Pre-Equalization for MIMO Wireless Channels with Delay Spread*

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Abstract

We consider a downlink finite impulse response (FIR) multi-input multi-output (MIMO) wireless channel with L taps. It is shown that such a channel can be pre-equalized with an FIR MIMO transmit filter with only L taps, if the angle spread due to the different multipaths is sufficiently large at the transmitter. The filter taps are derived for the cases where the transmitter has perfect and partial channel knowledge, respectively. Finally, we present a pre-filter structure which converts the available frequency diversity into spatial diversity. The resulting spatial diversity can then be exploited using conventional receivers designed for frequency-flat fading channels

1 Introduction and Outline

The use of multiple antennas at both the transmitter and the receiver of a wireless link has recently been shown to have the potential of achieving extraordinary bit rates [1, 2] and drastically improving link reliability through spatial diversity [3, 4, 5].

A major issue facing a multi-input multi-output (MIMO) wireless system designer is how to deal with delay spread channels. One option is to use Orthogonal Frequency Division Multiplexing (OFDM) (see for example [6, 7]) which converts a delay spread channel into a number of parallel frequency-flat fading subchannels. Single carrier systems typically perform linear or nonlinear equalization in the receiver to mitigate intersymbol interference (ISI). In applications like fixed wireless, wireless local loop systems with feedback, and time division duplexing systems, channel knowledge can be made available at the transmitter. A natural question to ask is how to use this channel information for pre-equalization of the MIMO channel. Pre-equalization can typically be employed by the base station for downlink transmission to significantly reduce the equalizer complexity in the mobiles.

In this paper, we address the problem of designing finite impulse response (FIR) MIMO pre-equalization filters. In the past, MIMO FIR equalizers have been used primarily in the receiver (see for example [8, 9]). The idea of pre-equalization using block processing of data was considered previously in [10, 11].

Contributions. Employing a parametric MIMO channel model introduced in [12, 6] our contributions are as follows:

- We show that a MIMO wireless channel with L taps can be pre-equalized by an FIR MIMO transmit filter with only L taps, if the angle spread due to the different multipaths is large at the transmitter.
- We derive solutions for the cases where the transmitter has perfect and partial channel knowledge.
- We present a pre-filter structure which converts the frequency diversity available in the channel into spatial diversity. This resulting spatial diversity can then be exploited using receivers designed for the frequency-flat fading case.

Organization of the paper. The rest of this paper is organized as follows. In Section 2, we present the system model and the wireless channel model. In Section 3, we introduce the problem formulation and provide solutions for varying degrees of channel knowledge in the transmitter. In Section 4, we present simulation results. Section 5 contains our conclusions.

2 System and Channel Model

In this section, we describe our system model and the parametric MIMO channel model.

2.1 System Model

Consider a downlink wireless MIMO system (see Fig. 1) with M_T base-station antennas and M_R terminal antennas where $M_T \geq M_R$ is assumed throughout the paper. We consider transmission of M_R data streams $s_n^{(i)}$ $(i = 0, 1, ..., M_R - 1)$ which are preprocessed by the $M_T \times M_R$ space-time FIR filter[1] $\mathbf{T}(z) = \sum_{l=-L'+1}^{0} \mathbf{T}_l \, z^{-l}$ (see Fig. 1). Here, the $M_T \times M_R$ matrix $\mathbf{T}_l$ $(l =$

*This work was supported in part by Lucent Technologies, NJ and by FWF grant J1868-TEC.

[1] We assume that the pre-equalization filter is anticausal in order to simplify the presentation in the subsequent developments.

$-L'+1, -L'+2, \ldots, 0)$ denotes the l-th tap of the pre-equalization filter. The space-time filtered signal is then launched into the $M_R \times M_T$ MIMO channel

$$\mathbf{H}(z) = \sum_{l=0}^{L-1} \mathbf{H}_l \, z^{-l}.$$

The $M_R \times 1$ received vector signal can now be written as

$$\hat{\mathbf{s}}_n = \sum_{k=-\infty}^{\infty} \mathbf{H}_{n-k} \sum_{p=-\infty}^{\infty} \mathbf{T}_{k-p} \mathbf{s}_p + \mathbf{v}_n, \qquad (1)$$

