E_k\right).$$

In the large relay limit $K \to \infty$, defining $\mu = (1/K)\sum_{k=1}^{K}\mathcal{E}\{E_k\}$, we furthermore have

$$C_{\text{upper}} \xrightarrow{\text{w.p. 1}} C_{\text{upper}}^{\infty} = \frac{M}{2}\log\left(\frac{KN\mu}{M\sigma_n^2}\right) = \frac{M}{2}\log(K) + O(1). \tag{3}$$

Proof: Separating the source terminal $\mathcal{S}$ from the rest of the network (see Fig. 3), using a broadcast cut [35], and applying the cut-set theorem [35, Theorem 14.10.1], it follows that the capacity of the MIMO relay network is upper bounded by

$$C_u = \mathcal{E}_{\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^K}\left\{\frac{1}{2}I(\mathbf{s};\mathbf{r}_1,\ldots,\mathbf{r}_K,\mathbf{y}\,|\,\mathbf{t}_1,\ldots,\mathbf{t}_K)\right\}$$

where the factor $1/2$ results from the fact that data is transmitted over two time slots. Applying the chain rule for mutual information [35], we obtain

$$\begin{aligned} I(\mathbf{s};\mathbf{r}_1,\ldots,\mathbf{r}_K,\mathbf{y}\,|\,\mathbf{t}_1,\ldots,\mathbf{t}_K) &= I(\mathbf{s};\mathbf{r}_1,\ldots,\mathbf{r}_K\,|\,\mathbf{t}_1,\ldots,\mathbf{t}_K) \\ &\quad + I(\mathbf{s};\mathbf{y}\,|\,\mathbf{r}_1,\ldots,\mathbf{r}_K,\mathbf{t}_1,\ldots,\mathbf{t}_K) \end{aligned} \tag{4}$$

where conditioning on the $\mathbf{t}_k$ in the first term can be dropped since neither $\mathbf{s}$ nor the $\mathbf{r}_k$ depend on the $\mathbf{t}_k$. Conditioning on the $\mathbf{t}_k$ and assuming perfect knowledge of the $\mathbf{G}_k$ at the receiver results in the second term on the right-hand-side (RHS) of (4) being equal to $I(\mathbf{s};\mathbf{z}) = 0$ due to independence of $\mathbf{s}$ and $\mathbf{z}$. We can therefore summarize our result as

$$C_u = \mathcal{E}_{\{\mathbf{H}_k\}_{k=1}^K}\left\{\frac{1}{2}I(\mathbf{s};\mathbf{r}_1,\ldots,\mathbf{r}_K)\right\}.$$

Recalling that $\mathbf{s}$ was assumed circularly symmetric complex Gaussian with $\mathcal{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_M$, we get [36]

$$C_u = \mathcal{E}_{\{\mathbf{H}_k\}_{k=1}^K}\left\{\frac{1}{2}\log\det\left(\mathbf{I}_M + \frac{1}{\sigma_n^2}\sum_{k=1}^{K}\frac{E_k}{M}\mathbf{H}_k^H\mathbf{H}_k\right)\right\} \tag{5}$$

which upon application of Jensen's inequality [35] yields

$$C_u \le \frac{M}{2}\log\left(1 + \frac{N}{M\sigma_n^2}\sum_{k=1}^{K}E_k\right) = C_{\text{upper}}$$

and hence completes the proof of the first part of the theorem.

In order to prove the second part, we start by noting that due to the assumption of the $E_k\,(k = 1, 2, \ldots, K)$ being bounded it follows that $\text{VAR}(E_k)$ is bounded as well for $k = 1, 2, \ldots, K$, and hence the Kolmogorov condition

$$\sum_{k=1}^{\infty}\frac{\text{VAR}(E_k)}{k^2} < \infty \tag{6}$$

is satisfied. We can therefore use [37, Theorem 1.8.D] to obtain

$$\sum_{k=1}^{K}\frac{E_k}{K} - \sum_{k=1}^{K}\frac{\mathcal{E}\{E_k\}}{K} \xrightarrow{\text{w.p. 1}} 0$$

which upon application of [37, Theorem 1.7], noting that $\log(x)$ is a continuous function, yields

$$C_{\text{upper}} \xrightarrow{\text{w.p. 1}} \frac{M}{2}\log\left(\frac{KN\mu}{M\sigma_n^2}\right) = C_{\text{upper}}^{\infty}. \tag{7}$$

$$y_i = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \frac{\sqrt{\frac{P_k}{N}}\|\mathbf{g}_{k,i}\|\left(\sqrt{\frac{E_k}{M}}\left(\|\mathbf{h}_{k,i}\|^2 s_i + \sum_{j=1,j\neq i}^{M}\mathbf{h}_{k,i}^H\mathbf{h}_{k,j}s_j\right)+\mathbf{h}_{k,i}^H\mathbf{n}_k\right)}{\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}} + \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{\sqrt{\frac{P_k}{N}}\mathbf{g}_{k,i}^T\mathbf{g}_{k,m}^*\left(\sqrt{\frac{E_k}{M}}\left(\|\mathbf{h}_{k,m}\|^2 s_m + \sum_{j=1,j\neq m}^{M}\mathbf{h}_{k,m}^H\mathbf{h}_{k,j}s_j\right)+\mathbf{h}_{k,m}^H\mathbf{n}_k\right)}{\|\mathbf{g}_{k,m}\|\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right) + z_i \quad (11)$$

