Multiple-Access Strategies for Frequency-Selective MIMO Channels

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***Abstract*—In this paper, we consider frequency-selective coherent multiple-input multiple-output (MIMO) multiple-access fading channels. Assuming that each of the users employs orthogonal frequency-division multiplexing (OFDM), we introduce a multiple-access scheme that gradually varies the amount of user collision in signal space by assigning different subsets of the available OFDM tones to different users. The corresponding multiple-access schemes range from frequency-division multiple access (FDMA) (each OFDM tone is assigned to at most one user) to CDMA (each OFDM tone is assigned to all the users). We quantify the effect of signal space collision between the users by computing the ergodic capacity region for the entire family of multiple-access schemes. It is shown that the ergodic capacity region obtained by a fully collision-based scheme (CDMA) is an outer bound to that corresponding to any other multiple-access strategy. In practice, however, minimizing the amount of user collision in frequency is desirable as this minimizes the receiver complexity incurred by having to separate the interfering (colliding) signals. Our analysis shows that the impact of collision on spectral efficiency depends critically on the channel's spatial fading statistics and the number of antennas.**

***Index Terms*—Code-division multiple access (CDMA), frequency-division multiple access (FDMA), frequency-selective fading, multiple-access channel, multiple-input multiple-output (MIMO), orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

THE USE of multiple-input multiple-output (MIMO) wireless systems has been shown to significantly increase the spectral efficiency of point-to-point wireless links [1]–[5]. The performance limits of MIMO multiple access (MA) and broadcast channels are considerably less understood and have recently attracted significant interest [6]–[12].

Contributions and relation to previous work: In this paper, we focus on MIMO MA channels with frequency-selective fading (spatially correlated at the receiver) assuming perfect channel state information (CSI) at the multiple-antenna receiver and no channel knowledge at the multiple antenna transmitters. Each of the users employs orthogonal frequency-division multiplexing (OFDM) [13]. We consider a MA scheme, which implements a variable amount of user collision in frequency (signal space) by assigning (potentially overlapping) subsets of the available OFDM tones to different users. The resulting family of MA schemes encompasses the extreme cases of frequency-division multiple access (FDMA), where each tone is assigned to at most one user, and code-division multiple access (CDMA), where each tone is assigned to all the users. Following [14], we use the term CDMA solely to indicate that all the users occupy the entire frequency band; the effect of redundancy-introducing spreading will not be considered.

Besides developing a framework for the analysis of MA schemes realizing a variable amount of collision in signal space, our main contributions can be summarized as follows.

- We show that, irrespective of spatial receive fading correlation and the number of antennas, the ergodic capacity region obtained for a fully collision-based (CDMA) scheme is an outer bound to the ergodic capacity region for any other MA strategy, where users collide only on subsets of the available tones or do not collide at all (FDMA). This result generalizes the main result in [14] showing the strict superiority of CDMA over FDMA (two extremes of our MA scheme) in single-antenna frequency-selective fading MA channels. Further results comparing the performance of CDMA and FDMA in the single-antenna case can be found in [15]–[18]. In particular, [16] and [17] discuss a TDMA scheme implementing a variable amount of collision and show that in the presence of cochannel interference full collision will in general be suboptimal from a capacity point-of-view. Finally, we note that the capacity region for deterministic MA channels with ISI (with perfect CSI both at transmitter and receiver) has been computed in [19]. It is furthermore shown in [19], that FDMA, with optimally selected frequency bands for each user, achieves the total capacity of the Gaussian MA channel with ISI.
- In practice, minimizing the amount of collision in signal space is desirable as this minimizes the receiver complexity incurred by having to separate the interfering (colliding) signals. We therefore study the joint decoding performance loss due to suboptimum (i.e., not fully collision-based) multiple accessing in a systematic fashion. It is first shown that in the low Signal to Noise Ratio (SNR) regime the amount of signal space collision has a vanishing impact on the ergodic capacity. Therefore, we focus on the high SNR regime and perform an analysis based on the notion of the multiplexing gain region [20]. Our analysis indicates that for rich scattering and a small number of receive antennas, very little collision is needed to realize a significant fraction of the available sum capacity. A detailed discussion of this aspect is provided for the 2-user case. Extending results reported in [21] for the single-antenna case, we further quantify the performance difference between CDMA (full collision) and FDMA (no collision) by computing their

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asymptotic (large number of users limit) sum capacity difference as a function of the number of antennas.

Further work on MIMO MA fading channels has been reported previously in [7], [20], [22], [24], [25]. In particular, in [24] a comparison between CDMA (with random spreading) employing MMSE and matched-filter (MF) receiver frontends and orthogonal accessing is provided. It is demonstrated in [24] that in terms of spectral efficiency CDMA with no spreading and a MF frontend outperforms orthogonal accessing for a large enough number of receive antennas. This result further strengthens the case for a fully-collision based MA scheme which outperforms orthogonal accessing even in the presence of a suboptimum receiver frontend.

Finally, we note that the problem statements considered in this paper were mostly inspired by previous work in [14] and [24].

Organization of the paper: The remainder of this paper is organized as follows. Section II introduces the channel and signal model and describes the concept of MA with variable amount of collision in signal space. In Section III, we derive the ergodic capacity region for arbitrary collision patterns. Section IV characterizes the shape of the multiplexing gain region as a function of the collision pattern. Section V provides a detailed discussion of the two-user case. In Section VI, we perform an asymptotic (large number of users limit) analysis of the sum capacity difference between CDMA and FDMA. We conclude in Section VII. Appendix I contains results on the rank of the sum of Gaussian random matrices. The proofs of the main theorems are provided in Appendices II and III.

Notation: $\mathcal{E}$ denotes the expectation operator. The superscripts ^{T}, ^{H} and * stand for transposition, conjugate transpose and elementwise conjugation, respectively. $r(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$, $\mathrm{span}\{\mathbf{A}\}$, and $\lambda_i(\mathbf{A})$ denote the rank, trace, column space (i.e., the vector space spanned by the columns of $\mathbf{A}$) and ith eigenvalue[1] of the matrix $\mathbf{A}$, respectively. $\mathbf{I}_m$ stands for the $m \times m$ identity matrix. For equal size matrices $\mathbf{A}_0, \mathbf{A}_1, \ldots, \mathbf{A}_{K-1}$, $\mathrm{diag}\{\mathbf{A}_i\}_{i=0}^{K-1}$ denotes the block diagonal matrix with $\mathbf{A}_i$ as the ith block diagonal entry. $\mathbf{A} \otimes \mathbf{B}$ stands for the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$. Let $\mathcal{C}$ denote a set, then $|\mathcal{C}|$ stands for the size of this set. If x and y are random variables, $x \sim y$ denotes equivalence in distribution. An m-variate circularly symmetric zero-mean complex Gaussian random vector is a random vector $\mathbf{z} = \mathbf{x} + j\mathbf{y} \sim \mathcal{CN}_m(\mathbf{0}, \mathbf{\Sigma})$, where the real-valued random vectors $\mathbf{x}$ and $\mathbf{y}$ are jointly Gaussian, $\mathcal{E}\{\mathbf{z}\} = \mathbf{0}$, $\mathcal{E}\{\mathbf{z}\mathbf{z}^H\} = \mathbf{\Sigma}$, and $\mathcal{E}\{\mathbf{z}\mathbf{z}^T\} = \mathbf{0}$. Throughout the paper rates are specified in bit per second per hertz (bit/s/Hz).

II. Signal and Channel Models and Multiple-Access Scheme

In this section, we introduce the MA MIMO channel and signal model and the MA scheme.

A. MA MIMO Channel Model

We consider a MA MIMO channel with U users, each of which is equipped with M_T transmit antennas; the receiver employs M_R antennas. The individual users' channels are assumed frequency-selective with the ith user's matrix-valued transfer function given by

$$\mathbf{H}_i(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_{i,l} e^{-j2\pi l\theta}, \quad 0 \le \theta < 1. \tag{1}$$

We restrict ourselves to purely Rayleigh block-fading channels with the elements of $\mathbf{H}_{i,l}$ $(i = 0,1,\ldots,U-1; l = 0,1,\ldots,L-1)$ being circularly symmetric zero mean complex Gaussian random variables, constant within a block and changing in an ergodic fashion from block to block [26]. Furthermore, the matrices $\mathbf{H}_{i,l}$ are assumed to be uncorrelated across users (indexed by i) and across taps (indexed by l). We also assume spatially uncorrelated fading at the transmit arrays. Spatial fading correlation at the receive array is modeled by decomposing the taps $\mathbf{H}_{i,l}$ according to $\mathbf{H}_{i,l} = \mathbf{R}_{i,l}^{1/2}\mathbf{H}_{w,i,l}$ with $\mathbf{H}_{w,i,l}$ denoting a random matrix with i.i.d. $\mathcal{CN}_1(0,1)$ entries and $\mathbf{R}_{i,l} = \mathbf{R}_{i,l}^{1/2}\mathbf{R}_{i,l}^{1/2}$ is the receive correlation matrix for the lth tap of the ith user. We note that the power delay profiles of the individual channels are incorporated into the correlation matrices $\mathbf{R}_{i,l}$. This channel model corresponds to a non-line-of-sight propagation scenario where the individual users are located in rich scattering environments (accounted for by uncorrelated spatial fading at the transmitters). Finally, we assume that the receiver knows all the channels perfectly whereas the transmitters have no channel state information.

B. Signal Model

We assume that each of the users employs OFDM [13] with N tones and the length of the cyclic prefix (CP) satisfies $L_{cp} \ge L$. The receive signal vector for the kth tone is consequently given by

$$\mathbf{r}_k = \sum_{i=0}^{U-1} \mathbf{H}_i(e^{j2\pi \frac{k}{N}})\mathbf{c}_{i,k} + \mathbf{n}_k, \quad k = 0,1,\ldots,N-1 \tag{2}$$

where $\mathbf{c}_{i,k} = [c_{i,k}^{(0)}\; c_{i,k}^{(1)} \ldots c_{i,k}^{(M_T-1)}]^T$ with $c_{i,k}^{(l)}$ denoting the data symbol transmitted by the ith user from the lth antenna on the kth tone and $\mathbf{n}_k \sim \mathcal{CN}_{M_R}(\mathbf{0}, \mathbf{I}_{M_R})$ is white noise uncorrelated across tones (indexed by k). The power allocated to the kth tone of the ith user is denoted as $P_{i,k} = \mathrm{Tr}(\mathcal{E}\{\mathbf{c}_{i,k}\mathbf{c}_{i,k}^H\})(i = 0,1,\ldots,U-1, k = 0,1,\ldots,N-1)$ and the total transmit power of user i is given by $P_i = \sum_{k=0}^{N-1} P_{i,k}(i = 0,1,\ldots,U-1)$.

We next state an important property which will be used frequently in what follows. Under the assumptions stated in Section II-A, using (1) we can conclude that the channel matrices for user i are identically distributed for all tones $k = 0,1,\ldots,N-1$, i.e.

$$\mathbf{H}_i(e^{j2\pi\frac{k}{N}}) \sim \mathbf{H}_i, \qquad i = 0,1,\ldots,U-1, \quad k = 0,1,\ldots,N-1. \tag{3}$$

In particular, we have

$$\mathbf{H}_i = \mathbf{R}_i^{1/2}\mathbf{H}_{i,w} \tag{4}$$

where $\mathbf{R}_i = \sum_{l=0}^{L-1} \mathbf{R}_{i,l}$ and $\mathbf{H}_{i,w}$ is a random matrix with i.i.d. $\mathcal{CN}_1(0,1)$ entries. The majority of results in the paper depends

[1]Eigenvalues of Hermitian matrices $\mathbf{A}$ are sorted in descending order with $\lambda_0(\mathbf{A})$ denoting the largest eigenvalue.

