

SPACE-FREQUENCY CODED BROADBAND OFDM SYSTEMS

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Abstract—Space-time coding for fading channels is a communication technique that realizes the diversity benefits of multiple transmit antennas. Previous work in this area has focused on the narrowband flat fading case where spatial diversity only is available. In this paper, we investigate the use of space-time coding in OFDM-based broadband systems where both spatial and frequency diversity are available. We consider a strategy which basically consists of coding across OFDM tones and will therefore be called *space-frequency coding*. For a spatial broadband channel model taking into account physical propagation parameters and antenna spacing, we derive the design criteria for space-frequency codes and we show that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not achieve full space-frequency diversity. Specifically, we show that the Alamouti scheme across tones fails to exploit frequency diversity. For a given set of propagation parameters and given antenna spacing, we establish the maximum achievable diversity order. Finally, we provide simulation results studying the influence of delay spread, propagation parameters, and antenna spacing on the performance of space-frequency codes.

1. INTRODUCTION AND OUTLINE

Two of the major impairments of wireless communications systems are *fading* caused by destructive addition of multipaths in the propagation medium and *interference from other users*. Diversity provides the receiver with several (ideally independent) replica of the transmitted signal and is therefore a powerful means to combat fading and interference. Common forms of diversity are time diversity (due to Doppler spread) and frequency diversity (due to delay spread). In recent years the use of spatial (or antenna) diversity has become increasingly popular. Spatial diversity is particularly attractive since it can be provided without loss in spectral efficiency. *Receive diversity*, i.e., the use of multiple antennas on the

receive side is a well-studied subject [1]. Driven by mobile wireless applications, where it is difficult to deploy multiple antennas in the handset, *transmit diversity* or equivalently the use of multiple antennas on the transmit side has become an active area of research [2]-[8]. *Space-time coding* evolved as one of the most promising transmit diversity techniques [9]-[13].

Most of the previous work on transmit diversity has been restricted to single-carrier systems operating over narrowband channels¹ where spatial diversity only is available. In this paper, we consider multi-antenna (multiple transmit and one or several receive antennas) broadband channels (i.e. channels with delay spread) where both spatial diversity (due to multiple antennas) and frequency diversity (due to delay spread) are available. *Orthogonal frequency division multiplexing* (OFDM) [14]-[16] significantly reduces receiver complexity in wireless broadband multi-antenna systems [17, 18]. We therefore study the use of space-time coding in OFDM-based multi-antenna systems.

Contributions. We consider a strategy which basically consists of employing a space-time code across OFDM tones and will therefore be called *space-frequency coding*. Our contributions are as follows.

- For a spatial delay spread channel model taking into account physical propagation parameters and antenna spacing, we derive the *design criteria for space-frequency codes* and we show that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not yield full space-frequency diversity. The requirement of exploiting frequency diversity as well imposes additional constraints on the codes and makes the design considerably more complicated than in the narrowband case.
- For a given set of channel parameters and given antenna spacing, we derive the *maximum achievable diversity order* and we discuss the impact of antenna spacing and propagation parameters on diversity.
- Using the design criteria established in this paper, we show that the *Alamouti scheme across tones fails to*

This work was supported in part by FWF grants J1629-TEC and J1868-TEC. H. Bölcskei is on leave from the Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien, Vienna, Austria. A. Paulraj is on part-time leave at Gigabit Wireless Inc., 3099 N. First Street, San Jose, CA.

¹Unless explicitly stated otherwise, throughout the paper when talking about the narrowband case we will actually mean the frequency-flat *slow fading* case.

exploit the frequency-diversity available in the delay spread case.

- We provide simulation results demonstrating the performance of known space-time codes employed as space-frequency codes under various propagation conditions.

Organization of the paper. The rest of this paper is organized as follows. In Section 2, we introduce the channel model, we briefly describe broadband OFDM-based multi-antenna systems and we discuss space-frequency coding. In Section 3, we derive the design criteria for space-frequency codes and we discuss their relation to previously established design criteria for the narrowband (slow and fast fading) case. In Section 4, we provide some simulation results. Finally, Section 5 contains our conclusions.