Noting that M and N are finite and $\mathcal{E}\{E_k\}$ being finite for $k = 1, 2, \ldots, K$ implies that μ is finite as well, we get

$$C_{\text{upper}}^{\infty} = \frac{M}{2}\log(K) + O(1) \quad (8)$$

which completes the proof of Theorem 1. ■

Discussion: It is straightforward to see that, operationally, the RHS of (5) results when all the relay terminals can fully cooperate (and perform joint decoding) so as to effectively form a point-to-point coherent (i.e., perfect receive CSI) MIMO channel with M transmit and KN receive antennas. Equation (8) explicitly reveals that for fixed M network capacity scales at most logarithmically in K with pre-log $M/2$. We finally note that the SNR in (8) has been absorbed in the $O(1)$-term. Throughout the discussion of the coherent relaying architecture, we shall isolate the SNR in the $O(1)$-term in order to emphasize the scaling behavior as a function of K.

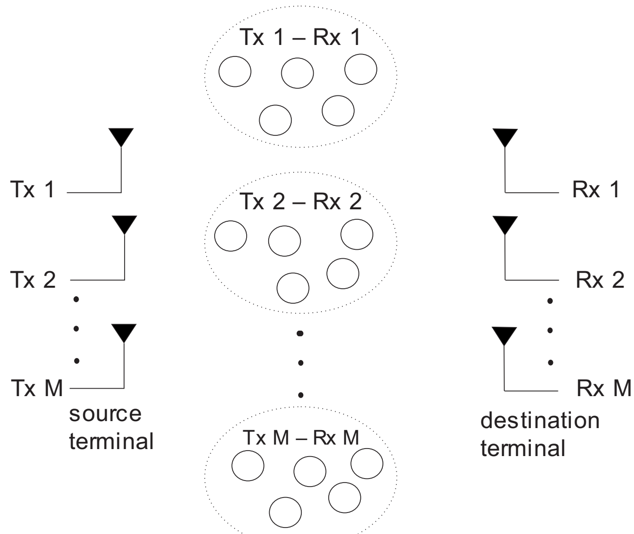

Fig. 4. Relay partitioning. Each transmit-receive antenna pair has an associated group of relay terminals.

IV. Coherent Relaying and Network Capacity

In this section, we present a simple relaying protocol, which does not require any cooperation between terminals and employs single-antenna (i.e., independent) decoding at the destination terminal. The capacity achieved by this protocol will be shown to asymptotically (in K) approach the cut-set upper bound (up to an $O(1)$-term), thus establishing the coherent MIMO relay network capacity scaling behavior.

A. The Relaying Protocol

The essence of the proposed protocol lies in assigning each of the relay terminals to one out of the M transmit-receive antenna pairs. Without loss of generality, we assume that the ith transmit antenna and the ith receive antenna constitute the ith pair. Our assignment policy needs to ensure that for increasing K each transmit-receive antenna pair is served by an increasing number of relay terminals. Consequently, the relay terminals are partitioned into groups with each group being responsible for one of the M source-destination antenna pairs (see Fig. 4). In the following, the set of relay terminals assigned to the ith transmit-receive antenna pair will be denoted by $\mathcal{X}_i$.

We start by assuming that the kth relay terminal is assigned to the lth transmit-receive antenna pair. Upon reception of $\mathbf{r}_k$, the relay performs *matched filtering* with respect to the assigned *backward channel* to obtain

$$u_k = \mathbf{h}_{k,l}^H\mathbf{r}_k = \sqrt{\frac{E_k}{M}}\left(\|\mathbf{h}_{k,l}\|^2 s_l + \sum_{j=1,j\neq l}^{M}\mathbf{h}_{k,l}^H\mathbf{h}_{k,j}s_j\right) + \mathbf{h}_{k,l}^H\mathbf{n}_k. \quad (9)$$

Using[2] $\mathcal{E}\{|u_k|^2\} = (E_k/M)\left(N^2+MN\right) + N\sigma_n^2$, the processed received signal is scaled to unit average energy and *transmit matched filtering* with respect to the assigned *forward channel* is performed so that

$$\mathbf{t}_k = \frac{u_k}{\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\frac{\mathbf{g}_{k,l}^*}{\|\mathbf{g}_{k,l}\|} \quad (10)$$

which is easily seen to satisfy the transmit power constraint $\mathcal{E}\{\|\mathbf{t}_k\|^2\} = N$.

Now, the signal at the ith receive antenna is given by

$$y_i = \sum_{m=1}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\sqrt{\frac{P_k}{N}}\mathbf{g}_{k,i}^T\mathbf{t}_k\right) + z_i, \quad i = 1, 2, \ldots, M$$

which upon insertion of (10) and (9) yields (11) shown at the top of the page.

[2]Note that we are averaging $|u_k|^2$ over the random channel, noise, and signal.

$$I_i = \frac{1}{2}\log\left(1 + \frac{\left|h_i^{\mathrm{sig}}\right|^2}{\sum_{j=1,j\neq i}^{M}\left|h_{i,j}^{\mathrm{int}}\right|^2 + \sigma_n^2\left(1 + \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} a_{k,i} + \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m} b_{k,i,m}\right)\right)}\right) \tag{14}$$

We observe that y_i consists of a contribution from the desired data stream[3] s_i, interference signal terms from the data streams s_m with $m \neq i$, forwarded noise from the relay terminals, and receiver noise. In order to set the stage for the main result of this section (the capacity lower bound in Theorem 2), we shall next identify the signal, interference, and noise contributions to y_i in detail. In particular, we seek a representation of the form

$$y_i = h_i^{\mathrm{sig}} s_i + \sum_{j=1,j\neq i}^{M} h_{i,j}^{\mathrm{int}} s_j + N_i, \quad i = 1, 2, \ldots, M \tag{12}$$

where h_i^{sig} denotes the effective scalar channel gain for the data stream transmitted from the ith source terminal antenna, $h_{i,j}^{\mathrm{int}}$ stands for the effective channel seen by the interference from s_j to s_i, and N_i denotes the effective noise term at the ith receive antenna.