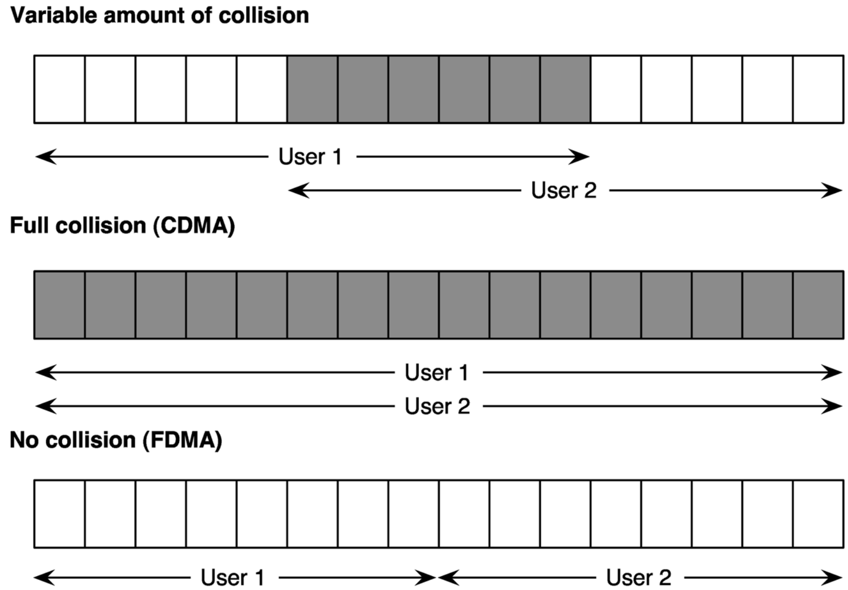

Fig. 1. Variable amount of user collision in frequency (signal space).

on property (3) which does not hold in the case of a more general channel model accounting for transmit correlation and/or Ricean fading.

We finally note that the assumption of the individual users and the receiver employing OFDM modulation and demodulation, respectively, essentially results in a periodic signal model, or more precisely the action of the channel on the transmitted signal is described by circular convolution rather than linear convolution. Our results are therefore not restricted to OFDM modulation, but hold for a system employing single-carrier modulation as well (with the notion of tone assignment becoming one of frequency band assignment). The exposition, however, is drastically simplified in the circulant case.

C. Multiple-Access (MA) Scheme

We consider a family of MA schemes obtained by assigning each OFDM tone $k = 0, 1, \ldots, N-1$ to a subset of users $\mathcal{U}_k$. Throughout the paper, we assume that $N \geq U$. A fully collision-based[2] MA scheme where all tones are assigned to each user (i.e., $\mathcal{U}_k = \{0, 1, \ldots, U-1\}$ for $k = 0, 1, \ldots, N-1$) is referred to as CDMA. FDMA is characterized by a tone assignment pattern satisfying $|\mathcal{U}_k| \leq 1$, for $k = 0, 1, \ldots, N-1$. Different tone assignment strategies are depicted in Fig. 1. As already mentioned earlier, we use the terminology CDMA solely to indicate that all users collide on all tones. Since the effect of redundancy-introducing spreading is not accounted for in our analysis, the capacity region we obtain for CDMA is an outer bound on the capacity region of CDMA systems employing spreading, such as multi-carrier CDMA [27].

III. Ergodic Capacity Regions

In this section, we derive the ergodic capacity regions for the entire family of MA schemes introduced in the previous section. For the sake of simplicity of exposition, we ignore the loss in spectral efficiency due to the presence of a CP. We start by noting that the tone assignment pattern is indirectly specified by the power allocation $P_{i,k} (i = 0, 1, \ldots, U-1, k = 0, 1, \ldots, N-1)$. The ergodic capacity region for per-user per-tone power allocation $P_{i,k}$ under the assumptions stated in Section II follows easily from results in [4], [28], [29] as the set of rates $R_i (i = 0, 1, \ldots, U-1)$ satisfying

$$\left\{(R_0, R_1, \ldots, R_{U-1}) : 0 \leq \sum_{i \in \mathcal{S}} R_i \leq \mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1} \log_2 \det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}} \frac{P_{i,k}}{M_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}, \forall \mathcal{S} \subseteq \mathcal{U}\right\} \quad (5)$$

with the $\mathbf{H}_i$ $(i = 0, 1, \ldots, U-1)$ defined in (3) and $\mathcal{U} = \{0, 1, \ldots, U-1\}$. Note that (5) shows (implicitly) that under a per-tone per-user power constraint uniform power allocation across transmit antennas on a tone-by-tone basis is optimum (due to the spatial fading at the transmitters being uncorrelated). This result reflects the assumption of spatially uncorrelated Rayleigh fading at the transmit arrays.

We shall next prove the first main result stating that the ergodic capacity region for variable amount of collision is always outer bounded by the ergodic capacity region for CDMA.

Theorem 1: Under the per-user power constraint $\sum_{k=0}^{N-1} P_{i,k} = P_i$ $(i = 0, 1, \ldots, U-1)$, the ergodic capacity region bounds in (5) are jointly maximized $\forall \mathcal{S} \subseteq \mathcal{U}$ if and only if $P_{i,k} = \frac{1}{N} P_i$ $\forall i, \forall k$. The corresponding ergodic capacity region is characterized by the set of rates satisfying

$$\left\{(R_0, R_1, \ldots, R_{U-1}) : 0 \leq \sum_{i\in\mathcal{S}} R_i \leq \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}} \frac{P_i}{NM_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}, \forall \mathcal{S} \subseteq \mathcal{U}\right\}. \quad (6)$$

Proof: See Appendix II. □

[2]Note that collision takes place in frequency.

Besides applying to MIMO channels, Theorem 1 generalizes the well known result by Gallager [14] in two additional ways: 1) it is not restricted to "block-fading" in frequency (i.e., a fading channel with bandwidth W and diversity order L is approximated by L independently fading chunks of bandwidth W/L where the tones (frequencies) within a block fade in a fully correlated fashion), and 2) it establishes the superiority of CDMA over any other (frequency) collision-based MA scheme including FDMA as a special case.

The main conclusion of Theorem 1 is as follows. In order to maximize system performance in terms of ergodic capacity, every user should split its total available transmit power uniformly between all tones and transmit antennas. In practice, however, minimizing the amount of user collision in frequency is desirable as this minimizes the receiver complexity incurred by having to separate the interfering (colliding) signals. It is therefore important to understand the (joint decoding) performance loss resulting from suboptimal (i.e., not fully collision-based) multiple accessing. When $P_i \ll 1$ for $i = 0, 1, \ldots, U-1$, we have

$$\begin{aligned}\mathcal{E}&\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\det\left(\mathbf{I}_{M_R}+\sum_{i\in\mathcal{S}}\frac{P_{i,k}}{M_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}\\ &\approx\frac{\log_2 e}{N}\sum_{k=0}^{N-1}\mathcal{E}\left\{\mathrm{Tr}\left(\sum_{i\in\mathcal{S}}\frac{P_{i,k}}{M_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}\\ &=\frac{\log_2 e}{N}\sum_{i\in\mathcal{S}}P_i\mathrm{Tr}(\mathbf{R}_i)\end{aligned} \tag{7}$$

for all $\mathcal{S} \subseteq \mathcal{U}$, which shows that in the low-SNR regime the amount of collision in signal space between the individual users has a negligible impact on the ergodic capacity region. Before proceeding in the next section with a detailed investigation of the high-SNR regime, we hasten to add that a refined low-SNR analysis using the concept of the capacity wideband slope introduced in [36] will reveal the superiority (in the low-SNR regime) of collision-based schemes over orthogonal accessing schemes. More specifically, recent results reported in [31] demonstrate, under quite general assumptions on the channel, the suboptimality of orthogonal accessing (time-division multiple access (TDMA) in the case of [31]).

IV. Multiplexing Gain Region

The aim of this section is to first introduce the notion of multiplexing gain regions as a function of the tone assignment and then quantify the impact of spatial receive fading correlation, number of transmit and receive antennas and amount of collision in frequency on these regions.

We first need the following formal definitions. The total power (over all users) is $\bar{P} = \sum_{i=0}^{U-1} P_i$; we assume $P_i = d_i\bar{P}$, where $d_i > 0$ is a constant not depending on $\bar{P}$, and $\sum_{i=0}^{U-1} d_i = 1$. Consequently, $\bar{P} \to \infty$ implies $P_i \to \infty$ $(i = 0, 1, \ldots, U-1)$ and d_i is the fraction of total power assigned to user i. Throughout the remainder of the paper, we assume that irrespective of the tone assignment (and hence the MA scheme used) all the users allocate their total available power uniformly across their assigned tones and the M_T transmit antennas. We emphasize that the assumption of uniform power allocation across the used tones is conceptual as it will not be optimum for all possible tone assignments, although it can be shown to be optimum for CDMA and FDMA.

A. Definition of the Multiplexing Gain Region

We adapt the definition of a multiplexing gain region, first proposed in a nonergodic setting in [20], to account for variable amount of collision in signal space (frequency) and for the ergodic nature of the channel considered in this paper.

Definition 1: For a given tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ (and hence MA scheme), denote the corresponding capacity region limit in (5) for $\mathcal{S} \subseteq \mathcal{U}$ as $C_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$. With the individual rates $R_i(\bar{P})$ define the multiplexing gain as

$$m_i = \lim_{\bar{P}\to\infty}\frac{R_i(\bar{P})}{\log_2(\bar{P})} \tag{8}$$

and[3]

$$\begin{aligned}&\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})\\ &\qquad= \lim_{\bar{P}\to\infty}\frac{C_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})}{\log_2(\bar{P})}.\end{aligned} \tag{9}$$

The multiplexing gain region for tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ is then characterized as the set of multiplexing gains m_i satisfying

$$\left\{(m_0, m_1, \ldots, m_{U-1}) : \sum_{i\in\mathcal{S}} m_i \leq \mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}),\ \forall\mathcal{S}\subseteq\{0, 1, \ldots, U-1\}\right\}. \tag{10}$$

For a point-to-point link the multiplexing gain equals the (ergodic) capacity pre-log in the high-SNR regime and is often used to quantify the capacity increase due to the use of multiple transmit and receive antennas. In a MA channel, the multiplexing gain region characterizes the set of simultaneously achievable multiplexing gains. In particular, $\mathcal{M}_{\mathcal{U}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$ is the sum-capacity prelog and will henceforth be called multiuser multiplexing gain. The quantities $\mathcal{M}_{\{i\}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$ for $i = 0, 1, \ldots, U-1$, are termed marginal (or single-user) multiplexing gains. From Theorem 1, we can conclude that $\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$ for $\mathcal{S} \subseteq \mathcal{U}$ is maximized for CDMA, i.e., $\mathcal{U}_k = \mathcal{U}$ for $k = 0, 1, \ldots, N-1$. The corresponding maximum value for $\mathcal{M}_{\mathcal{U}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$ will subsequently be denoted as $\mathcal{M}_{\mathrm{CDMA},\mathcal{S}}$ and serves as a reference when computing the multiplexing gain region for any other MA scheme with a variable amount of collision in frequency.

We note that a more refined analysis of the high-SNR behavior taking into account the high-SNR power offset [30], albeit desirable, seems to yield closed-form expressions for the high-SNR power offset only in the cases where all users have the same receive correlation matrices or the receive correlation

[3]Recall that the noise variance was set to 1 so that taking $\bar{P}$ in (9) to infinity is equivalent to taking the SNR to infinity.

matrices are mutually orthogonal. Finally, we note that even though the definition of multiplexing gain in [32] and [20] is for a non-ergodic setting, in the ergodic case, considered here, the notion of "high-SNR capacity prelog" [23] would be more appropriate from a purely technical perspective. It has, however, become customary in the literature to use the notion of "multiplexing gain" for the ergodic capacity high-SNR prelog as defined in (9) as well.