where $\mathbf{v}_n$ is the noise vector sequence[2], $\mathbf{s}_n = [s_n^{(0)} \; s_n^{(1)} \; \ldots \; s_n^{(M_R-1)}]^T$ is the $M_R \times 1$ transmitted vector sequence and $\hat{\mathbf{s}}_n$ is defined similarly. In the z-transform domain, (1) can be rewritten as

$$\hat{\mathbf{s}}(z) = \mathbf{H}(z)\mathbf{T}(z)\mathbf{s}(z) + \mathbf{v}(z) \qquad (2)$$

with $\mathbf{H}(z) = \sum_{l=0}^{L-1} \mathbf{H}_l \, z^{-l}$, $\mathbf{T}(z) = \sum_{l=-L'+1}^{0} \mathbf{T}_l \, z^{-l}$, and $\mathbf{s}(z), \hat{\mathbf{s}}(z)$ and $\mathbf{v}(z)$ defined similarly.

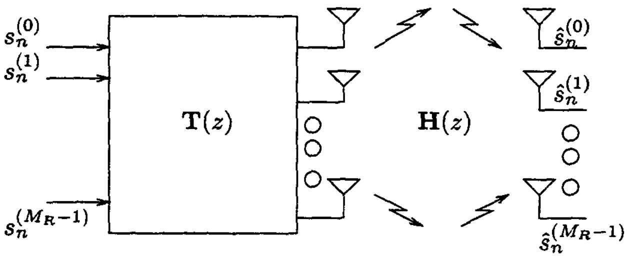

Fig. 1. Pre-equalized space-time delay spread channel.

2.2 Channel Model

We employ the parametric delay spread MIMO wireless channel model introduced in [12, 6], which is briefly reviewed below.

The signal launched into the MIMO channel propagates along L multipaths created by reflection and scattering in the wireless environment (see Fig. 2). We define $\theta_{T,l}$ to be the angle of departure of the l-th multipath at the transmitter and $\theta_{R,l}$ to be the angle of arrival of the l-th multipath at the receiver. The array response vectors for the l-th path at the transmitter and the receiver, respectively, are denoted as $\mathbf{a}_{T,l}(\theta_{T,l})$ and $\mathbf{a}_{R,l}(\theta_{R,l})$. For uniform linear arrays we have

$$\mathbf{a}_{R,l}(\theta_{R,l}) = [1 \;\; e^{j2\pi\Delta\sin(\theta_{R,l})} \;\; \ldots \;\; e^{j2\pi(M_R-1)\Delta\sin(\theta_{R,l})}]^T, \qquad (3)$$

and similarly for $\mathbf{a}_{T,l}(\theta_{T,l})$, where $\Delta = \frac{d}{\lambda}$ is the relative antenna spacing, d is the absolute antenna spacing, and $\lambda = \frac{c}{f_c}$ is the wavelength of a narrowband signal with center frequency f_c. We note that the following developments are not restricted to uniform linear arrays.

The transfer function of the MIMO wireless channel can now be written as

$$\mathbf{H}(z) = \sum_{l=0}^{L-1} \beta_l \mathbf{a}_{R,l}(\theta_{R,l}) \, \mathbf{a}_{T,l}^T(\theta_{T,l}) \, z^{-l} = \sum_{l=0}^{L-1} \mathbf{H}_l \, z^{-l}, \qquad (4)$$

[2]The superscripts $^T, ^H$, and * stand for transposition, conjugate transposition, and elementwise conjugation, respectively.

where β_l denotes the complex path gain of the l-th path, and $\mathbf{H}_l = \beta_l \mathbf{a}_{R,l}(\theta_{R,l}) \, \mathbf{a}_{T,l}^T(\theta_{T,l})$.

Now, for a sufficiently large number of transmit antennas, assuming that the angles of departure at the transmitter are fairly well separated, the corresponding array response vectors $\mathbf{a}_{T,l}(\theta_{T,l})$ will be approximately orthogonal to each other, i.e.,

$$\mathbf{a}_{T,l}^H(\theta_{T,l})\mathbf{a}_{T,l'}(\theta_{T,l'}) \approx 0 \qquad \text{for} \quad l \neq l'. \qquad (5)$$

This assumption is justified in cases where the base station is unobstructed and highly elevated and the number of multipaths L does not significantly exceed M_T.