2. CHANNEL MODEL, OFDM, AND SPACE-FREQUENCY CODING

We shall first introduce the channel model, and then briefly describe OFDM-based multi-antenna systems and space-frequency coding.

2.1. The Channel Model

In the following M_T and M_R denote the number of transmit and receive antennas, respectively. We assume that the channel consists of L matrix taps (each of size M_R by M_T) with the matrix-valued transfer function given by

$$\mathbf{H}(e^{j2\pi\theta}) = \sum_{l=0}^{L-1} \mathbf{H}_l e^{-j2\pi l\theta}, \quad 0 \leq \theta < 1. \quad (1)$$

We restrict ourselves to Rayleigh fading channels. Hence the elements of the \mathbf{H}_l ($l = 0, 1, \dots, L-1$) are (possibly correlated) circularly symmetric zero mean complex gaussian random variables² with variance 1. We furthermore assume that heavy scattering occurs around the transmitter whereas the receiver is unobstructed. A schematic representation of the channel model is provided in Fig. 1. The l -th tap in the channel impulse response is assumed to correspond to the l -th scatterer cluster with mean angle of arrival at the receive array of $\bar{\theta}_l$ (derived from the position of the scatterer cluster), angular spread δ_l (proportional to the scattering radius of the cluster), and a (complex) path gain β_l . Since heavy scattering occurs around the transmitter, different transmit antennas will be uncorrelated which will be taken into account by assuming that different columns of \mathbf{H}_l are uncorrelated. Due to the lack of scattering at the receiver different receive antennas will be correlated which corresponds to assuming that different rows of \mathbf{H}_l are correlated. We furthermore assume that the different scatterer clusters are uncorrelated.

We assume a uniform linear array at both the transmitter and the receiver. The relative antenna spacing is denoted as $\Delta = \frac{d}{\lambda}$, where d is the absolute antenna spacing and $\lambda = c/f_c$ is the wavelength of a narrowband signal

²A circularly symmetric complex gaussian random variable is a random variable $z = (x + jy) \sim \mathcal{CN}(0, \sigma^2)$, where x and y are i.i.d. $\mathcal{N}(0, \sigma^2/2)$.

with center frequency f_c . We furthermore assume that the angle of arrival for the l -th path cluster at the receiver is gaussian distributed around the mean angle of arrival $\bar{\theta}_l$, i.e., the actual angle of arrival is given by³ $\theta_l = \bar{\theta}_l + \hat{\theta}_l$ with $\hat{\theta}_l \sim \mathcal{N}(0, \sigma_{\theta_l})$. The correlation matrix of the k -th column of the zero mean matrix \mathbf{H}_l given by⁴ $\mathbf{R}_l = \mathcal{E}\{\mathbf{h}_{l,k}\mathbf{h}_{l,k}^H\}$ with $\mathbf{h}_{l,k} = [h_{l,k}^{(0)} \ h_{l,k}^{(1)} \ \dots \ h_{l,k}^{(M_R-1)}]^T$ is independent of k , or equivalently the fading statistics are the same for all transmit antennas. Defining $\rho_l(s\Delta, \bar{\theta}_l, \sigma_{\theta_l}) = \mathcal{E}\{h_{l,k}^{(r)}h_{l,k}^{(r+s)*}\}$ for $l = 0, 1, \dots, L-1$, $k = 0, 1, \dots, M_T-1$ to be the fading correlation between two receive antenna elements spaced $s\Delta$ wavelengths apart, the correlation matrix \mathbf{R}_l ($l = 0, 1, \dots, L-1$) can be written as

$$[\mathbf{R}_l]_{m,n} = |\beta_l|^2 \rho_l((n-m)\Delta, \bar{\theta}_l, \sigma_{\theta_l}), \quad (2)$$

where we have absorbed the power delay profile of the channel into the correlation matrices. For small angular spread the correlation function in (2) is given by [19]

$$\rho_l(s\Delta, \bar{\theta}_l, \sigma_{\theta_l}) \approx e^{-j2\pi s\Delta \cos(\bar{\theta}_l)} e^{-\frac{1}{2}(2\pi s\Delta \sin(\bar{\theta}_l)\sigma_{\theta_l})^2}.$$