B. Multiplexing Gain Region for CDMA

Next, we will study the multiplexing gain region for CDMA in detail. Throughout this section, we will use the notation $C_{\mathcal{S}}$ to denote $C_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \dots, \mathcal{U}_{N-1}\})$ with the tone assignment corresponding to CDMA. For arbitrary $\mathcal{S} \subseteq \mathcal{U}$ with $P_i = d_i \bar{P}$ it follows from (6) that

$$C_{\mathcal{S}} = \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \bar{P}\sum_{i\in\mathcal{S}} \frac{d_i}{NM_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} \quad (11)$$

where the $\mathbf{H}_i$ were defined in (3). Denoting the eigenvalues of the matrix $\sum_{i\in\mathcal{S}} \frac{d_i}{NM_T}\mathbf{H}_i\mathbf{H}_i^H$ as $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{M_R-1}$, we can rewrite (11) as

$$C_{\mathcal{S}} = \mathcal{E}\left\{Q_{\mathcal{S}}\log_2(\bar{P}) + \sum_{l=0}^{Q_{\mathcal{S}}-1}\log_2\left(\frac{1}{\bar{P}} + \lambda_l\right)\right\} \quad (12)$$

with the random variable $Q_{\mathcal{S}} = r(\sum_{i\in\mathcal{S}} \frac{d_i}{NM_T}\mathbf{H}_i\mathbf{H}_i^H)$. Using (9), it follows immediately from (12) that

$$\mathcal{M}_{\text{CDMA},\mathcal{S}} = \mathcal{E}\{Q_{\mathcal{S}}\}.$$

Next, we define $\mathbf{H}_{\mathcal{S}} = [\mathbf{H}_{\mathcal{S},1}\ \mathbf{H}_{\mathcal{S},2}\ \cdots\ \mathbf{H}_{\mathcal{S},|\mathcal{S}|}]$ so that

$$\sum_{i\in\mathcal{S}} \frac{d_i}{NM_T}\mathbf{H}_i\mathbf{H}_i^H = \mathbf{H}_{\mathcal{S}}\mathbf{D}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H$$

where $\mathbf{D}_{\mathcal{S}} = \frac{1}{NM_T}\text{diag}\{d_i\}_{i\in\mathcal{S}} \otimes \mathbf{I}_{M_T}$. Since $r(\mathbf{H}_{\mathcal{S}}\mathbf{D}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H) = r(\mathbf{H}_{\mathcal{S}}\mathbf{D}_{\mathcal{S}}^{1/2})$ and $d_i > 0$ $(i = 0, 1, \dots, U-1)$ which implies $r(\mathbf{H}_{\mathcal{S}}\mathbf{D}_{\mathcal{S}}^{1/2}) = r(\mathbf{H}_{\mathcal{S}})$, we can conclude that $\mathcal{M}_{\text{CDMA},\mathcal{S}} = \mathcal{E}\{r(\mathbf{H}_{\mathcal{S}})\}$. Theorem 3 in Appendix I shows that the circularly symmetric complex Gaussian assumption on the $\mathbf{H}_i$ $(i = 0, 1, \dots, U-1)$ implies that $r(\mathbf{H}_{\mathcal{S}})$ is a constant with probability 1 (w.p.1) and hence $\mathcal{M}_{\text{CDMA},\mathcal{S}}$ is simply given by the value that $r(\mathbf{H}_{\mathcal{S}})$ takes on w.p.1.

When $|\mathcal{S}| = 1$, $\mathbf{H}_{\mathcal{S}} = \mathbf{H}_i$ for some $i \in \mathcal{U}$ and therefore[4] $r(\mathbf{H}_{\mathcal{S}}) = r(\mathbf{H}_i) = \min(r(\mathbf{R}_i), M_T)$ w.p.1. We can therefore conclude that the marginal multiplexing gains for CDMA are given by

$$\mathcal{M}_{\text{CDMA},\{i\}} = \min(r(\mathbf{R}_i), M_T), \quad i = 0, 1, \dots, U-1.$$

For $|\mathcal{S}| > 1$, $\mathcal{M}_{\text{CDMA},\mathcal{S}}$ is determined by the rank of a sum of matrices, and hence a general expression for $\mathcal{M}_{\text{CDMA},\mathcal{S}}$ in terms of $\mathbf{R}_i$, M_T, M_R and $|\mathcal{S}|$ can not be given. An exception is the case $\mathbf{R}_0 = \mathbf{R}_1 = \dots = \mathbf{R}_{U-1} = \mathbf{R}$, where it is straightforward to show that

$$\mathcal{M}_{\text{CDMA},\mathcal{S}} = \min(r(\mathbf{R}), |\mathcal{S}|M_T). \quad (13)$$

Another exception is the case $M_R \leq M_T$, with general receive correlation matrices, discussed next. We start by decomposing $\mathbf{H}_{\mathcal{S}}$ according to

$$\mathbf{H}_{\mathcal{S}} = \tilde{\mathbf{R}}_{\mathcal{S}}\tilde{\mathbf{H}}_{\mathcal{S}}$$

where $\tilde{\mathbf{R}}_{\mathcal{S}} = [\mathbf{R}_{\mathcal{S},1}^{1/2}\ \mathbf{R}_{\mathcal{S},2}^{1/2}\ \cdots\ \mathbf{R}_{\mathcal{S},|\mathcal{S}|}^{1/2}]$ with $\mathbf{H}_{\mathcal{S},q_i} = \mathbf{R}_{\mathcal{S},q_i}^{1/2}\mathbf{H}_{i,w}$ and $\tilde{\mathbf{H}}_{\mathcal{S}} = \text{diag}\{\mathbf{H}_{i,w}\}_{i\in\mathcal{S}}$. Next, we note that $\tilde{\mathbf{H}}_{\mathcal{S}}$ is of full row rank (recall that $M_R \leq M_T$) w.p.1. Hence, it follows that $r(\mathbf{H}_{\mathcal{S}}) = r(\tilde{\mathbf{R}}_{\mathcal{S}})$ w.p.1. Since $r(\tilde{\mathbf{R}}_{\mathcal{S}}) = r(\tilde{\mathbf{R}}_{\mathcal{S}}\tilde{\mathbf{R}}_{\mathcal{S}}^H) = r(\sum_{i\in\mathcal{S}}\mathbf{R}_i)$, we have

$$\mathcal{M}_{\text{CDMA},\mathcal{S}} = r\left(\sum_{i\in\mathcal{S}}\mathbf{R}_i\right) \quad (14)$$

for $M_R \leq M_T$. In the case $M_R > M_T$, a trivial lower bound on $\mathcal{M}_{\text{CDMA},\mathcal{S}}$ follows from the fact that $r(\mathbf{H}_i) = \min(r(\mathbf{R}_i), M_T)$ w.p.1 and hence

$$\mathcal{M}_{\text{CDMA},\mathcal{S}} \geq \max_{i\in\mathcal{S}}(\min(r(\mathbf{R}_i), M_T)). \quad (15)$$

Still assuming that $M_R > M_T$, an upper bound on $\mathcal{M}_{\text{CDMA},\mathcal{S}}$ is obtained by noting that $r(\mathbf{H}_{i,w}) = M_T$ w.p.1 and hence $r(\tilde{\mathbf{H}}_{\mathcal{S}}) = |\mathcal{S}|M_T$ w.p.1, which using $r(\mathbf{H}_{\mathcal{S}}) \leq \min(r(\tilde{\mathbf{R}}_{\mathcal{S}}), r(\tilde{\mathbf{H}}_{\mathcal{S}}))$ [34, p. 13] finally yields

$$\mathcal{M}_{\text{CDMA},\mathcal{S}} \leq \min\left(r\left(\sum_{i\in\mathcal{S}}\mathbf{R}_i\right), |\mathcal{S}|M_T\right). \quad (16)$$

For the case $M_R > M_T$, the exact expressions (13) and (14) and the upper bound (16) show that the multiuser multiplexing gain obtained by setting $\mathcal{S} = \mathcal{U}$ can be significantly higher than any of the marginal (or single-user) multiplexing gains $\mathcal{M}_{\text{CDMA},\{i\}}$ $(i = 0, 1, \dots, U-1)$, i.e., the capacity growth resulting from MIMO technology can be significantly higher in the sum capacity than in the marginal capacities. This is due to the fact that for $\mathcal{S} = \mathcal{U}$ all the users contribute to the multiuser multiplexing gain. More specifically, the presence of multiple users increases the effective number of transmit antennas from M_T to $M_T U$. Assuming that U is sufficiently large, (14) and (16) show that $r(\sum_{i=0}^{U-1}\mathbf{R}_i)$ limits the multiuser multiplexing gain. Intuitively, this means that the multiuser multiplexing gain is limited by the dimensionality of the receive signal space induced by the entire collection of users rather than the dimensionality of the receive signal space induced by a single user and given by $r(\mathbf{R}_i)$.

In practice, in order to obtain a high-rank sum-correlation matrix $\sum_{i=0}^{U-1}\mathbf{R}_i$, we either need the receive antenna spacing to be large so that the individual correlation matrices $\mathbf{R}_i$ are high-rank or alternatively the $\mathbf{R}_i$ have to span different subspaces. In practice, the latter requirement tends to be satisfied if the individual users are well separated in space, which is typically the case in a cellular system. We finally note that for large M_R and rich scattering/large user separation, $\mathcal{M}_{\text{CDMA},\mathcal{U}}$ can be U times higher than any of the single-user multiplexing gains.

[4]This standard result can be easily shown using, for example, the technique presented in [33, proof of Theorem 3.1.4].

C. Multiplexing Gain Region for General Tone Assignments

Next, we characterize the impact of user collision in frequency (or lack thereof) on the multiplexing gain region. We start by introducing some notation. For $\mathcal{D} \subseteq \mathcal{U}$, $\varrho(\mathcal{D})$ denotes the rank that $\sum_{i\in\mathcal{D}} \mathbf{H}_i\mathbf{H}_i^H$ assumes w.p.1. Theorem 3 in Appendix I shows the existence of such a $\varrho(\mathcal{D})\ \forall D \subseteq \mathcal{U}$. As before, we define the total user power as $\bar{P} = \sum_{i=0}^{U-1} P_i$ with $P_i = d_i\bar{P}$ and $d_i > 0$ for $i = 0, 1, \ldots, U-1$. We furthermore set $P_{i,k} = b_{i,k}P_i$ $(i = 0, 1, \ldots, U-1, k = 0, 1, \ldots, N-1)$, where $\sum_{k=0}^{N-1} b_{i,k} = 1$ for $i = 0, 1, \ldots, U-1$. Since we assumed that the total available power P_i is split uniformly between the N_i tones assigned to user i, we have

$$b_{i,k} = \begin{cases} \frac{1}{N_i}, & i \in \mathcal{U}_k \\ 0, & i \notin \mathcal{U}_k. \end{cases}$$

With these definitions the capacity region limit in (5) corresponding to $\mathcal{S} \subseteq \mathcal{U}$ and a general tone assignment $\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ can be written as

$$C_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) = \frac{1}{N}\sum_{k=0}^{N-1} \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{\bar{P}}{M_T}\sum_{i\in\mathcal{S}} d_i b_{i,k}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}.$$

Denoting the lth eigenvalue of the random matrix $\frac{1}{M_T}\sum_{i\in\mathcal{S}} d_i b_{i,k}\mathbf{H}_i\mathbf{H}_i^H$ as $\lambda_{l,k}$, it follows that

$$\begin{aligned} C_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) &= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{\varrho(\mathcal{U}_k\cap\mathcal{S})-1} \mathcal{E}\left\{\log_2\left(1+\bar{P}\lambda_{l,k}\right)\right\} \\ &= \frac{\log_2(\bar{P})}{N}\sum_{k=0}^{N-1}\varrho(\mathcal{U}_k\cap\mathcal{S}) \\ &\quad + \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{\varrho(\mathcal{U}_k\cap\mathcal{S})-1}\mathcal{E}\left\{\log_2\left(\frac{1}{\bar{P}}+\lambda_{l,k}\right)\right\}. \end{aligned} \tag{17}$$