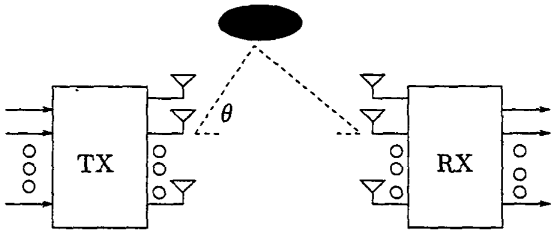

Fig. 2. Parametric MIMO wireless channel model.

3 MIMO Pre-Equalization

In this section, we consider the problem of pre-equalization and provide the solution for three different cases (discussed in Secs. 3.1-3.3) corresponding to different requirements on the equalizer and varying degrees of channel knowledge. In Sec. 3.4, we introduce a measure for quantifying the error incurred by making the assumption in (5).

3.1 Space-Time Pre-Equalization

Assuming that the transmitter has full channel knowledge we next derive an analytic expression for the taps of the zero-forcing pre-equalizer satisfying

$$\mathbf{H}(z)\mathbf{T}(z) = \alpha \, \mathbf{I}_{M_R}, \qquad (6)$$

where $\alpha > 0$ is a (real) constant chosen such that the constraint $\mathrm{Tr}\left\{\sum_{l=-L'+1}^{0} \mathbf{T}_l \mathbf{T}_l^H\right\} = 1$ is satisfied. Now, for $M_T = M_R$ assuming that $\mathbf{H}(z)$ is invertible the unique solution of (6) is given by $\mathbf{T}(z) = \alpha \, \mathbf{H}^{-1}(z) = \alpha \, \frac{adj \, \mathbf{H}(z)}{det \, \mathbf{H}(z)}$, where *adj* denotes the adjoint matrix and *det* stands for the determinant of a matrix. If $M_T > M_R$ there is in principle an infinite number of solutions. We shall, however, restrict our attention to the pseudo-inverse, which is given by

$$\mathbf{T}(z) = \alpha \, \tilde{\mathbf{H}}(z) \, [\mathbf{H}(z)\tilde{\mathbf{H}}(z)]^{-1} = \alpha \, \tilde{\mathbf{H}}(z) \, \frac{adj \, [\mathbf{H}(z)\tilde{\mathbf{H}}(z)]}{det \, [\mathbf{H}(z)\tilde{\mathbf{H}}(z)]}, \qquad (7)$$

where $\tilde{\mathbf{H}}(z) = \mathbf{H}^H\left(\frac{1}{z^*}\right)$. Now, from (4) it follows that

$$\mathbf{H}(z)\tilde{\mathbf{H}}(z) = \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} \mathbf{H}_l \mathbf{H}_{l'}^H \, z^{-l+l'}.$$

Invoking (5), we obtain $\mathbf{H}_l\mathbf{H}_{l'}^H \approx \mathbf{0}_{M_R}$ for $l \neq l'$ and therefore

$$\mathbf{H}(z)\tilde{\mathbf{H}}(z) \approx \sum_{l=0}^{L-1} \mathbf{H}_l\mathbf{H}_l^H = \mathbf{H}_c.$$

Assuming that $L \geq M_R$, the matrix $\mathbf{H}_c$ will generally be full rank and hence we can write

$$\mathbf{T}(z) = \alpha\,\tilde{\mathbf{H}}(z)\mathbf{H}_c^{-1} \tag{8}$$

or equivalently

$$\mathbf{T}_{-l} = \alpha\,\mathbf{H}_l^H\mathbf{H}_c^{-1}.$$

We have thus found an analytic expression of the zero-forcing pre-equalization filter which happens to equal the matched filter multiplied by $\mathbf{H}_c^{-1}$ from the right. We note that this result basically says that if the transmit array signature vectors are orthogonal to each other, the MIMO channel can be perfectly pre-equalized using an L-tap MIMO filter. Finally, we note that $\mathbf{H}_c$ can be further simplified to yield

$$\mathbf{H}_c = \sum_{l=0}^{L-1} |\beta_l|^2 \mathbf{a}_{R,l}(\theta_{R,l})\mathbf{a}_{R,l}^H(\theta_{R,l}), \tag{9}$$

where we have assumed that the transmit array signature vectors have unit norm.

