Realizing MIMO Gains Without User Cooperation in Large Single-Antenna Wireless Networks

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Abstract — **We consider wireless networks where L single-antenna source-destination terminal pairs communicate concurrently through a common set of K single-antenna relay terminals using one-hop relaying. It is shown that asymptotically in K, the sum capacity of this network scales as $C = (L/2)\log(K) + O(1)$ and can be achieved without cooperation between any of the terminals.**

I. Assumptions on the network

We consider a wireless network consisting of $K+2L$ single-antenna terminals with L designated source-destination terminal pairs denoted as $\mathcal{S}_l$ and $\mathcal{D}_l\,(l = 1, ..., L)$, respectively, and K single-antenna relay terminals denoted as $\mathcal{R}_k\,(k = 1, ..., K)$. We assume that 1) source terminal $\mathcal{S}_l$ intends to communicate solely with destination terminal $\mathcal{D}_l$, 2) no cooperation between any of the terminals is allowed, 3) no direct link between source and destination terminals exists, 4) the terminals cannot transmit and receive simultaneously, 5) communication takes place in two hops over two separate time slots (i.e., one-hop relaying), 6) all channels are independent, frequency-flat Rayleigh block-fading with independent realizations across blocks, and 7) transmission/reception between the terminals is perfectly synchronized.

In the first time slot the source terminals $\mathcal{S}_l\,(l = 1, ..., L)$ broadcast their information to all the relay terminals. The corresponding input-output relation is given by

$$r_k = \sum\nolimits_{l=1}^{L} \sqrt{E_{k,l}}\, h_{k,l} s_l + n_k, \quad k = 1, ..., K,$$

where r_k denotes the received signal at the k-th relay terminal, $E_{k,l}$ is an energy normalization factor accounting for path loss and shadowing in the $\mathcal{S}_l \rightarrow \mathcal{R}_k$ link, $h_{k,l}$ denotes the corresponding[1] $\mathcal{CN}(0, 1)$ complex-valued channel gain, s_l is the temporally i.i.d. $\mathcal{CN}(0, 1)$ data signal transmitted by $\mathcal{S}_l$ and satisfying $\mathcal{E}\{s_l s_k^*\} = \delta[l-k]$, and n_k is $\mathcal{CN}(0, \sigma_n^2)$ temporally and spatially (across relay terminals) white noise. The relay terminals process r_k to produce t_k subject to the power constraint $\mathcal{E}\{|t_k|^2\} \leq 1$ and simultaneously broadcast the t_k to all destination terminals during the second time slot while the source terminals are silent. The l-th destination terminal receives the signal

$$y_l = \sum\nolimits_{k=1}^{K} \sqrt{P_{l,k}}\, g_{l,k} t_k + z_l, \quad l = 1, ..., L,$$

where $P_{l,k}$ is an energy normalization factor accounting for path loss and shadowing in the $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link, $g_{l,k}$ is the corresponding $\mathcal{CN}(0, 1)$ complex-valued channel gain, and z_l is $\mathcal{CN}(0, \sigma_n^2)$ temporally and spatially (across destination terminals) white noise at destination terminal $\mathcal{D}_l$. The random variables $E_{k,l}$ and $P_{l,k}\,(k = 1, ..., K,\ l = 1, ..., L)$ are i.i.d., strictly positive (dense network), bounded, and remain constant over the entire time period of interest. Finally, we note that for $L = 1$ our setup essentially reduces to the relay network considered in [2].

[1] Notation: $\mathcal{CN}(0, \sigma^2)$ denotes a circularly-symmetric complex Gaussian random variable with mean 0 and variance σ^2, $\mathcal{E}$ stands for the expectation operator, and $\delta[l] = 1$ for $l = 0$ and 0 otherwise.

II. Capacity of the Network

We assume that there is no channel state information at the source terminals $\mathcal{S}_l$, the k-th relay terminal has perfect knowledge of the individual compound channels (including path loss and shadowing factors) corresponding to $\mathcal{S}_l \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}_l$ for $l = 1, ..., L$, and the receive terminal $\mathcal{D}_l$ knows the compound single-input single-output channel $\mathcal{S}_l \rightarrow \mathcal{D}_l$.

Theorem [1]. *For a fixed number of source-destination pairs L, in the large relay limit $K \rightarrow \infty$, the sum capacity of the network described above is upper bounded by*

$$C_u = \frac{L}{2}\log(K) + O(1). \tag{1}$$

This bound is a direct consequence of the cut-set bound and is achieved when all the relay terminals can fully cooperate and can further convey the relays' transmit signals t_k in a lossless fashion to the cooperating destination terminals $\mathcal{D}_l$.

A protocol which achieves the upper bound (1) and hence network capacity (up to the $O(1)$-term) without requiring any terminal cooperation whatsoever is described in detail in [1]. The essence of this protocol lies in partitioning the relay terminals into L subsets $\mathcal{M}_l\,(l = 1, ..., L)$ each of which is assigned to one of the L source-destination pairs. The relaying strategy for $\mathcal{R}_k \in \mathcal{M}_l$ is as follows. The received signal r_k is matched-filtered with respect to the (backward) channel $\mathcal{S}_l \rightarrow \mathcal{R}_k$ followed by matched-filtering with respect to the (forward) channel $\mathcal{R}_k \rightarrow \mathcal{D}_l$ subject to an energy normalization such that $\mathcal{E}\{|t_k|^2\} = 1$. When the number of relay terminals $K \rightarrow \infty$, we need to ensure that $|\mathcal{M}_l| \rightarrow \infty$ for $l = 1, ..., L$.

We can conclude that the network described above under the capacity achieving protocol introduced in [1] behaves like a multiple-input multiple-output (MIMO) link with spatial multiplexing gain $L/2$ where each of the multiplexed data streams experiences a distributed array gain of K. The factor $1/2$ in the multiplexing gain stems from the fact that communication takes place over two time slots. Moreover, we note that our protocol completely eliminates *multi-user interference* in a fully decentralized fashion thus diagonalizing the effective MIMO channel. Consequently, sum capacity is achieved without cooperation between the destination terminals.

References

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