Applying (9) to (17), we obtain

$$\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) = \frac{1}{N}\sum_{k=0}^{N-1}\varrho(\mathcal{U}_k\cap\mathcal{S}). \tag{18}$$

Since $\varrho(\mathcal{U}_k\cap\mathcal{S}) \leq \varrho(\mathcal{S})$, it follows immediately that $\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) \leq \varrho(\mathcal{S}) = \mathcal{M}_{\mathcal{S},\mathrm{CDMA}}$ showing that the multiplexing gain region for any tone assignment (and hence any amount of signal space collision) is outer-bounded by the multiplexing gain region obtained for CDMA (full collision). Moreover, for the individual multiplexing gains, we have

$$\begin{aligned} \mathcal{M}_{\{i\}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) &= \min(r(\mathbf{R}_i), M_T)\frac{N_i}{N} \\ &\leq \min(r(\mathbf{R}_i), M_T) = M_{\{i\},\mathrm{CDMA}} \end{aligned} \tag{19}$$

showing that suboptimal (i.e., not fully collision-based) multiple accessing results in strictly smaller individual multiplexing gains than in the CDMA-case. However, (18) also shows that one does not have to enforce full collision in frequency to achieve $\mathcal{M}_{\mathcal{S},\mathrm{CDMA}}$ for $|\mathcal{S}| > 1$; it suffices to choose tone assignments that result in $\varrho(\mathcal{U}_k\cap\mathcal{S}) = \varrho(\mathcal{S})$ for $k = 0, 1, \ldots, N-1$. Intuitively, this ensures that the spatial degrees of freedom offered by the MA channel are indeed exploited.[5] This suggests a tone assignment strategy for maximization of the multiplexing gain region. Suppose that user i employs a total of N_i tones. Since the statistics of $\mathbf{H}_i(e^{j2\pi\frac{k}{N}})$ are independent of k (c.f. (3)), the marginal multiplexing gains are given by $\min(r(\mathbf{R}_i), M_T)\frac{N_i}{N}$ independently of the tone assignment. The quantities $\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\})$ for $|\mathcal{S}| > 1$ do, however, critically depend on the tone assignment. Therefore, if a certain tone k has already been allocated to users indexed by $\mathcal{U}_k$, it is worthwhile (in terms of multiplexing gain) to assign this tone to user i only if $\varrho((\mathcal{U}_k\cap\mathcal{S})\cup\{i\}) > \varrho(\mathcal{U}_k\cap\mathcal{S})$. For example, when $r(\mathbf{R}_i) \leq M_T$ for $i = 0, 1, \ldots, U-1$, only users with different $\mathrm{span}\{\mathbf{R}_i\}$ should collide. In general, the extent to which a tone assignment realizing $\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) = \mathcal{M}_{\mathrm{CDMA},\mathcal{S}}$ is possible depends on the number of users, the channel's spatial fading statistics and the number of transmit and receive antennas. In the case of FDMA, we can be more specific and state the following.

Theorem 2: Assume that $|\mathcal{U}_k| = 1$ for $k = 0, 1, \ldots, N-1$ (i.e., FDMA where every tone gets assigned to exactly one user) and $N_i \geq 1$ for $i = 0, 1, \ldots, U-1$, then the following results hold.

1) The corresponding multiplexing gain region is strictly outer-bounded by the multiplexing gain region achieved by CDMA, i.e.,

$$\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) < \mathcal{M}_{\mathcal{S},\mathrm{CDMA}} \quad \forall\mathcal{S}\subset\mathcal{U}$$

except for the multiuser multiplexing gain that satisfies

$$\mathcal{M}_{\mathcal{U}}(\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}) \leq \mathcal{M}_{\mathcal{U},\mathrm{CDMA}}. \tag{20}$$

Equality in (20) is achieved if and only if $\mathrm{span}\{\mathbf{R}_0\} = \mathrm{span}\{\mathbf{R}_1\} = \cdots = \mathrm{span}\{\mathbf{R}_{U-1}\}$ and $r(\mathbf{R}_0) \leq M_T$.

2) If $N_i = \frac{N}{U} \in \mathbb{N}$ for $i = 0, 1, \ldots, U-1$ (every user gets the same number of tones), we have the following inner bound on the multiplexing gain region:

$$\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0,\mathcal{U}_1,\ldots,\mathcal{U}_{N-1}\}) \geq \frac{1}{U}\mathcal{M}_{\mathcal{S},\mathrm{CDMA}} \quad \forall\mathcal{S}\subseteq\mathcal{U}. \tag{21}$$

Equality in (21) is obtained $\forall\mathcal{S}\subseteq\mathcal{U}$ if $r(\mathbf{R}_i) \geq UM_T\ \forall i$, or $\mathbf{R}_i\mathbf{R}_j = \mathbf{0}\ \forall i \neq j$.

Proof: See Appendix III. □

Part 1) of Theorem 2 states that FDMA, irrespectively of the specific tone assignment strategy, always yields a strictly smaller multiplexing gain region than CDMA. The two regions

[5]Recall that in point-to-point MIMO links, in order to realize spatial multiplexing gain it is crucial that the signals transmitted from the individual antennas are co-channel (or equivalently collide in signal space).

can only meet in the multiuser multiplexing gain point. The necessary and sufficient condition for this to happen stated in Theorem 2 essentially amounts to the channel not providing any spatial separation between the users. Part 2) of Theorem 2 shows that when the dimensionality of the receive signal space is large enough, or the channel provides full spatial separation, collision is needed to excite all the spatial dimensions. Based on these observations, we conclude this section by noting that in practice, for good spatial separation between the users and for large M_R, collision in frequency (signal space) is critical to achieve a high multiuser multiplexing gain. On the other hand, for poor spatial separation and/or small M_R little or no collision is needed to achieve $\mathcal{M}_{\mathcal{U},\mathrm{CDMA}}$.

V. The Two-User Case

The purpose of this section is to analyze the ergodic capacity region and the multiplexing gain region in detail in the two-user case. In particular, we shall quantify the impact of the amount of collision in frequency and spatial receive fading correlation. For the sake of simplicity of exposition, throughout the section, we restrict ourselves to a simplified scenario where collision in frequency can be described by one collision parameter.

We assume that all the available tones are used, the two users employ the same number of tones $N_u \geq \frac{N}{2} \in \mathbb{N}$ out of which bN tones with[6] $b = \frac{2i}{N}$, $i \in \{0, 1, \ldots, N/2\}$, are assigned to both users. Consequently, $b = 0$ corresponds to FDMA (no collision in frequency) and $b = 1$ yields a fully collision-based MA scheme (CDMA). The total power is the same for both users, i.e., $P_0 = P_1$. As before, it is also assumed that each user allocates the total power uniformly over the tones it has been assigned.

A. Multiplexing Gain Region

Under the assumptions stated above, bN tones are assigned to both users and in addition each user obtains $\frac{1-b}{2}N$ tones individually. In the following, for the sake of simplicity of notation, we shall simply write $\mathcal{M}_{\mathcal{S}}$ instead of $\mathcal{M}_{\mathcal{S}}(\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\})$. Noting that the total number of tones assigned to each of the 2 users is $\frac{1+b}{2}N$ and using (18) the multiplexing gain region is characterized by the following set of inequalities:

$$
\begin{aligned}
m_0 &\leq \frac{1+b}{2} Q_0 \\
m_1 &\leq \frac{1+b}{2} Q_1 \\
m_0 + m_1 &\leq b\left(Q - \frac{Q_0 + Q_1}{2}\right) + \frac{1}{2}(Q_0 + Q_1)
\end{aligned} \tag{22}
$$

where $r([\mathbf{H}_0\ \mathbf{H}_1]) = Q$, $r(\mathbf{H}_0) = Q_0$ and $r(\mathbf{H}_1) = Q_1$, all w.p.1. Recall that $Q_i = \min(r(\mathbf{R}_i), M_T)$ for $i = 0, 1$. We can now immediately conclude that in the case of orthogonal accessing where $b = 0$ (i.e., no collision), the individual multiplexing gain limits are reduced by a factor of 2 compared to the CDMA case (where $b = 1$). This result is intuitively clear since for $\bar{P} \to \infty$, we operate in the degree-of-freedom limited regime and FDMA results in each of the two users getting assigned only half of the total available degrees of freedom (bandwidth in our case).

The behavior of the sum multiplexing gain is somewhat different. The third inequality in (22) shows that the sum multiplexing gain limit depends on b through the quantity $Q - \frac{1}{2}(Q_0 + Q_1)$. Since $Q_i \leq Q$, the sum multiplexing gain limit is independent of b (and minimum for given b) if and only if $Q_0 = Q_1 = Q$. Theorem 4 in Appendix I shows that this happens if and only if $\mathrm{span}\{\mathbf{R}_0\} = \mathrm{span}\{\mathbf{R}_1\}$ and $r(\mathbf{R}_0) \leq M_T$, corresponding to the case of no spatial separation between the users. On the other hand, the multiuser multiplexing gain is maximum (for given b) in at least the following two cases (see Appendix I, Theorem 4):

1) $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$;
2) $r(\mathbf{R}_i) \geq 2M_T$ for $i = 0, 1$.

The first case corresponds to full spatial separation (induced by the channel) since the users' spatial signatures span orthogonal subspaces. Moreover, we note that under 1) the multiplexing gain region is rectangular. The second condition essentially guarantees that the receiver has enough spatial degrees of freedom to perfectly separate the two users (spatially). We can conclude that the multiuser multiplexing gain depends critically on the spatial separation of the users. If the spatial separation is poor (i.e., the subspaces spanned by $\mathbf{R}_0$ and $\mathbf{R}_1$ tend to be aligned), we achieve full multiuser multiplexing gain with any amount of collision. On the other hand, if the spatial separation of the users is good (i.e., the subspaces spanned by $\mathbf{R}_0$ and $\mathbf{R}_1$ tend to be orthogonal), the multiuser multiplexing gain depends critically on the amount of collision. In summary, we can conclude that in the high-SNR regime, collision is required either in the spatial dimension (i.e., the channel induced signatures collide) or in frequency in order to realize a high multiuser multiplexing gain. Fig. 2 depicts the multiplexing gain regions for a system with $M_T = M_R = 2$ and receive correlation matrices $\mathbf{R}_0$ and $\mathbf{R}_1$ satisfying $r(\mathbf{R}_0) = r(\mathbf{R}_1) = 1$.

B. Capacity Region

An analysis of the multiplexing gain region reveals the fundamental performance limits in terms of only the prelog in the capacity expression. We shall next provide a refined analysis for the extreme cases of no and full spatial separation by computing the high-SNR capacity region as a function of b. Under the assumptions stated in the previous section, the capacity region is characterized by the following set of inequalities:

$$
\begin{aligned}
R_i &\leq \frac{1+b}{2}\, \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\},\ i = 0, 1 \\
R_0 + R_1 &\\
&\leq b\, \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\left(\mathbf{H}_0\mathbf{H}_0^H + \mathbf{H}_1\mathbf{H}_1^H\right)\right)\right\} \\
&\quad + \frac{(1-b)}{2}\, \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_0\mathbf{H}_0^H\right)\right\} \\
&\quad + \frac{(1-b)}{2}\, \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{2\rho}{b+1}\mathbf{H}_1\mathbf{H}_1^H\right)\right\}
\end{aligned} \tag{23}
$$

where $\mathbf{H}_0$ and $\mathbf{H}_1$ have been defined in (3) and $\rho = \frac{P}{NM_T}$.

[6] The assumptions imply that the number of tones assigned to both users is even.