3.2 Time-Only Pre-Equalization

In many applications, it is sufficient to equalize the MIMO channel up to a *mixing* (flat-fading) matrix. This can be achieved by setting

$$\mathbf{T}(z) = \alpha\,\tilde{\mathbf{H}}(z). \tag{10}$$

Again exploiting the orthogonality of the transmit signature vectors, we get

$$\hat{\mathbf{s}}(z) = \alpha\,\mathbf{H}_c\mathbf{s}(z) + \mathbf{v}(z). \tag{11}$$

This result has an interesting interpretation. Using (9), we can see that the resultant flat-fading channel matrix $\mathbf{H}_c$ has contributions from all L multipaths, which implies that the pre-equalizer in (10) not only eliminates ISI, but also converts the available frequency diversity into spatial diversity. This resulting spatial diversity can then be exploited using conventional receivers designed for the flat-fading case.

3.3 Partial Channel Knowledge

In practical systems, it is often difficult to obtain full channel knowledge at the transmitter. This problem typically arises if the feedback link is too slow or the channel changes rapidly. However, knowledge of the angles of departure of the L multipaths is rather easy to obtain since these angles depend only on the geometry of the environment and will thus be relatively stationary over long periods of time. We shall next show that this information suffices to eliminate ISI by pre-equalization.

Suppose we choose the precoding filter according to

$$\mathbf{T}(z) = \alpha \sum_{l=0}^{L-1} \mathbf{a}_{T,l}^*(\theta_{T,l})\mathbf{\Phi}_l^T z^l \tag{12}$$

where the $\mathbf{\Phi}_l\,(l = 0, 1, ..., L-1)$ are unit norm $M_R \times 1$ vectors chosen such that the matrix $[\mathbf{\Phi}_0\ \mathbf{\Phi}_1\ ...\ \mathbf{\Phi}_{L-1}]$ has full rank. Using the approximate orthogonality of the transmit array signature vectors (5), it can readily be verified that

$$\mathbf{H}(z)\mathbf{T}(z) \approx \sum_{l=0}^{L-1} \beta_l\,\mathbf{a}_{R,l}(\theta_{R,l})\,\mathbf{\Phi}_l^T.$$

We can thus see that choosing the pre-equalizer according to (12) provides the required ISI mitigation, even though the transmitter knows only the angles of departure of the L multipaths.

3.4 A Measure of Performance

In practice, the transmit array response vectors corresponding to different angles of departure at the transmitter will not be entirely orthogonal to each other. This problem will typically occur if there is not enough angle spread or if the number of multipath components significantly exceeds the number of transmit antennas. As a consequence, the performance of the pre-equalization filters designed in Secs. 3.1-3.3 will degrade. We shall next introduce a measure quantifying the degree of nonorthogonality of the transmit array response vectors. We shall provide this measure for the case of time-only pre-equalization. The developments for the other two cases discussed in Secs. 3.1 and 3.3 is similar.

Substituting $\mathbf{T}(z)$ in (10) into (1) we can write

$$\hat{\mathbf{s}}_n = \alpha\mathbf{H}_c\mathbf{s}_n + \alpha \sum_{l,l'=0,l\neq l'}^{L-1} \mathbf{H}_l\mathbf{H}_{l'}^H\mathbf{s}_{n-l+l'} + \mathbf{v}_n, \tag{13}$$

where the first term is the desired signal and the second term corresponds to the distortion incurred by lack of orthogonality of the transmit array signature vectors. We now define the following measure

$$\mathcal{D}_{ms} = \frac{\mathrm{Tr}\{\sum_{l,l'=0,l\neq l'}^{L-1} \mathbf{H}_l\mathbf{H}_{l'}^H\mathbf{H}_{l'}\mathbf{H}_l^H\}}{\mathrm{Tr}\{\mathbf{H}_c\mathbf{H}_c^H\}} \tag{14}$$

which is the energy in the ISI-terms in (13) divided by the energy in the ISI-free component $\mathbf{H}_c$.

4 Simulations

Simulation Example 1. In the first simulation example we study the behavior of $\mathcal{D}_{ms}$ as a function of the angle spread and M_T. We chose $L = 2$ with $\theta_{T,0} = 0$. The two paths were separated by one symbol period and $|\beta_0|^2 = |\beta_1|^2=1$. Fig. 3 shows $\mathcal{D}_{ms}$ as a function of $\theta_{T,1}$, where $\theta_{R,0} = 0$, $\theta_{R,1}$ was drawn from a uniform distribution in the interval $[0, \frac{\pi}{2}]$, and $\mathcal{D}_{ms}$ was computed by averaging over 5,000 independent Monte Carlo runs for each value of $\theta_{T,1}$ (which was varied in steps of 2 degrees). We can clearly see that for a fixed number of transmit antennas $\mathcal{D}_{ms}$ decreases for increasing angle separation at the transmitter. We furthermore observe that for fixed angle spread at the transmitter $\mathcal{D}_{ms}$ generally decreases as M_T increases. We thus conclude that the assumption (5) will be justified if M_T and the angle separation at the transmitter are large.