Signal contribution: From (11) it follows by inspection that

$$h_i^{\mathrm{sig}} = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} d_{k,i} + \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m} f_{k,i,m}\right)$$

where

$$d_{k,i} = \sqrt{\frac{P_k}{N}}\left(\frac{\sqrt{\frac{E_k}{M}}\|\mathbf{g}_{k,i}\|\|\mathbf{h}_{k,i}\|^2}{\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right)$$

$$f_{k,i,m} = \sqrt{\frac{P_k}{N}}\left(\frac{\sqrt{\frac{E_k}{M}}\mathbf{g}_{k,i}^T\mathbf{g}_{k,m}^*\mathbf{h}_{k,m}^H\mathbf{h}_{k,i}}{\|\mathbf{g}_{k,m}\|\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right).$$

Interference contribution: Again, by inspection of (11), we get

$$h_{i,j}^{\mathrm{int}} = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} x_{k,i,j} + \sum_{k,\mathcal{R}_k\in\mathcal{X}_j} w_{k,i,j} + \sum_{m=1,m\neq i,j}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m} v_{k,i,j,m}\right)$$

where

$$x_{k,i,j} = \sqrt{\frac{P_k}{N}}\left(\frac{\sqrt{\frac{E_k}{M}}\|\mathbf{g}_{k,i}\|\mathbf{h}_{k,i}^H\mathbf{h}_{k,j}}{\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right)$$

$$w_{k,i,j} = \sqrt{\frac{P_k}{N}}\left(\frac{\sqrt{\frac{E_k}{M}}\mathbf{g}_{k,i}^T\mathbf{g}_{k,j}^*\|\mathbf{h}_{k,j}\|^2}{\|\mathbf{g}_{k,j}\|\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right)$$

$$v_{k,i,j,m} = \sqrt{\frac{P_k}{N}}\left(\frac{\sqrt{\frac{E_k}{M}}\mathbf{g}_{k,i}^T\mathbf{g}_{k,m}^*\mathbf{h}_{k,m}^H\mathbf{h}_{k,j}}{\|\mathbf{g}_{k,m}\|\sqrt{\frac{E_k}{M}(N+M)+\sigma_n^2}}\right).$$

[3]Recall that single-antenna decoding is performed at the receiver and the ith receive antenna is designated to decode the signal transmitted from the ith source terminal antenna.

Noise contribution: The total noise term N_i follows from (11) as

$$N_i = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i} \widetilde{n}_{k,i} + \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m} \widetilde{q}_{k,i,m}\right) + z_i$$

where $\widetilde{n}_{k,i}|\,\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K \sim \mathcal{CN}(0, a_{k,i}\sigma_n^2)$ and $\widetilde{q}_{k,i,m}|\,\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K \sim \mathcal{CN}(0, b_{k,i,m}\sigma_n^2)$ with

$$a_{k,i} = \frac{P_k\|\mathbf{g}_{k,i}\|^2\|\mathbf{h}_{k,i}\|^2}{N\left(\frac{E_k}{M}(N+M)+\sigma_n^2\right)}$$

$$b_{k,i,m} = \frac{P_k\|\mathbf{h}_{k,m}\|^2|\mathbf{g}_{k,i}^T\mathbf{g}_{k,m}^*|^2}{N\left(\frac{E_k}{M}(N+M)+\sigma_n^2\right)\|\mathbf{g}_{k,m}\|^2}.$$

B. Lower Bound on MIMO Relay Network Capacity

Based on (12), we are now ready to derive the asymptotic (in K) capacity achieved by the relaying protocol described in Section IV-A. Our main result is summarized as follows.

Theorem 2: For a fixed number of source-destination antenna pairs M and fixed N, in the $K \to \infty$ limit such that[4] $|\mathcal{X}_1| = |\mathcal{X}_2| = \cdots = |\mathcal{X}_M| = K/M$, assuming perfect knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K$ at each of the receive antennas, the relay network capacity scales at least as

$$C_{\mathrm{lower}}^{\infty} = \frac{M}{2}\log(K) + O(1). \tag{13}$$

For i.i.d. $\{E_k\}$ and i.i.d. $\{P_k\}$ knowledge of h_i^{sig} and $h_{i,j}^{\mathrm{int}}$ $(j = 1, 2, \ldots, M, j \neq i)$ at the ith receive antenna is sufficient for (13) to hold.