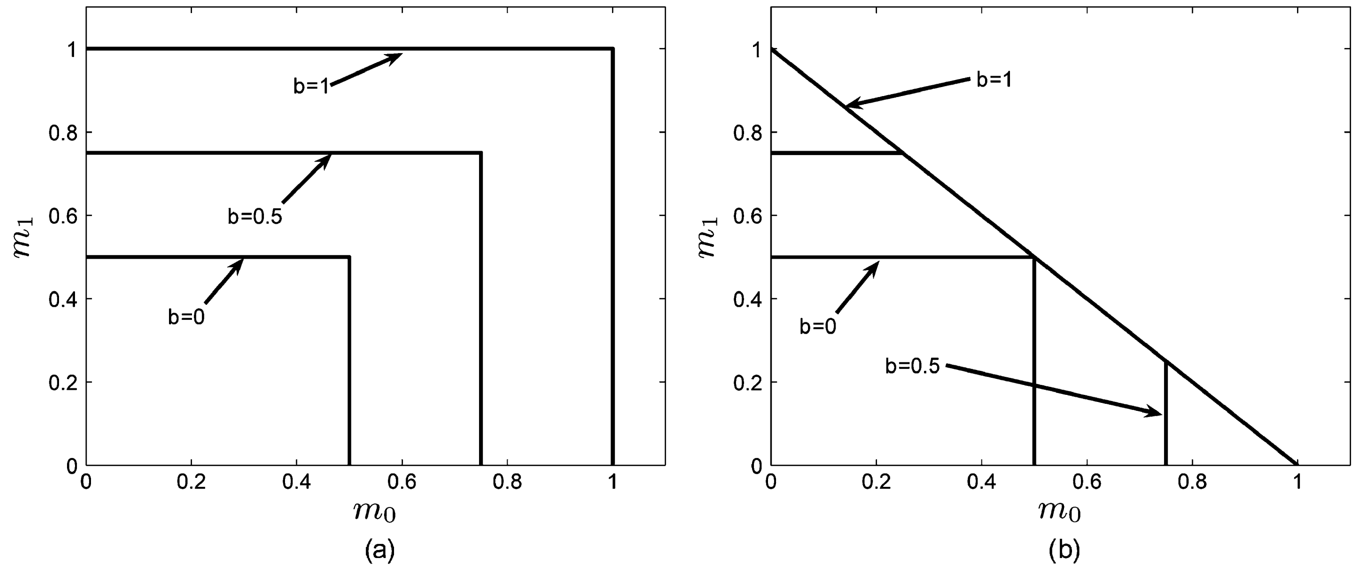

Fig. 2. Multiplexing gain regions for $M_T = M_R = 2$. (a) $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$ (perfect spatial separation). (b) $\mathbf{R}_0 = \mathbf{R}_1$ (no spatial separation).

Assume next that $M_R \geq 2$ and $r(\mathbf{R}_0) = r(\mathbf{R}_1) = 1$ (i.e., fully correlated fading at the receive array) with $\mathrm{Tr}(\mathbf{R}_0) = \mathrm{Tr}(\mathbf{R}_1) = 1$. Let us start with the case of no spatial separation where $\mathbf{R}_0 = \mathbf{R}_1$. Applying the high-SNR approximation for ergodic MIMO capacity reported in [35] to each of the individual terms on the right-hand side (RHS) of (23) we obtain (24) shown at the bottom of the page where $\gamma \approx 0.5722$ denotes Euler's constant. Since $\sum_{p=M_T}^{2M_T-1} \frac{1}{p}$ is strictly decreasing as a function of M_T, we can conclude that the impact of b on sum capacity reduces for increasing M_T. In fact, we have $\lim_{M_T \to \infty}[C_S(1) - C_S(0)] = 0$ which implies that asymptotically (in the number of transmit antennas) in the high-SNR regime a fully collision-based scheme achieves the same sum capacity as orthogonal accessing (i.e., FDMA). Since $\sum_{p=1}^{M_T-1} \frac{1}{p}$ is strictly increasing in M_T, it follows that the impact of the collision parameter on the marginal rates becomes more dominant for increasing M_T. In the case of full spatial separation, i.e., when $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$, again applying results from [35], we have

$$R_i \leq \frac{1+b}{2}\left(\log_2\left(\frac{2\rho}{b+1}\right) + \frac{1}{\ln 2}\left(\sum_{p=1}^{M_T-1}\frac{1}{p} - \gamma\right)\right), \quad i = 0,1$$

$$R_0 + R_1 \leq (1+b)\log_2\left(\frac{2\rho}{1+b}\right) + \frac{1+b}{\ln 2}\left(\sum_{p=1}^{M_T-1}\frac{1}{p} - \gamma\right). \quad (25)$$

In contrast to the case of no spatial separation, the sum-rate depends on the collision parameter through the prelog $(1+b)$ in the first term as well. Hence, in the high-SNR regime, collision is mandatory to achieve a high sum-rate. The marginal rates in (25) are identical to the marginal rates in the case of no spatial separation. Consequently, the impact of b on the marginal capacities is the same as in the case of no spatial separation. Finally, we note that the multiplexing gain region is readily obtained from the pre-log terms in the capacity region expression.

C. Numerical Results

We shall next provide numerical results describing the impact of collision on the two-user capacity region. We simulated a system with $M_T = M_R = 2$ and receive correlation matrices $\mathbf{R}_0$ and $\mathbf{R}_1$ satisfying $r(\mathbf{R}_0) = r(\mathbf{R}_1) = 1$ and normalized such that $\mathrm{Tr}(\mathbf{R}_0) = \mathrm{Tr}(\mathbf{R}_1) = 1$. Again, two different scenarios were considered, namely $\mathbf{R}_0 = \mathbf{R}_1$ (no spatial separation) and $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$ (full spatial separation). Fig. 3 shows the capacity regions (obtained through 10 000 Monte Carlo runs) for two different SNR values ρ. In accordance with (25), the simulation results demonstrate that for high-spatial separation, the difference between the capacity regions for full collision (CDMA) and for orthogonal multiple accessing (FDMA) becomes more pronounced for increasing SNR. Moreover, we can see that for high spatial separation the capacity region is rectangular. In the case of poor spatial separation between the users, FDMA achieves a significant fraction of the maximum available sum capacity (as suggested by (24) and the ensuing discussion).

$$R_i \leq \frac{1+b}{2}\left(\log_2\left(\frac{2\rho}{b+1}\right) + \frac{1}{\ln 2}\left(\sum_{p=1}^{M_T-1}\frac{1}{p} - \gamma\right)\right), \quad i = 0,1$$

$$R_0 + R_1 \leq \log_2\left(\frac{2\rho}{1+b}\right) + \frac{1}{\ln 2}\left(\sum_{p=1}^{M_T-1}\frac{1}{p} - \gamma\right) + \frac{b}{\ln 2}\sum_{p=M_T}^{2M_T-1}\frac{1}{p} = C_S(b) \quad (24)$$

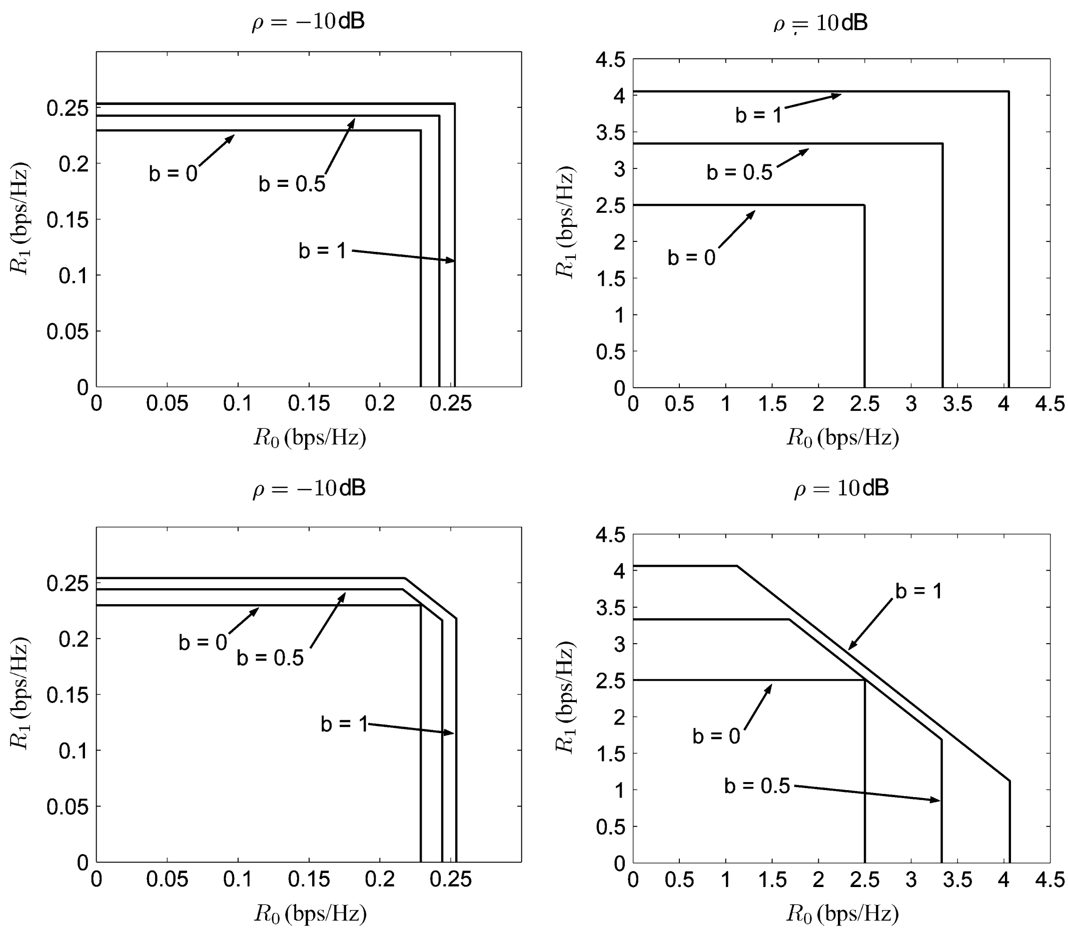

Fig. 3. Ergodic capacity regions for $M_T = M_R = 2$. First row: $\mathbf{R}_0\mathbf{R}_1 = \mathbf{0}$ (perfect spatial separation). Second row: $\mathbf{R}_0 = \mathbf{R}_1$ (no spatial separation). First column: $\rho = -10$ dB. Second column: $\rho = 10$ dB.

We can furthermore observe that in accordance with (7) the collision parameter b has little impact on the low-SNR capacity region. Finally, we note that as SNR increases, the shape of the high-SNR capacity region approaches the shape of the multiplexing gain region shown in Fig. 2.

VI. Asymptotic Analysis

The results in the previous sections can be further quantified through an asymptotic (in the number of users) comparison of FDMA and CDMA inspired by the approach in [15] used to analyze the sum capacity gap between FDMA and CDMA in single-antenna MA fading channels. In the following, for the sake of simplicity, we assume $\mathbf{R}_0 = \mathbf{R}_1 = \ldots = \mathbf{R}_{U-1} = \mathbf{I}_{M_R}$ and equal user powers, i.e., $P_0 = P_1 = \cdots = P_{U-1} = P$. Furthermore, we set $N = KU$ with the constant $K \in \mathbb{N}$ independent of the number of users U. With these assumptions, the sum capacity for CDMA (full collision) is given by

$$C_{\mathcal{U},\mathrm{CDMA}} = \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{P}{NM_T}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\}. \tag{26}$$

Since $N = KU$ and $\frac{1}{U}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H \xrightarrow{\text{w.p.1}} M_T\mathbf{I}_{M_R}$ as[7] $U \to \infty$, we have

$$\mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{P}{NM_T}\sum_{i=0}^{U-1}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} \longrightarrow M_R \log_2\left(1 + \frac{P}{K}\right)$$

as $U \to \infty$. Therefore, in the large number of users limit for P large, we have

$$C_{\mathcal{U},\mathrm{CDMA}} \approx M_R \log_2(P/K)$$

and consequently $\mathcal{M}_{\mathcal{U},\mathrm{CDMA}} = M_R$. For FDMA, assuming that each user employs K tones, the sum capacity is independent of the number of users and given by

$$C_{\mathcal{U},\mathrm{FDMA}} = \mathcal{E}\left\{\log_2 \det\left(\mathbf{I}_{M_R} + \frac{P}{KM_T}\mathbf{H}_0\mathbf{H}_0^H\right)\right\}.$$

[7]We note that $U \to \infty$ implies $N \to \infty$. As in [15], the underlying assumption is, however, that the total bandwidth remains constant.