Simulation Example 2. In this simulation example we corroborate our statement that time-only pre-equalization turns frequency-diversity into spatial diversity if M_T and the angle spread are sufficiently large. For $M_T = 6, M_R = 2$ and varying L we study the performance of a time-only pre-equalized system in terms of bit error rate. The transmit array signature vectors were chosen to be the columns of an $M_T \times L$ unitary matrix. For $L = 2, 3, ..., 6$ we therefore have perfect orthogonality of the $\mathbf{a}_{T,l}(\theta_{T,l})$. The SNR defined as the total transmit power (over all transmit antennas) divided by the noise power at one receive antenna was set to 10dB. We performed 50,000 Monte Carlo runs for each value of L. In the receiver we employed a uniform linear array and drew the angles of arrival from independent uniform distributions in the interval $\left[0, \frac{\pi}{2}\right]$. We employed 4−QAM modulation and performed ML decoding on the 2×2 matrix $\mathbf{H}_c$. From Fig. 4 it follows that the performance of the pre-equalized system improves for increasing L up to $L = 6$. The pre-equalizer along with the ML receiver operating on $\mathbf{H}_c$ therefore indeed converts the frequency-diversity into spatial diversity. For $L > M_T$ the transmit array signature separation is poor and hence (5) will not be satisfied. In other words, the pre-equalizer increasingly fails to mitigate ISI which results in a performance degradation since the receiver performs ML decoding on $\mathbf{H}_c$ only and does not take into account the resulting ISI terms.

5 Conclusion

Employing a parametric MIMO wireless channel model, we found that for applications where the angle separation at the transmitter is large and the number of multipath components does not significantly exceed the number of transmit antennas, an L tap MIMO filter is sufficient to pre-equalize an L tap MIMO channel. We derived the zero-forcing pre-equalization filter for the cases of perfect and partial channel knowledge. Finally, we provided an L tap filter structure which converts the frequency diversity available in the channel into spatial diversity. The resulting spatial diversity can then be exploited using conventional receivers for the frequency-flat fading case.

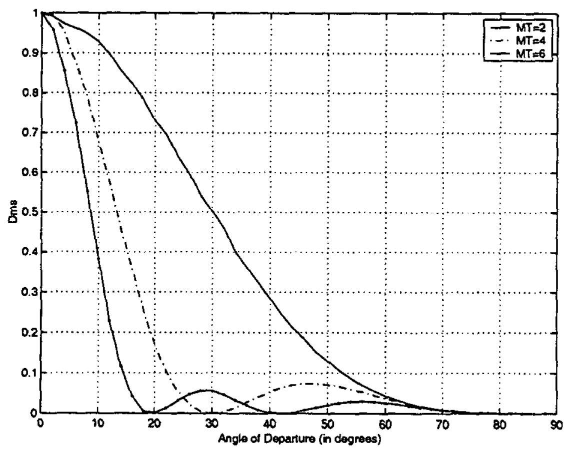

Fig. 3. Behavior of $\mathcal{D}_{ms}$ as a function of angle separation at the transmitter for different values of M_T.

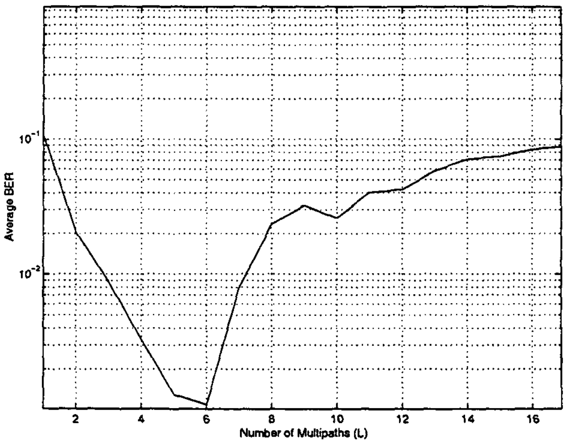

Fig. 4. Average bit error rate as a function of the number of multipaths for SNR = 10dB, $M_T = 6$, and $M_R = 2$.

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