Proof: Recalling that we assumed independent decoding for each of the multiplexed data streams, and using (12), we get[5]

$$C_{\mathrm{lower}} = \sum_{i=1}^{M}\mathcal{E}_{\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^K}\{I_i\}$$

where I_i is given in (14) shown at the top of the page. Here, we used the fact that perfect knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K$ implies circularly symmetric complex Gaussianity of N_i and results in perfect knowledge of $h_{i,j}^{\mathrm{int}}$ $(i = 1, 2, \ldots, M,\ j = 1, 2, \ldots, M,\ j \neq i)$, which renders the noise plus interference contribution in (12) circularly symmetric complex Gaussian. Next, multiplying and dividing the argument of the logarithm in (14) by $(K/M)^2$, we obtain (15) [see following page]. Using the assumption that the E_k and P_k are positive and bounded $\forall k$, it is straightforward but tedious to show that the variances of each of the RVs, $d_{k,i}$, $f_{k,i,m}$, $x_{k,i,j}$, $w_{k,i,j}$,

[4]The assumption of equal size relay clusters is made for the sake of simplicity of exposition only. The scaling result (13) is valid more generally for $|\mathcal{X}_i|$ $(i = 1, 2, \ldots, M)$ proportional to K.

[5]Note that the E_k and P_k were assumed to remain constant over the entire time period of interest so that ergodic capacity is obtained by averaging over the $\mathbf{H}_k$ and $\mathbf{G}_k$ only.

$$I_i = \frac{1}{2}\log\left(1 + \frac{\left|\frac{h_i^{\mathrm{sig}}}{K/M}\right|^2}{\sum_{j=1,j\neq i}^{M}\left|\frac{h_{i,j}^{\mathrm{int}}}{K/M}\right|^2 + \frac{\sigma_n^2}{K/M}\left(\frac{1}{K/M} + \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{a_{k,i}}{K/M} + \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{b_{k,i,m}}{K/M}\right)\right)}\right) \tag{15}$$

$v_{k,i,j,m}$, $a_{k,i}$, and $b_{k,i,m}$ are bounded $\forall\, i,j,k,m$. Hence, the Kolmogorov condition (6) is satisfied for each of these sequences (as a function of k) and it follows from [37, Theorem 1.8.D] that

$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{d_{k,i}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{\mathcal{E}\{d_{k,i}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{f_{k,i,m}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{\mathcal{E}\{f_{k,i,m}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{x_{k,i,j}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{\mathcal{E}\{x_{k,i,j}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_j}\frac{w_{k,i,j}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_j}\frac{\mathcal{E}\{w_{k,i,j}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{v_{k,i,j,m}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{\mathcal{E}\{v_{k,i,j,m}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{a_{k,i}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{\mathcal{E}\{a_{k,i}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$
$$\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{b_{k,i,m}}{K/M} - \sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{\mathcal{E}\{b_{k,i,m}\}}{K/M} \xrightarrow{\text{w.p. 1}} 0$$

as $K \to \infty$. Next, noting that $\mathcal{E}\{f_{k,i,m}\} = \mathcal{E}\{x_{k,i,j}\} = \mathcal{E}\{w_{k,i,j}\} = \mathcal{E}\{v_{k,i,j,m}\} = 0\ \forall\, i,j,k,m$, and applying [37, Theorem 1.7], we obtain

$$I_i \xrightarrow{\text{w.p. 1}} \frac{1}{2}\log\left(1 + \frac{K\delta_i^2}{M\sigma_n^2\left(\frac{M}{K} + \alpha_i + \beta_i\right)}\right) \tag{16}$$

for $K \to \infty$, where

$$\delta_i = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{\mathcal{E}\{d_{k,i}\}}{K/M}$$
$$\alpha_i = \sum_{k,\mathcal{R}_k\in\mathcal{X}_i}\frac{\mathcal{E}\{a_{k,i}\}}{K/M}$$
$$\beta_i = \sum_{m=1,m\neq i}^{M}\left(\sum_{k,\mathcal{R}_k\in\mathcal{X}_m}\frac{\mathcal{E}\{b_{k,i,m}\}}{K/M}\right).$$

Hence, the total capacity is given by

$$C \xrightarrow{\text{w.p. 1}} \frac{1}{2}\sum_{i=1}^{M}\log\left(1 + \frac{K\delta_i^2}{M\sigma_n^2\left(\frac{M}{K} + \alpha_i + \beta_i\right)}\right)$$

and for $K \to \infty$

$$C_{\mathrm{lower}}^{\infty} = \frac{M}{2}\log(K) + O(1).$$

This concludes the proof of (13).

The proof for i.i.d. $\{E_k\}$ and i.i.d. $\{P_k\}$ starts by noting that i.i.d. $\{E_k\}_{k=1}^K$, $\{P_k\}_{k=1}^K$, $\{\mathbf{G}_k\}_{k=1}^K$, and $\{\mathbf{H}_k\}_{k=1}^K$ implies that $\tilde{n}_{k,i}$ and $\tilde{q}_{k,i}$ are i.i.d. (across k) $\forall i$, which, combined with the fact that the corresponding variances are bounded, allows to apply the Lindeberg-Levy theorem [37, Theorem 1.9.1.A] to conclude that asymptotically in K the noise terms N_i are circularly symmetric complex Gaussian for all i. Since h_i^{sig} and $h_{i,j}^{\mathrm{int}}$ $(j = 1,2,\ldots,M,\ j \neq i)$ are assumed perfectly known at the ith receive antenna, the receive signal y_i is circularly symmetric complex Gaussian. Applying steps similar to those leading to (16) and the steps thereafter completes the proof. ∎

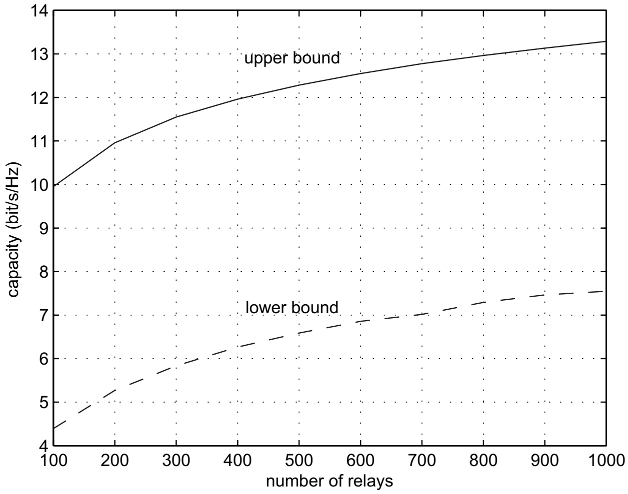

Fig. 5. Capacity upper and lower bounds vs. number of relays for the coherent case.