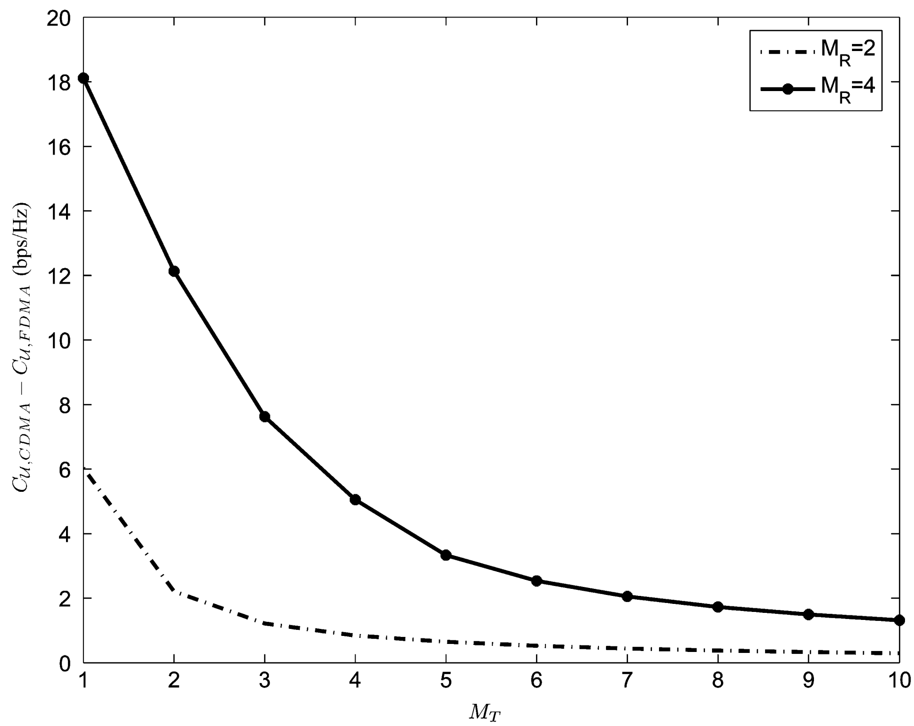

Fig. 4. Asymptotic (in the number of users) sum capacity difference between CDMA and FDMA for different values of M_R.

Again, considering the large P regime, we have [35]

$$C_{\mathcal{U},\text{FDMA}} \approx \min(M_R, M_T)\log_2\left(\frac{P}{KM_T}\right) + \frac{1}{\ln 2}\left(\sum_{j=1}^{\min(M_R,M_T)}\sum_{p=1}^{\max(M_T,M_R)-j}\frac{1}{p} - \gamma\min(M_R,M_T)\right).$$

The corresponding multiuser multiplexing gain is given by $\mathcal{M}_{\mathcal{U},\text{FDMA}} = \min(M_R, M_T)$.

Combining our results, it follows that in the large P regime asymptotically in the number of users, the sum capacity difference between CDMA and FDMA is given by

$$\begin{aligned} C_{\mathcal{U},\text{CDMA}} &- C_{\mathcal{U},\text{FDMA}} \\ &\approx (M_R - \min(M_R, M_T))\log_2(P/K) \\ &\quad + \min(M_R, M_T)\log_2(M_T) \\ &\quad - \frac{1}{\ln 2}\left(\sum_{j=1}^{\min(M_R,M_T)}\sum_{p=1}^{\max(M_T,M_R)-j}\frac{1}{p} - \gamma\min(M_R,M_T)\right). \end{aligned} \tag{27}$$

The performance difference between CDMA and FDMA in terms of multiuser multiplexing gain is

$$\mathcal{M}_{\mathcal{U},\text{CDMA}} - \mathcal{M}_{\mathcal{U},\text{FDMA}} = M_R - \min(M_R, M_T). \tag{28}$$

In the SISO case $(M_T = M_R = 1)$ (27) specializes to $C_{\mathcal{U},\text{CDMA}} - C_{\mathcal{U},\text{FDMA}} \approx \gamma/\ln 2$, which was found previously in [15]. Fig. 4 shows $C_{\mathcal{U},\text{CDMA}} - C_{\mathcal{U},\text{FDMA}}$ for $M_R = 2, 4$ and $P/K = 20$ dB as a function of M_T. In the regime $M_T < M_R$ we observe that the asymptotic sum capacity performance benefits significantly from collision in frequency. As M_T increases the performance gap between CDMA and FDMA closes.

Extending the analysis above to the case of general receive correlation matrices seems difficult. In the case $\mathbf{R}_0 = \mathbf{R}_1 = \ldots = \mathbf{R}_{U-1} = \mathbf{R}$, one can, however, show that for $UM_T > r(\mathbf{R})$, we have

$$\mathcal{M}_{\mathcal{U},\text{CDMA}} - \mathcal{M}_{\mathcal{U},\text{FDMA}} = r(\mathbf{R}) - \min\left(r(\mathbf{R}), M_T\right). \tag{29}$$

For $M_T \geq r(\mathbf{R})$, we can therefore conclude that FDMA achieves the same multiuser multiplexing gain as CDMA. This is due to the fact that for $M_T \geq r(\mathbf{R})$ the multiuser multiplexing gain is "bottle-necked" by $r(\mathbf{R})$ and collision of the transmit signals across the M_T antennas of an individual user is sufficient to achieve full multiuser multiplexing gain. For $\mathbf{R} = \mathbf{I}_{M_R}$ and fixed $M_T < M_R$, the performance gap between CDMA and FDMA, as quantified in (29), increases with M_R, which can be attributed to the fact that increasing M_R opens up more spatial dimensions and hence collision in frequency becomes mandatory to "excite these dimensions" and achieve full multiuser multiplexing gain.

VII. Conclusion

We introduced a family of MIMO MA schemes which allows to gradually vary the amount of user collision in frequency (signal space) by assigning different portions of the available frequency band to different users. The performance of the proposed class of MA schemes, ranging from FDMA to CDMA, was assessed by computing the corresponding ergodic capacity and multiplexing gain regions. Conditions for FDMA to achieve full multiuser multiplexing gain (sum capacity pre-log) were provided. We further quantified the performance gap between CDMA and FDMA through asymptotic (in the number of users) expressions for the corresponding high-SNR sum capacities and multiuser multiplexing gains. Besides introducing an entire family of MA schemes and a framework for studying the impact of user collision in signal space (frequency, in our case), our main findings are summarized as follows: Generalizing a well known result by Gallager [14], we showed that the ergodic capacity region for any amount of user collision in signal space is always outer bounded by the ergodic capacity region for a fully collision-based (CDMA) MA scheme. This result was shown to hold irrespective of the number of antennas (at the transmitters and the receiver) and the spatial receive correlation matrices. Moreover, it was found that in the low-SNR regime the amount of collision has a negligible impact on the capacity region. For high SNR the spatial separation between the users and the number of receive antennas govern the shape of the ergodic capacity region. When the users are spatially well separated (as measured by the spatial signatures induced by the different users) and for large M_R, collision in frequency is crucial to maximize the ergodic capacity region. On the other hand, for poor spatial separation and/or small M_R the impact of collision on the ergodic capacity region is small. Hence, when sum capacity (or equivalently multiuser multiplexing gain) is the limiting factor, collision is not necessarily needed in the latter case. Minimizing the amount of user collision in signal space is desirable in practice, as it minimizes the receiver complexity incurred by having to separate the interfering (colliding) signals. We finally note that even though from a sum capacity (or multiuser multiplexing gain) point-of-view the number of receive (base station) antennas is typically the limiting factor, there is still strong motivation for using multiantenna transmitters (terminals) since this will result in higher individual data rates.

We conclude by pointing out further avenues of research in the context of MA with variable amount of collision. One of the main findings in this paper shows that fully collision-based MA (i.e., CDMA) is optimum in the sense of maximizing the ergodic capacity region. The picture can be expected to change if (out-of-cell) interference is taken into account. In fact, it was shown previously in [16] and [17] that the presence of interference renders MA schemes implementing full collision in signal space (time, in this case) suboptimum. Another interesting topic for future research is a refined low-SNR analysis using the concept of the capacity wideband slope introduced in [36].

APPENDIX I
RESULTS ON THE RANK OF (A SUM OF) GAUSSIAN RANDOM MATRICES

This Appendix provides some results on the rank of (a sum of) Gaussian random matrices that will be used frequently in the paper.

Lemma 1: Let $\mathbf{x} \sim \mathcal{CN}_M(\mathbf{0}, \boldsymbol{\Sigma})$ and let $S^{(p)}$ be a p-dimensional subspace of $\mathbb{C}^M$. Then

1) $\Pr\{\mathbf{x} \in S^{(p)}\}$ is either 0 or 1;
2) If $p < r(\boldsymbol{\Sigma})$, then $\Pr\{\mathbf{x} \in S^{(p)}\} = 0$.

Proof: The result 1) is obvious for $p = M$ since in this case $\Pr\{\mathbf{x} \in S^{(p)}\} = 1$. In order to prove 1) for $p < M$, denote the projection matrix onto the orthogonal complement of $S^{(p)}$ as $\boldsymbol{\Gamma}$. Then $\mathbf{x} \in S^{(p)}$ if and only if $\boldsymbol{\Gamma}\mathbf{x} = \mathbf{0}$. From $\boldsymbol{\Gamma}\mathbf{x} \sim \mathcal{CN}_M(\mathbf{0}, \boldsymbol{\Gamma}\boldsymbol{\Sigma}\boldsymbol{\Gamma}^H)$, it follows that

$$\Pr\left\{\mathbf{x} \in S^{(p)}\right\} = \Pr\left\{\boldsymbol{\Gamma}\mathbf{x} = \mathbf{0}\right\} = \begin{cases} 1, & \boldsymbol{\Gamma}\boldsymbol{\Sigma}^{1/2} = \mathbf{0} \\ 0, & \boldsymbol{\Gamma}\boldsymbol{\Sigma}^{1/2} \neq \mathbf{0} \end{cases}$$

which concludes the proof of statement 1). The proof of 2) follows by noting that $r(\boldsymbol{\Gamma}) = M - p$ which combined with $r(\boldsymbol{\Sigma}) > p$ necessarily yields $\boldsymbol{\Gamma}\boldsymbol{\Sigma}^{1/2} \neq \mathbf{0}$ and by 1) allows to conclude that $\Pr\{\mathbf{x} \in S^{(p)}\} = 0$. □

Lemma 2: Let $\mathbf{x}_i \sim \mathcal{CN}_M(\mathbf{0}, \boldsymbol{\Sigma}_i)$, $i = 1, 2, \ldots, K$, and assume that the $\mathbf{x}_i$ are independently drawn. Then the probability that $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K$ are linearly dependent is either 0 or 1.