We note that although this approximation is accurate only for small angular spread, it does provide the correct trend for large angular spread, namely uncorrelated spatial fading. It is interesting to observe that for $\sigma_{\theta_l} = 0$ the correlation matrix \mathbf{R}_l collapses to a rank-1 matrix and can be written as $\mathbf{R}_l = |\beta_l|^2 \mathbf{a}(\bar{\theta}_l)\mathbf{a}^H(\bar{\theta}_l)$ with the array response vector of a uniform linear array given by

$$\mathbf{a}(\theta) = [1 \ e^{j2\pi\Delta \cos(\theta)} \ \dots \ e^{j2\pi(M_R-1)\Delta \cos(\theta)}]^T. \quad (3)$$

The rank of the correlation matrix \mathbf{R}_l will therefore be driven by the antenna spacing at the receiver and the angle spread of the l -th scatterer cluster.

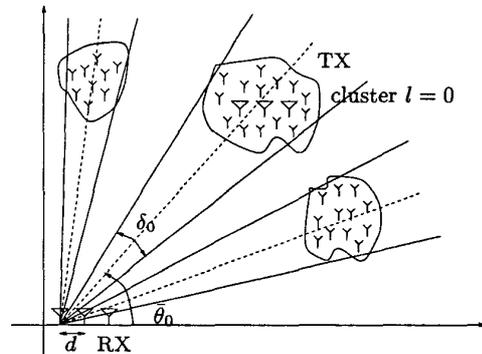


Fig. 1. Space-time delay spread channel composed of multiple clustered paths. Each path cluster has a mean angle of arrival $\bar{\theta}_l$ and an angle spread δ_l .

Factoring the $M_R \times M_R$ correlation matrix \mathbf{R}_l according to $\mathbf{R}_l = \mathbf{R}_l^{1/2}\mathbf{R}_l^{H/2}$, where $\mathbf{R}_l^{1/2}$ is of size $M_R \times M_R$, the $M_R \times M_T$ matrices \mathbf{H}_l can be written as

$$\mathbf{H}_l = \mathbf{R}_l^{1/2}\mathbf{H}_{w,l}, \quad l = 0, 1, \dots, L-1 \quad (4)$$

³The variance σ_{θ_l} is proportional to the angular spread δ_l and hence the scattering radius of the l -th path cluster.

⁴The superscripts T and H stand for transposition and conjugate transposition, respectively.

with the $\mathbf{H}_{w,l}$ being uncorrelated $M_R \times M_T$ matrices with i.i.d. $\mathcal{CN}(0,1)$ entries.

2.2. OFDM-based Multi-Antenna Systems

In an OFDM-based multi-antenna system the data streams are OFDM-modulated before transmission. The OFDM modulator applies an N -point IFFT to N consecutive data symbols and then prepends the cyclic prefix (CP) (which is a copy of the last L samples of the symbol) to the symbol, so that the overall OFDM symbol length is $M = N + L$. In the receiver, the individual signals are passed through an OFDM demodulator which first discards the CP and then applies an FFT. Organizing the transmitted data symbols into frequency vectors $\mathbf{c}_k = [c_k^{(0)} c_k^{(1)} \dots c_k^{(M_T-1)}]^T$ with $c_k^{(i)}$ denoting the data symbol transmitted from the i -th antenna on the k -th tone, the reconstructed data vector for the k -th tone is given by

$$\mathbf{r}_k = \sqrt{E_s} \mathbf{H}(e^{j\frac{2\pi}{N}k}) \mathbf{c}_k + \mathbf{n}_k, \quad k = 0, 1, \dots, N-1, \quad (5)$$

where \mathbf{n}_k is complex-valued additive white gaussian noise satisfying

$$\mathcal{E}\{\mathbf{n}_k \mathbf{n}_l^H\} = \sigma_n^2 \mathbf{I}_{M_R} \delta[k-l] \quad (6)$$

with \mathbf{I}_{M_R} denoting the identity matrix of size M_R . The data symbols $c_k^{(i)}$ are taken from a finite complex alphabet chosen such that the average energy of the constellation elements is 1.