C. Network Capacity and Discussion of Results

Network capacity: We can now combine Theorems 1 and 2 to state that asymptotically in K the coherent network capacity (under the assumptions in Theorem 2) is given by

$$C = \frac{M}{2}\log(K) + O(1). \tag{17}$$

We hasten to add that the $O(1)$-terms in the upper and lower bounds in Theorems 1 and 2 are in general different so that we "sandwich" the exact network capacity up to an $O(1)$-term. Consequently, our relaying protocol is optimal in the sense that it achieves the right capacity scaling law, or equivalently, it is optimal in the sense that it achieves network capacity up to an $O(1)$-term. The difference between the $O(1)$-terms in the lower and upper bounds can be significant (see, e.g., Fig. 5). Moreover, (17) implies that the proposed protocol asymptotically turns the network into a point-to-point MIMO link with multiplexing gain $M/2$ and distributed per-stream array gain K. From (16) we can furthermore conclude that in the large K limit each of the M single-input single-output (SISO) channels (with corresponding input-output relation (12)) made up of the individual transmit-receive antenna pairs approaches an AWGN channel. This effect can be attributed to distributed diversity gain. The factor $1/2$ penalty in the multiplexing gain comes from the fact that communication takes place over two time slots ("half-duplex protocol"). The dependence of network capacity on SNR in (17) is through the quantities δ_i, α_i, and β_i, and has been absorbed in the $O(1)$-term.

Channel state information: We note that for our relaying protocol to be asymptotically (in K) optimal (up to an $O(1)$-term) we need perfect knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K$ at each of the receive antennas in the general case, and perfect knowledge of h_i^{sig} and $h_{i,j}^{\text{int}}$ $(j = 1, 2, \ldots, M,\ j \neq i)$ at the ith receive antenna in the case of i.i.d. $\{E_k\}$ and i.i.d. $\{P_k\}$. In the general case, perfect knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K$ is a sufficient condition for (13) to hold. More generally, we need to ensure that the conditions for the Lindeberg-Feller theorem [37, Theorem 1.9.2.A] to hold are satisfied. Acquiring knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K$ at each of the receive antennas will be challenging in practice and will lead to significant training overhead. Knowledge of the h_i^{sig} and $h_{i,j}^{\text{int}}$ $(j = 1, 2, \ldots, M,\ j \neq i)$ at the ith receive antenna can be obtained through standard multiple-input single-output (MISO) training schemes since the overall effective channel for each receive antenna becomes a MISO channel. Acquiring knowledge of the relays' backward channels can be done through standard training approaches; obtaining knowledge of the forward channels is more challenging and equivalent to acquiring transmit CSI in point-to-point wireless links. At first sight it may appear that accurate CSI at the relays is critical for the scaling law (17) to hold. However, it was shown recently [38] (for $N = 1$) that (17) remains valid under very mild conditions on the knowledge of the phases of the backward and forward channels; knowledge of the magnitudes of the fading coefficients is not required. It was found, however, that the price to be paid for inaccurate channel (phase) knowledge at the relays is an increase (w.r.t. the perfect CSI case) in the number of relays needed to achieve a given network capacity [38].

Distributed orthogonalization: Besides realizing multiplexing gain, distributed array gain, and distributed diversity gain, the proposed relaying protocol also *orthogonalizes the effective MIMO channel* between source and destination terminals and hence *multistream interference* is *completely eliminated* without cooperation between the relay terminals. We can therefore conclude that even though multistream interference at the relay terminals is not canceled (recall that our result holds for any $N \geq 1$) as was done in [39] through zero-forcing, in the large relay limit *multistream interference cancellation* is achieved in a *completely decentralized fashion* by exploiting differences in the (distributed) spatial signatures of the interfering data signals across the relay terminals. The resulting coherent combining of the data signals at their intended destination terminal antennas raises the received power of the desired signal by a factor of K over the noise and interference (distributed array gain) so that asymptotically in K all M source-destination pairs can communicate as though only one pair was active. Since the overall MIMO channel is orthogonalized, single-antenna (i.e., independent) decoding at the receive terminal achieves capacity in the large K limit. This implies that there is no need for sophisticated joint decoding schemes such as successive cancellation [40], [41] or sphere decoding [42]. In contrast, in an $M \times M$ point-to-point MIMO link, achieving a multiplexing gain of M requires cooperation (i.e., joint processing) between the receive antennas.

