

Semicoherent PPM for Wideband Communications

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Abstract — We quantify the impact of coherence on pulse position modulation (PPM) over wideband fading channels by computing achievable rates and an upper bound on uncoded symbol error probability. We study the influence of channel estimation accuracy on the optimum diversity order and furthermore find that a near-optimum receiver typically needs to estimate a few channel taps only.

I. SYSTEM AND CHANNEL MODEL

We employ a discrete-time, frequency-selective block-fading channel model with baseband equivalent input-output relation

$$y_n = \sum_{l=0}^{L-1} h_l x_{n-l} + w_n$$

where x_n and h_l denote the transmit signal and the channel impulse response, respectively, and w_n is i.i.d. proper Gaussian noise with variance \mathcal{N}_0 . The underlying continuous-time channel has finite delay spread τ_0 . The *semicoherent* receiver knows the channel statistics and has partial knowledge of the L channel taps, modeled by decomposing the vector¹ $\mathbf{h} = [h_0 \ h_1 \ \dots \ h_{L-1}]^T$ into a known and an unknown part as $\mathbf{h} = \mathbf{h}_k + \mathbf{h}_u$.

The modulation scheme analyzed is M -ary PPM, with a single pulse of length T_p and amplitude $\sqrt{\mathcal{E}_s}$ transmitted in one of the M time slots of the PPM symbol. A guard interval of length τ_0 is appended after each time slot to ensure orthogonality of the received symbols. We assume interleaved transmission such that each PPM symbol is subject to independent fading.

II. ERROR PERFORMANCE

In the following, P_{pep} denotes the pairwise error probability (PEP), averaged over the channel. Due to the orthogonality of the symbols, all PEPs are equal, and the average symbol error probability is upper-bounded by $P_e \leq (M-1)P_{pep}$.

Theorem 1 (PEP for Semicoherent PPM)

Consider M -ary PPM over a frequency-selective Rayleigh-fading channel with L taps partially known to the receiver according to $\mathbf{h} = \mathbf{h}_k + \mathbf{h}_u$, where \mathbf{h}_k and \mathbf{h}_u are independent zero-mean proper Gaussian random vectors with $\mathbb{E}[\mathbf{h}_k \mathbf{h}_k^H] = \text{diag}_{l=0}^{L-1} \{\sigma_{k,l}^2\}$ and $\mathbb{E}[\mathbf{h}_u \mathbf{h}_u^H] = \text{diag}_{l=0}^{L-1} \{\sigma_{u,l}^2\}$. The uncoded PEP for semicoherent maximum likelihood (ML) detection is upper-bounded as

$$P_{pep} \leq \prod_{l=0}^{L-1} \frac{1 + \frac{\mathcal{E}_s}{\mathcal{N}_0} \sigma_{u,l}^2}{1 + \frac{\mathcal{E}_s}{2\mathcal{N}_0} (2\sigma_{u,l}^2 + \sigma_{k,l}^2) + \left(\frac{\mathcal{E}_s}{2\mathcal{N}_0}\right)^2 \sigma_{u,l}^2 (\sigma_{u,l}^2 + \sigma_{k,l}^2)}$$

The proof is based on a technique introduced in [1].

For MMSE channel estimation with U averages, $\sigma_{k,l}^2$ and $\sigma_{u,l}^2$ equal the variances of the estimate and the estimation error of the l th tap. Fig. 1 shows the corresponding PEP for uniform power delay profile (PDP) as a function of L for different values of U . We can see that there is always an optimum

¹Our notation is as follows: T, H denote transpose and conjugate transpose, respectively; $\mathbb{E}[\cdot]$ is the expectation operator.

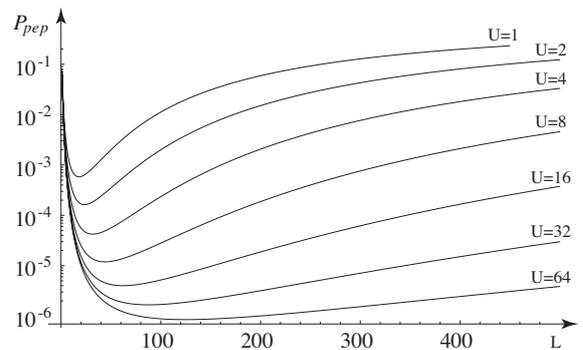


Fig. 1: PEP upper bound for MMSE channel estimation with U averages, $\mathcal{E}_s/\mathcal{N}_0 = 15$ dB, and uniform PDP.

diversity order L^* [2], [3], corresponding to the minimum P_{pep} . Moreover, L^* increases with U and therefore with estimation accuracy. Since the number of resolvable taps increases (typically linearly) with bandwidth, Theorem 1 allows to numerically compute the optimum bandwidth for given target error probability, PDP, and U .

A reduced complexity receiver only estimates a subset of channel taps. In this case, Theorem 1 permits to determine the number of taps that need to be estimated for a given target error probability. This number is in general quite small for exponential PDPs, which are typical for real-world channels.

III. ACHIEVABLE RATES

The impact of coherence, PDP, channel order, and SNR on the performance of coded PPM systems can be quantified through

Theorem 2 (Achievable Rates for Semicoherent PPM) Denote the bound on the PEP in Theorem 1 as Q_{pep} . For M -ary semicoherent PPM, rates $R \leq R_0$ are achievable with

$$R_0 = \frac{1}{M(T_p + \tau_0)} \log_2 \frac{M}{1 + (M-1)Q_{pep}}$$

For MMSE channel estimation and fixed modulation size M , the low-SNR slope of the R_0 vs. SNR curve always equals the slope for noncoherent detection, irrespective of the number of averages U . When compared to the coherent case, this implies that performance degrades significantly faster with decreasing SNR. Consequently, in the low-SNR regime, channel estimates can no longer be modeled as perfect. In order to achieve close to coherent performance, M has to be increased for decreasing SNR, resulting in “peakier” signals.

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