Proof: Define the event that the $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K$ are linearly dependent as A. If $K > M$, we obviously have $\Pr\{A\} = 1$. Assume now that $K \leq M$ and $\Pr\{A\} > 0$. Then, by Theorem 4.2 in [37], for some p with $1 \leq p \leq K$ the following assertion holds: there exist p vectors $\{\mathbf{x}_{i_\alpha} : \alpha = 1, 2, \ldots, p\}$ and there exists a $(p-1)$-dimensional subspace $S^{(p-1)}$ of $\mathbb{C}^M$ such that $\Pr\{\mathbf{x}_{i_\alpha} \in S^{(p-1)}\} > 0$ for $\alpha = 1, 2, \ldots, p$. By Lemma 1 this immediately implies that $\Pr\{\mathbf{x}_{i_\alpha} \in S^{(p-1)}\} = 1$ for $\alpha = 1, 2, \ldots, p$. We can therefore conclude that $\Pr\{A\} > 0$ implies $\Pr\{A\} = 1$. □

Theorem 3: Let $\mathcal{D} \subseteq \mathcal{U} = \{0, 1, \ldots, U-1\}$, $\mathcal{D} \neq \emptyset$, with elements $i_1, i_2, \ldots, i_{|\mathcal{D}|}$ and denote $\mathbf{H}_{\mathcal{D}} = [\mathbf{H}_{i_1}\ \mathbf{H}_{i_2}\ \cdots\ \mathbf{H}_{i_{|\mathcal{D}|}}]$. Then, for any $\mathcal{D}$, $\exists\ \varrho(\mathcal{D}) \in \mathbb{N}$ such that $r(\mathbf{H}_{\mathcal{D}}) = \varrho(\mathcal{D})$ w.p.1.

Proof: We start by noting that the columns of the $M_R \times M_T|\mathcal{D}|$ matrix $\mathbf{H}_{\mathcal{D}}$ constitute a set of independent but not necessarily identically distributed, circularly symmetric complex Gaussian random vectors. Obviously we have $\Pr\{1 \leq r(\mathbf{H}_{\mathcal{D}}) \leq \min(M_R, M_T|\mathcal{D}|)\} = 1$ so that the random variable $r(\mathbf{H}_{\mathcal{D}})$ is distributed on the set $\{1, 2, \ldots, \min(M_R, M_T|\mathcal{D}|)\}$. Next, we note that

$$\Pr\{r(\mathbf{H}_{\mathcal{D}}) \leq \alpha\} = \max_{\{i_1, i_2, \ldots, i_{\alpha+1}\} \in \mathcal{D}} \left(\Pr\left\{\mathbf{h}_{i_1}, \mathbf{h}_{i_2}, \ldots, \mathbf{h}_{i_{\alpha+1}} \text{are linearly dependent}\right\}\right). \quad (30)$$

Using Lemma 2 it follows that the RHS of (30) can only be equal to 0 or 1. Now, starting with $\Pr\{r(\mathbf{H}_{\mathcal{D}}) \leq \min(M_R, M_T|\mathcal{D}|)\} = 1$ and noting that $\Pr\{r(\mathbf{H}_{\mathcal{D}}) \leq \min(M_R, M_T|\mathcal{D}|) - i\}$ is either 0 or 1 for $i = 1, 2, \ldots, \min(M_R, M_T|\mathcal{D}|) - 1$ we find the smallest value of i that yields

$$\Pr\{r(\mathbf{H}_{\mathcal{D}}) \leq \min(M_R, M_T|\mathcal{D}|) - i\} = 0.$$

Denoting this value of i as $i_{\min}$, we can immediately conclude that the entire probability mass of the random variable $r(\mathbf{H}_{\mathcal{D}})$ is concentrated at $\varrho(\mathcal{D}) = \min(M_R, M_T|\mathcal{D}|) - i_{\min} + 1$ which shows that $r(\mathbf{H}_{\mathcal{D}}) = \varrho\mathcal{D}$ w.p.1. □

The following Lemma will be needed in the proof of Theorem 4.

Lemma 3: Let $\mathbf{x}_i \sim \mathcal{CN}_M(\mathbf{0}, \mathbf{\Sigma}_i)$, $i = 1, 2, \ldots, K$ with $K \leq M$, and assume that the $\mathbf{x}_i$ are independently drawn. Then $r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) < K$ w.p.1 if and only if the following assertion holds: there exist $1 \leq p \leq K$ vectors $\{\mathbf{x}_{i_\alpha} : \alpha = 1, 2, \ldots, p\}$ and there exists a $(p-1)$-dimensional subspace $S^{(p-1)}$ of $\mathbb{C}^M$ such that $\Pr\{\mathbf{x}_{i_\alpha} \in S^{(p-1)},\ \alpha = 1, 2, \ldots, p\} = 1$.

Proof: From the arguments employed in the proof of Theorem 3, we can conclude that

$$r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) = \beta \quad \text{w.p.1}$$

for some $1 \leq \beta \leq K$. Next, we note that β equals the dimensionality of the subspace spanned by the vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K$. Applying Theorem 4.2 in [37] with $j = k = K$, we can conclude that $\Pr\{r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) \leq K-1\} > 0$ is equivalent to the assertion: there exist $1 \leq p \leq K$ vectors $\{\mathbf{x}_{i_\alpha} : \alpha = 1, 2, \ldots, p\}$ and there exists a $(p-1)$-dimensional subspace $S^{(p-1)}$ such that

$$\Pr\left\{\mathbf{x}_{i_\alpha} \in S^{(p-1)}, \alpha = 1, 2, \ldots, p\right\} > 0.$$

Noting that

$$\begin{aligned}\Pr\{r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) \leq K-1\} \\ = \Pr\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K \text{ are linearly dependent}\}\end{aligned}$$

and applying Lemma 2, we can conclude that

$$\Pr\{r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) \leq K-1\} > 0$$

implies

$$\Pr\{r([\mathbf{x}_1\, \mathbf{x}_2\, \cdots\, \mathbf{x}_K]) \leq K-1\} = 1.$$

Finally, applying Lemma 1, we can conclude that

$$\Pr\{\mathbf{x}_{i_\alpha} \in S^{(p-1)}, \alpha = 1, 2, \ldots, p\} > 0$$

implies

$$\Pr\{\mathbf{x}_{i_\alpha} \in S^{(p-1)}, \alpha = 1, 2, \ldots, p\} = 1$$

which concludes the proof. □

Theorem 4: Use the notation from Theorem 3. The following statements hold:

1) $\frac{1}{U} \leq \frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} \leq 1$.

2) $\frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} = 1$ if and only if $\text{span}\{\mathbf{R}_0\} = \text{span}\{\mathbf{R}_1\} = \cdots = \text{span}\{\mathbf{R}_{U-1}\}$ and $r(\mathbf{R}_0) \leq M_T$.

3) $\frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} = \frac{1}{U}$ if $r(\mathbf{R}_i) \geq UM_T\ \forall i$, or $\mathbf{R}_i\mathbf{R}_j = \mathbf{0}$ $\forall i \neq j$.

Proof: We start with the proof of statement 1). Since adding more columns to a given matrix can only lead to a rank increase, we have $\varrho(\{i\}) \leq \varrho(\mathcal{U})\ \forall i$. Since trivially $\varrho(\mathcal{U}) \leq \sum_{i=0}^{U-1} \varrho(\{i\})$ we can conclude that

$$\frac{1}{U} \leq \frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} \leq 1$$

which completes the proof of 1).

For the proof of statement 2), we start by assuming that $\text{span}\{\mathbf{R}_0\} = \text{span}\{\mathbf{R}_1\} = \cdots = \text{span}\{\mathbf{R}_{U-1}\}$ and $r(\mathbf{R}_0) \leq M_T$. Since $r(\mathbf{H}_i) = \min(r(\mathbf{R}_i), M_T)$, we have $\varrho(\{i\}) = r\ \forall i$ and consequently $\varrho(\mathcal{U}) \geq r$. Since $\text{span}\{\mathbf{R}_0\} = \text{span}\{\mathbf{R}_1\} = \cdots = \text{span}\{\mathbf{R}_{U-1}\}$, any collection of $r+1$ columns of $\mathbf{H}_\mathcal{U}$ will satisfy $\Pr\{\mathbf{h}_{i_\alpha} \in \text{span}\{\mathbf{R}_0\}, \alpha = 1, 2, \ldots, r+1\} = 1$. Applying Lemma 3, we can conclude that $\varrho(\mathcal{U}) < r+1$ which in combination with $\varrho(\mathcal{U}) \geq r$ yields $\varrho(\mathcal{U}) = r$ and consequently $\frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} = 1$. This finishes the proof of the "if" part of statement 2). In order to prove the "only if" part, we start by noting that $\sum_{i=0}^{U-1} \varrho(\{i\}) \leq U\max_{i\in\mathcal{U}}(\varrho(\{i\})) \leq U\varrho(\mathcal{U})$ which implies that $\frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} = 1$ only if $\varrho(\{0\}) = \varrho(\{1\}) = \cdots = \varrho(\{U-1\})$. Since $\varrho(\{i\}) = \min(r(\mathbf{R}_i), M_T)$, there are three different possibilities for this equality to hold:

a) $r(\mathbf{R}_i) \geq M_T\ \forall i$ and there exists at least one index j such that $r(\mathbf{R}_j) > M_T$.

b) $r(\mathbf{R}_i) = r \leq M_T\ \forall i$ and there exists at least one pair $\{j, k\}$ such that $\text{span}\{\mathbf{R}_j\} \neq \text{span}\{\mathbf{R}_k\}$.

c) $r(\mathbf{R}_i) = r \leq M_T\ \forall i$ and $\text{span}\{\mathbf{R}_0\} = \text{span}\{\mathbf{R}_1\} = \cdots = \text{span}\{\mathbf{R}_{U-1}\}$.

In what follows, we will show that a) and b) imply $\frac{\sum_{i=0}^{U-1} \varrho(\{i\})}{U\varrho(\mathcal{U})} < 1$ hence only leaving part c).

Assume that a) holds and choose any j such that $r(\mathbf{R}_j) > M_T$. From Lemma 1, part 2), we can conclude that the M_T column vectors $\mathbf{h}_i$, $i = 1, 2, \ldots, M_T$ in $\mathbf{H}_j$ satisfy $\Pr\{\mathbf{h}_i \in S^{(p)}\} = 0$, where $S^{(p)}$ is a p-dimensional subspace of $\mathbb{C}^{M_R}$ with $p \leq M_T$. Now, applying Lemma 3 we can conclude that any $M_T + 1$ columns of $\mathbf{H}_\mathcal{U}$ that contain the M_T columns of $\mathbf{H}_j$ along with an arbitrarily chosen additional column in $\mathbf{H}_\mathcal{U}$ are linearly independent w.p.1, which implies that $\varrho(\mathcal{U}) \geq (M_T + 1)$ and consequently

$$U\varrho(\mathcal{U}) \geq U(M_T+1) > UM_T = \sum_{i=0}^{U-1} \varrho(\{i\}).$$

This concludes case a).

Assume next that b) holds and choose any pair $\{j, k\}$ such that $\text{span}\{\mathbf{R}_j\} \neq \text{span}\{\mathbf{R}_k\}$. Let

$$\{\mathbf{h}_{i_\alpha}, \alpha = 1, 2, \ldots, r+1\}$$

be any collection of $r+1$ columns of $\mathbf{H}_\mathcal{U}$ that contains r columns from $\mathbf{H}_j$ and one column from[8] $\mathbf{H}_k$. We will show that these columns are linearly independent w.p.1. By Lemma 3 and Lemma 1, part 2), any collection of r columns of $\mathbf{H}_\mathcal{U}$ is linearly independent w.p.1. Therefore, if $\{\mathbf{h}_{i_\alpha}, \alpha = 1, 2, \ldots, r+1\}$ would not be linearly independent w.p.1, by Lemma 3 there would exist an r-dimensional subspace $S^{(r)}$ of $\mathbb{C}^{M_R}$ such that $\Pr\{\mathbf{h}_i \in S^{(r)}, \alpha = 1, 2, \ldots, r+1\} = 1$. However, since $r(\mathbf{R}_j) = r(\mathbf{R}_k) = r$, this would imply that $\text{span}\{\mathbf{R}_j\} = \text{span}\{\mathbf{R}_k\}$, which is a contradiction. Consequently, we have $\varrho(\mathcal{U}) \geq r+1$ which implies $\sum_{i=0}^{U-1} \varrho(\{i\}) = Ur < U\varrho(\mathcal{U})$. This completes the proof of part 2).