Relation to interference channels: Since no cooperation is required between antennas at the transmitter (spatial multiplexing) and between antennas at the receiver (orthogonalization), the overall system can also be regarded as a network where M single-antenna source destination pairs communicate concurrently through a set of K common relays, as was done in [43]. The asymptotic (in K) sum-capacity of this network is given by (17). In the absence of relay terminals (assuming a direct link between source and destination) the overall system consisting of M single-antenna source terminals and M single-antenna noncooperating destination terminals constitutes an interference channel [44]. Assuming that the output signals of this interference channel are statistically equivalent [44], which is the case if the scalar channels between the source and the destination terminals are i.i.d., it follows that the sum capacity of this interference channel equals the sum capacity of any one of its multiple-access channels [44]. Consequently, in the absence of relays the lack of cooperation at the destination terminals results in a multiplexing gain of 1. We can therefore conclude that for $M > 2$ coherent relaying yields a higher multiplexing gain than that obtained in the no-relay case.

Comments on path loss and shadowing: We conclude this section with a few comments on the assumptions made on the scalar RVs E_k and P_k incorporating the effects of large-scale fading and path loss. Positivity of the E_k and P_k implies that as the network grows large the relay terminals populate a domain of fixed area (i.e., the network is dense). While this assumption may seem restrictive, we emphasize that the E_k and P_k can be arbitrarily small (but positive), which implies that the domain can be large. On the other hand, investigation of the upper bound in (3) reveals that increasing the size of the domain will result in a decrease of $\mu = (1/K)\sum_{k=1}^K \mathcal{E}\{E_k\}$ and hence in reduced network capacity. The asymptotic growth rate, however, remains unchanged as long as we have an infinite number of relay terminals with positive E_k and positive P_k. Finally, the assumption of the E_k and P_k being bounded implies that the source and destination terminals cannot communicate with the relay terminals in a lossless fashion. Clearly, relaxing this assumption can only increase network capacity.

D. Numerical Example

In the following, we provide a numerical result for $M = 2$, $N = 1$, and[6] $E_k/\sigma_n^2 = P_k/\sigma_n^2 = 10\text{dB}\,(k = 1, 2, \ldots, K)$. Fig. 5 shows the upper bound on coherent network capacity in Theorem 1 and the corresponding lower bound achieved by the matched-filtering based relaying protocol described in Section IV-A. While our relaying protocol can indeed be seen to exhibit logarithmic capacity scaling in K, we can also see that there is a *significant gap* between the bounds, which can be attributed to the differences in the $O(1)$-term. This gap is

[6]Note that the E_k and P_k were chosen to be deterministic for simplicity.

independent of K (as seen in Fig. 5) and becomes insignificant only for very large values of K.

V. NONCOHERENT MIMO RELAY NETWORKS

So far, we considered coherent relay networks, where each relay terminal knows its assigned backward and forward channels perfectly. In the following, we relax this assumption and study networks with no CSI at the relay terminals, i.e., noncoherent relay networks as defined in Section I-B. In particular, we investigate a simple amplify-and-forward protocol, where relay terminals simply forward a scaled version of the received signal without additional processing.

A. The AF Protocol

We assume that the kth relay terminal knows the average received signal plus noise energy $E_k+\sigma_n^2$ at each of its N antennas and performs the normalization $\mathbf{t}_k=(E_k+\sigma_n^2)^{-1/2}\mathbf{r}_k$. This ensures that the power constraint[7] $\mathcal{E}\{\|\mathbf{t}_k\|^2\}=N$ is satisfied. It now follows from (2) that the signal received at the destination terminal $\mathcal{D}$ is given by

$$\mathbf{y}=\sum_{k=1}^{K}\sqrt{\frac{P_k}{N(E_k+\sigma_n^2)}}\mathbf{G}_k\mathbf{r}_k+\mathbf{z}. \tag{18}$$

Inserting (1) into (2), dividing the RHS and left-hand side (LHS) of (18) by $\sqrt{K}$, and simplifying, we get

$$\frac{\mathbf{y}}{\sqrt{K}}=\underbrace{\frac{1}{\sqrt{K}}\sum_{k=1}^{K}\left(\sqrt{\frac{P_kE_k}{NM(E_k+\sigma_n^2)}}\mathbf{G}_k\mathbf{H}_k\right)\mathbf{s}}_{\mathbf{A}}+\underbrace{\frac{1}{\sqrt{K}}\sum_{k=1}^{K}\left(\sqrt{\frac{P_k}{N(E_k+\sigma_n^2)}}\mathbf{G}_k\mathbf{n}_k\right)+\frac{\mathbf{z}}{\sqrt{K}}}_{\mathbf{b}}$$

or equivalently $\mathbf{y}/\sqrt{K}=\mathbf{As}+\mathbf{b}$. In the following, similar to Theorem 2, again we need to distinguish the cases of arbitrary independent E_k and P_k and the case of i.i.d. $\{E_k\}_{k=1}^K$ and i.i.d. $\{P_k\}_{k=1}^K$. In the general case, we need to assume that the receiver has perfect knowledge[8] of $\{E_k,P_k,\mathbf{G}_k\}_{k=1}^K$. In the i.i.d. case knowledge of the compound channel matrix $\mathbf{A}$ in the receiver is sufficient. In order to simplify the exposition, we shall provide the detailed derivation for the i.i.d. case only. The general case requires minor modifications to establish the i.i.d. nature of the compound channel matrix $\mathbf{A}$ conditioned on perfect knowledge of $\{\mathbf{G}_k\}_{k=1}^K$. In the $K\to\infty$ limit, applying the Lindeberg-Feller theorem [37, Theorem 1.9.2.A] and [37, Theorem 1.8.D] in conjunction with the Kolmogorov conditions

$$\sum_{k=1}^{\infty}\frac{\mathrm{VAR}\left(\frac{P_kE_k}{E_k+\sigma_n^2}\right)}{k^2}<\infty,\qquad \sum_{k=1}^{\infty}\frac{\mathrm{VAR}\left(\frac{P_k}{E_k+\sigma_n^2}\right)}{k^2}<\infty \tag{19}$$

[7]Note that averaging is performed over the signal, noise, and backward channel $\mathbf{H}_k$.

[8]Again, this is only a sufficient condition. In general, we need to ensure that the Lindeberg condition [37] is satisfied.

it now follows that for $i,j=1,2,\ldots,M$,

$$[\mathbf{A}]_{i,j}\sim\mathcal{CN}\left(0,\frac{1}{K}\sum_{k=1}^{K}\mathcal{E}\left\{\frac{P_kE_k}{M(E_k+\sigma_n^2)}\right\}\right)$$

and for $i=1,2,\ldots,M$,

$$b_i\sim\mathcal{CN}\left(0,\frac{\sigma_n^2}{K}\left(\sum_{k=1}^{K}\mathcal{E}\left\{\frac{P_k}{E_k+\sigma_n^2}\right\}+1\right)\right)$$

where $[\mathbf{A}]_{i,j}$ denotes the element in the ith row and jth column of $\mathbf{A}$, and b_i is the ith element of $\mathbf{b}$. We shall next establish independence between the elements of the composite channel $\mathbf{A}$ by considering the covariance matrix $\mathbf{R_A}=\mathcal{E}_{\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^K}\{\mathbf{aa}^H\}$, where $\mathbf{a}=\mathrm{vec}(\mathbf{A})$. Using the identity $\mathrm{vec}(\mathbf{ABC})=(\mathbf{C}^T\otimes\mathbf{A})\,\mathrm{vec}(\mathbf{B})$ [45], we get

$$\begin{aligned}\mathbf{R_A}&=\frac{1}{K}\sum_{k=1}^{K}\frac{P_kE_k}{NM(E_k+\sigma_n^2)}\\&\quad\times\mathcal{E}\{(\mathbf{I}_M\otimes\mathbf{G}_k)\,\mathrm{vec}(\mathbf{H}_k)\,\mathrm{vec}^H(\mathbf{H}_k)(\mathbf{I}_M\otimes\mathbf{G}_k^H)\}\\&=\frac{1}{K}\sum_{k=1}^{K}\frac{P_kE_k}{M(E_k+\sigma_n^2)}\mathbf{I}_{M^2}\end{aligned}$$

where we used the fact that $(\mathbf{A}\otimes\mathbf{C})(\mathbf{B}\otimes\mathbf{D})=(\mathbf{AB})\otimes(\mathbf{CD})$, and the expectation is w.r.t. $\{\mathbf{H}_k,\mathbf{G}_k\}_{k=1}^K$. It follows from [37, Theorem 1.8.D] and (19) that for $K\to\infty$

$$\mathbf{R_A}\xrightarrow{\text{w.p. }1}\frac{1}{K}\sum_{k=1}^{K}\mathcal{E}\left\{\frac{P_kE_k}{M(E_k+\sigma_n^2)}\right\}\mathbf{I}_{M^2}.$$

Independence between the elements of $\mathbf{b}$ can be established similarly. Combining these results with the fact that $I(\mathbf{y};\mathbf{s})=I(\mathbf{y}/\sqrt{K};\mathbf{s})$, and assuming that the receiver has perfect knowledge of the compound channel matrix $\mathbf{A}$, we have proven

Theorem 3: For noncoherent MIMO relay networks the capacity achieved by the AF protocol as described in this section, in the large K limit, is given by

$$C_{\mathrm{AF}}^{\infty}=\frac{1}{2}\mathcal{E}_{\mathbf{H}_w}\left\{\log\det\left(\mathbf{I}_M+\frac{\rho}{M}\mathbf{H}_w\mathbf{H}_w^H\right)\right\} \tag{20}$$

where the elements of $\mathbf{H}_w$ are i.i.d. $\mathcal{CN}(0,1)$ and

$$\rho=\frac{\sum_{k=1}^{K}\mathcal{E}\left\{\frac{P_kE_k}{E_k+\sigma_n^2}\right\}}{\sigma_n^2\sum_{k=1}^{K}\mathcal{E}\left\{\frac{P_k}{E_k+\sigma_n^2}\right\}}$$

can be interpreted as an effective SNR. ■

It is interesting to observe that $C_{\mathrm{AF}}^{\infty}=(1/2)\,C_{M\times M}$, where $C_{M\times M}$ denotes the capacity of an $M\times M$ i.i.d. Gaussian MIMO channel with receive SNR ρ, no transmit CSI, and perfect receive CSI. In the high SNR regime $\rho\gg1$, we have from [36]

$$C_{\mathrm{AF}}^{\infty}=\frac{M}{2}\log\left(\frac{\rho}{M}\right)+O(1).$$

We can now draw a number of interesting conclusions.