[8]This specific choice of the number of columns will be exploited later in the proof.

Finally, for the proof of statement 3), we start by noting that $\frac{\sum_{i=0}^{U-1}\varrho(\{i\})}{U\varrho(\mathcal{U})} = \frac{1}{U}$ implies

$$\begin{aligned}\varrho(\mathcal{U}) &= r(\mathbf{H}_{\mathcal{U}}^H\mathbf{H}_{\mathcal{U}})\\ &= r(\mathrm{diag}\{\mathbf{H}_{i,w}^H\mathbf{R}_i\mathbf{H}_{i,w}\}_{i=0}^{U-1})\\ &= \sum_{i=0}^{U-1}\varrho(\{i\}) \text{ w.p.1}\end{aligned}$$

and consequently $\frac{\sum_{i=0}^{U-1}\varrho(\{i\})}{U\varrho(\mathcal{U})} = \frac{1}{U}$. If $r(\mathbf{R}_i) \geq UM_T\ \forall i$, $\varrho(\mathcal{U})$ necessarily has to equal UM_T. To show this assume that $\varrho(\mathcal{U}) < UM_T$. By Lemma 3 this would imply that there exists a p-dimensional subspace $S^{(p)}$ with $1 \leq p \leq UM_T - 1$ such that all the columns of $\mathbf{H}_{\mathcal{U}}$ belong to $S^{(p)}$ w.p.1. Since $r(\mathbf{R}_i) \geq UM_T\ \forall i$, this is a contradiction by part 2) of Lemma 1. The proof is completed by noting that $r(\mathbf{R}_i) \geq UM_T\ \forall i$ implies $\varrho(\{i\}) = \min(r(\mathbf{R}_i), M_T) = M_T\ \forall i$. □

APPENDIX II
PROOF OF THEOREM 1

Let $\mathcal{S} \subseteq \mathcal{U}$, and define the function[9] $\phi_{\mathcal{S}}(\boldsymbol{\rho}) : \mathbb{R}_+^U \to \mathbb{R}_+$

$$\phi_{\mathcal{S}}(\boldsymbol{\rho}) = \mathcal{E}\left\{\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_i\mathbf{H}_i\mathbf{H}_i^H\right)\right\}$$

where $\boldsymbol{\rho} = [\rho_0\ \rho_1\ \ldots\ \rho_{U-1}]^T$ and $\mathbf{H}_i$ was defined in (3). Let $\boldsymbol{\rho}^{(1)} \in \mathbb{R}_+^U$ and $\boldsymbol{\rho}^{(2)} \in \mathbb{R}_+^U$ and let $\boldsymbol{\rho}^{(i)} = [\rho_0^{(i)}\ \rho_1^{(i)} \ldots \rho_{U-1}^{(i)}]^T$ for $i = 1, 2$. Since $\log_2\det$ is strictly concave on the convex set of positive definite Hermitian matrices [34, Theorem 7.6.7], we get that for any $\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{U-1}$ and $\lambda \in [0, 1]$

$$\begin{aligned}&\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\left(\lambda\rho_i^{(1)} + (1-\lambda)\rho_i^{(2)}\right)\mathbf{H}_i\mathbf{H}_i^H\right)\\ &= \log_2\det\left(\lambda\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_i^{(1)}\mathbf{H}_i\mathbf{H}_i^H\right)\right.\\ &\qquad\left. + (1-\lambda)\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_i^{(2)}\mathbf{H}_i\mathbf{H}_i^H\right)\right)\\ &\geq \lambda\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_i^{(1)}\mathbf{H}_i\mathbf{H}_i^H\right)\\ &\qquad + (1-\lambda)\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\rho_i^{(2)}\mathbf{H}_i\mathbf{H}_i^H\right). \quad (31)\end{aligned}$$

Taking expectations on both sides of (31), we can conclude that $\phi_{\mathcal{S}}(\boldsymbol{\rho})$ is concave in $\boldsymbol{\rho}$. The corresponding capacity region bound in (5) can now be written as

$$\mathcal{E}\left\{\frac{1}{N}\sum_{k=0}^{N-1}\log_2\det\left(\mathbf{I}_{M_R} + \sum_{i\in\mathcal{S}}\frac{P_{i,k}}{M_T}\mathbf{H}_i\mathbf{H}_i^H\right)\right\} = \frac{1}{N}\sum_{k=0}^{N-1}\phi_{\mathcal{S}}(\boldsymbol{\rho}_k)$$

where $\boldsymbol{\rho}_k = [\frac{P_{0,k}}{M_T}\ \frac{P_{1,k}}{M_T}\ \cdots\ \frac{P_{U-1,k}}{M_T}]^T$. Since $\phi_{\mathcal{S}}(\boldsymbol{\rho})$ is concave in $\boldsymbol{\rho}$, Jensen's inequality [38] allows us to conclude that

$$\frac{1}{N}\sum_{k=0}^{N-1}\phi_{\mathcal{S}}(\boldsymbol{\rho}_k) \leq \phi_{\mathcal{S}}\left(\frac{1}{N}\sum_{k=0}^{N-1}\boldsymbol{\rho}_k\right). \quad (32)$$

Noting that $\frac{1}{N}\sum_{k=0}^{N-1}\boldsymbol{\rho}_k = [\frac{P_0}{NM_T}\ \frac{P_1}{NM_T}\ \cdots\ \frac{P_{U-1}}{NM_T}]^T$, we can infer that the capacity region bounds for all $\mathcal{S} \subseteq \mathcal{U}$ are jointly maximized if $P_{i,k} = \frac{1}{N}P_i\ \forall i, \forall k$. The corresponding expression for the capacity region is given in (6).

We next show that any other power allocation strategy leads to a strictly smaller ergodic capacity region. For this it is sufficient to show that $\phi_{\mathcal{U}}(\boldsymbol{\rho})$ is *strictly* concave in $\boldsymbol{\rho}$. This will be achieved by showing that for $\mathcal{S} = \mathcal{U}$ equality in (31) is achieved with probability 0 if $\boldsymbol{\rho}^{(1)} \neq \boldsymbol{\rho}^{(2)}$. Since $\log_2\det$ is strictly concave on the set of positive definite Hermitian matrices equality in (31), for $\mathcal{S} = \mathcal{U}$, is achieved if and only if $\sum_{i=0}^{U-1}\rho_i^{(1)}\mathbf{H}_i\mathbf{H}_i^H = \sum_{i=0}^{U-1}\rho_i^{(2)}\mathbf{H}_i\mathbf{H}_i^H$. Assume that $\boldsymbol{\rho}^{(1)} \neq \boldsymbol{\rho}^{(2)}$, let j be an index such that $\rho_j^{(1)} \neq \rho_j^{(2)}$ and denote the $(1, 1)$ element of $\mathbf{H}_i\mathbf{H}_i^H$ by y_i. Then[10]

$$\begin{aligned}&\Pr\left\{\sum_{i=0}^{U-1}\rho_i^{(1)}\mathbf{H}_i\mathbf{H}_i^H = \sum_{i=0}^{U-1}\rho_i^{(2)}\mathbf{H}_i\mathbf{H}_i^H\right\}\\ &\leq \Pr\left\{\sum_{i=0}^{U-1}\rho_i^{(1)}y_i = \sum_{i=0}^{U-1}\rho_i^{(2)}y_i\right\}\\ &= \mathcal{E}\left\{\Pr\left\{y_j = \frac{\sum_{i\neq j}(\rho_i^{(1)} - \rho_i^{(2)})y_i}{\rho_j^{(2)} - \rho_j^{(1)}}\right|\right.\\ &\qquad\left.\left. y_0, y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_{U-1}\right\}\right\}\\ &= \mathcal{E}\{0\} = 0\end{aligned}$$

where we used the fact that the probability that a continuous random variable takes on a specific value is always zero. Thus, $\phi_{\mathcal{U}}(\boldsymbol{\rho})$ is strictly concave in $\boldsymbol{\rho}$ which implies that equality in (32), for $\mathcal{S} = \mathcal{U}$, is achieved if and only if $P_{i,k} = \frac{1}{N}P_i\ \forall i, \forall k$. The proof is now complete.

APPENDIX III
PROOF OF THEOREM 2

In order to prove part 1), we note that since $N_i \geq 1\ \forall i$, for $\mathcal{S} \subset \mathcal{U}$, there has to exist at least one $k \in \{0, 1, \ldots, N-1\}$ such that $\mathcal{U}_k \cap \mathcal{S} = \emptyset$. Hence $\mathcal{M}_{\mathcal{S}}\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\} = \frac{1}{N}\sum_{k=0}^{N-1}\varrho(\mathcal{U}_k \cap \mathcal{S}) < \varrho(\mathcal{S}) = \mathcal{M}_{\mathcal{S},\mathrm{CDMA}}\ \forall \mathcal{S} \subset \mathcal{U}$. Since every tone is assigned to exactly one user, we have $\mathcal{M}_{\mathcal{U}}\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\} = \frac{1}{N}\sum_{i=0}^{U-1}N_i\varrho(\{i\})$. If the $\varrho(\{i\})$ for $i = 0, 1, \ldots, U-1$ are not all equal, $\frac{1}{N}\sum_{i=0}^{U-1}N_i\varrho(\{i\}) < \varrho(\mathcal{U}) = \mathcal{M}_{\mathcal{U},\mathrm{CDMA}}$ since $\varrho(\{i\}) \leq \varrho(\mathcal{U})\ \forall i$. If $\varrho(\{0\}) = \varrho(\{1\}) = \cdots = \varrho(\{U-1\})$, we have $\frac{1}{N}\sum_{i=0}^{U-1}N_i\varrho(\{i\}) = \frac{1}{U}\sum_{i=0}^{U-1}\varrho(\{i\})$ and by Theorem 4, $\frac{1}{U}\sum_{i=0}^{U-1}\varrho(\{i\}) \leq \varrho(\mathcal{U}) = \mathcal{M}_{\mathcal{U},\mathrm{CDMA}}$, where equality is achieved if and only if $\mathrm{span}\{\mathbf{R}_0\} = \mathrm{span}\{\mathbf{R}_1\} = \cdots = \mathrm{span}\{\mathbf{R}_{U-1}\}$ and $r(\mathbf{R}_0) \leq M_T$. This completes the proof of part 1).

[9]We note that the slight abuse of notation when defining the domain of $\phi_{\mathcal{S}}$ as $\mathbb{R}_+^U$ instead of $\mathbb{R}_+^{|S|}$ is done on purpose in order to keep the notation simple.

[10]The expectations are taken over the random variables $y_0, y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_{U-1}$.

For the proof of part 2), we start by noting that trivially $\forall \mathcal{S} \subseteq \mathcal{U}, \mathcal{M}_{\mathcal{S},\text{CDMA}} = \varrho(\mathcal{S}) \leq \sum_{i\in\mathcal{S}} \varrho(\{i\})$. When $N_i = \frac{N}{U}$ for $i = 0, 1, \ldots, U-1$, $\mathcal{M}_{\mathcal{S}}\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\} = \frac{1}{U}\sum_{i\in\mathcal{S}} \varrho(\{i\})$ and hence

$$\frac{1}{U}\mathcal{M}_{\mathcal{S},\text{CDMA}} \leq \mathcal{M}_{\mathcal{S}}\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}.$$

Theorem 4, part 3) implies that $\frac{1}{U}\mathcal{M}_{\mathcal{U},\text{CDMA}} = \mathcal{M}_{\mathcal{U}}\{\mathcal{U}_0, \mathcal{U}_1, \ldots, \mathcal{U}_{N-1}\}$ if $r(\mathbf{R}_i) \geq UM_T \ \forall i$, or $\mathbf{R}_i\mathbf{R}_j = \mathbf{0}$ $\forall i \neq j$. When $\mathcal{S} \subset \mathcal{U}$, the equality in (21) can easily be shown by using the arguments employed in Theorem 4, proof of part 3). The proof is now complete.

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