- In the large K limit, the simple AF protocol realizes a multiplexing gain of $M/2$ independently of the number of relay terminal antennas $N\geq1$. It is remarkable that multiplexing gain can be obtained even if the relays do

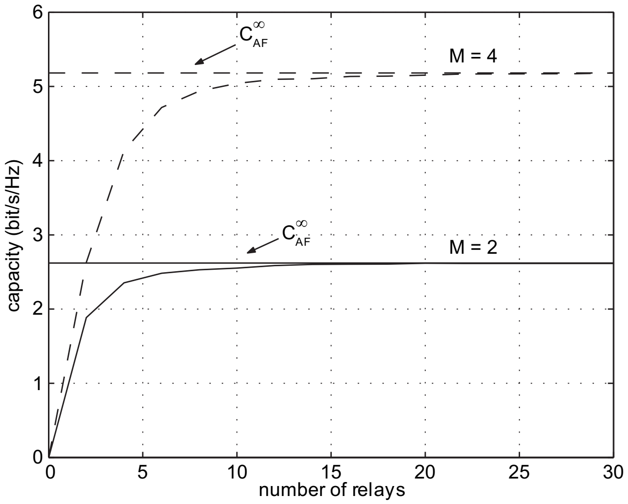

Fig. 6. Capacity vs. number of relays for the AF protocol.

not have channel knowledge. For $N = 1$, for example, each of the relay terminals induces an overall channel between the source and destination that is pin-hole [46] like (up to the forwarded noise contribution) and hence of rank 1 w.p. 1. This implies that each of the relays supports only one spatial data stream between source and destination. Asymptotically in K due to the independence of the individual $\mathbf{H}_k$ and $\mathbf{G}_k$ across k, the collection of relays ensures that all M data streams are being "served". This result demonstrates that employing relays as active scatterers can recover spatial multiplexing gain in poor scattering environments. Numerical evidence of this fact has been provided in [47].

- The absence of channel knowledge at the relay terminals results in a lack of distributed array gain reflected by the fact that C_{AF}^{∞} is independent of K. This is in stark contrast to the coherent case, where a distributed per-stream array gain of K was obtained. Note, however, that (20) implies traditional receive array gain and diversity gain (independent of K, i.e., there is no distributed diversity gain as in the coherent case) present in coherent point-to-point MIMO links (recall that the destination terminal knows the compound channel matrix $\mathbf{A}$ perfectly).
- Unlike the coherent case, the effective MIMO channel between $\mathcal{S}$ and $\mathcal{D}$ is not orthogonalized so that joint decoding at the destination terminal is crucial to achieve capacity.
- The capacity result obtained for the noncoherent case is not as strong as the result for the coherent case. The expression in (20) corresponds to the capacity achieved by a particular relaying strategy, namely amplify-and-forward. In contrast, in the coherent case, the cut-set bound provides the ultimate capacity upper bound achievable through any single-letter processing strategy at the relays.

B. Numerical Example

The purpose of this numerical example is to demonstrate that convergence (w.r.t. increasing K) to C_{AF}^{∞} depends critically on the number of source-destination terminal antenna pairs M and is generally very fast. Fig. 6 shows the capacity (obtained through Monte Carlo simulation) of the AF protocol described above as a function of the number of relays K for $N = 1$ and $M = 2, 4$ along with the large K capacity limit as predicted by (20). For finite K, knowledge of $\{E_k, P_k, \mathbf{H}_k, \mathbf{G}_k\}_{k=1}^{K}$ is assumed at the destination terminal, rendering the noise term conditionally Gaussian. We furthermore assume that E_k/σ_n^2 and P_k/σ_n^2 are drawn independently from a truncated log-normal distribution (modeling macroscopic fading) with mean $10\,\mathrm{dB}$, standard deviation (of the underlying log-normal distribution) $4\,\mathrm{dB}$, truncated such that P_k/σ_n^2 and E_k/σ_n^2 are bounded above by $15\,\mathrm{dB}$ and below by $5\,\mathrm{dB}$. We can see that the capacity of the AF protocol converges very quickly to C_{AF}^{∞} and that increasing M requires larger K to achieve the same fraction of (the larger) C_{AF}^{∞}. Fig. 6 also shows that for small K the capacity increases linearly in K, which is due to the fact that the corresponding effective channel matrix $\mathbf{A}$ is building up rank and hence multiplexing gain (reflected by a pre-log increase). For large K, once $\mathbf{A}$ is full-rank with high probability, the curve flattens out and increasing K mainly has the effect of rendering the elements of $\mathbf{A}$ Gaussian.

VI. Conclusions

We studied capacity scaling laws in MIMO relay networks under two-hop half-duplex relaying. For M antennas at the source and destination terminals and perfect channel state information at the relays, we showed that asymptotically in the number of relay terminals K, independently of the number of relay terminal antennas N, network capacity scales as $C = (M/2)\,\log(K) + O(1)$. In particular, we showed that this capacity can be realized by relay partitioning, backward and forward matched filtering at the relay terminals, and independent stream decoding (*thanks to distributed orthogonalization*) at the destination terminal. This is possible because the relays orthogonalize the effective MIMO channel in a distributed fashion. For noncoherent networks, we investigated a simple amplify-and-forward protocol and showed that for any $N \geq 1$, the large K capacity limit is given by $C = (M/2)\,\log(\mathrm{SNR}) + O(1)$. In the noncoherent case, cooperation between the receive terminal antennas is crucial to achieve capacity.

The aim of this paper was to introduce the idea of *distributed multistream interference cancellation* and to study the *impact of CSI at the relay terminals on the network capacity scaling behavior*. A number of interesting topics remain open such as: (i) the investigation of more sophisticated relaying strategies such as successive cancellation and Costa precoding in the coherent case, and applying nonlinear functions in the noncoherent case possibly also exploiting the spatial nature of the channel; (ii) an analytic investigation of the convergence properties of network capacity in the large K limit, and in particular the impact of (i) on this convergence behavior; (iii) the extension to more general channel models, such as frequency-selective fading.

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