2.3. Space-Frequency Coding

The bit stream to be transmitted is encoded by the space-frequency encoder into blocks of size $M_T \times N$. One data burst therefore consists of N vectors of size $M_T \times 1$ or equivalently one spatial OFDM symbol. The channel is assumed to be constant over at least one OFDM symbol. Assuming perfect channel state information, the maximum likelihood (ML) decoder computes the vector sequence $\hat{\mathbf{c}}_k$, $k = 0, 1, \dots, N-1$ according to

$$\hat{\mathbf{c}}_k = \arg \min_{\mathbf{c}} \sum_{k=0}^{N-1} \|\mathbf{r}_k - \sqrt{E_s} \mathbf{H}(e^{j\frac{2\pi}{N}k}) \mathbf{c}_k\|^2,$$

where $\mathbf{C} = [\mathbf{c}_0 \ \mathbf{c}_1 \ \dots \ \mathbf{c}_{N-1}]$ and the minimization is over all possible codewords. We finally note that an interesting experimental performance analysis of a space-frequency coded OFDM system appears in [20].

3. DESIGN CRITERIA

In this section, we shall derive the design criteria for space-frequency codes assuming that the receiver has perfect channel state information.

3.1. Pairwise Error Probability

Let $\mathbf{C} = [\mathbf{c}_0 \ \mathbf{c}_1 \ \dots \ \mathbf{c}_{N-1}]$ and $\mathbf{E} = [\mathbf{e}_0 \ \mathbf{e}_1 \ \dots \ \mathbf{e}_{N-1}]$ be two different space-frequency codewords of size $M_T \times N$ and assume that \mathbf{C} was transmitted. For a given channel realization $\mathbf{H}(e^{j\frac{2\pi}{N}k})$, the probability that the receiver decides erroneously in favor of the signal \mathbf{E} is given by [21]

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) = Q\left(\sqrt{\frac{E_s}{2\sigma_n^2}} d^2(\mathbf{C}, \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k}))\right),$$

where

$$d^2(\mathbf{C}, \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) = \sum_{k=0}^{N-1} \|\mathbf{H}(e^{j\frac{2\pi}{N}k})(\mathbf{c}_k - \mathbf{e}_k)\|^2$$

denotes the squared Euclidean distance between the two codewords \mathbf{C} and \mathbf{E} . Using the Chernoff bound $Q(x) \leq e^{-x^2/2}$ we get

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) \leq e^{-\frac{E_s}{4\sigma_n^2} d^2(\mathbf{C}, \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k}))}. \quad (7)$$

Next, we need to compute the expected pairwise error probability by averaging over all channel realizations taking into account the channel model presented in Sec. 2.1. For this we define $\mathbf{y}_k = \mathbf{H}(e^{j\frac{2\pi}{N}k})(\mathbf{c}_k - \mathbf{e}_k)$ for $k = 0, 1, \dots, N-1$ and

$$\mathbf{Y} = [\mathbf{y}_0^T \ \mathbf{y}_1^T \ \dots \ \mathbf{y}_{N-1}^T]^T.$$

With this notation we get $d^2(\mathbf{C}, \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) = \|\mathbf{Y}\|^2$ and hence (7) can be rewritten as

$$P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) \leq e^{-\frac{E_s}{4\sigma_n^2} \|\mathbf{Y}\|^2}. \quad (8)$$

Since the \mathbf{H}_l were assumed to be i.i.d. gaussian it follows from (1) that the $\mathbf{H}(e^{j\frac{2\pi}{N}k})$ for $k = 0, 1, \dots, N-1$ are gaussian as well and hence the $M_R N \times 1$ vector \mathbf{Y} is gaussian. The average over all channel realizations of the right-hand side in (8) is fully characterized by the eigenvalues of the covariance matrix of \mathbf{Y} [22] defined as $\mathbf{C}_Y = \mathcal{E}\{\mathbf{Y} \mathbf{Y}^H\}$. In [23] it is shown that

$$\mathbf{C}_Y = \sum_{l=0}^{L-1} \left[\mathbf{D}^l (\mathbf{C} - \mathbf{E})^T (\mathbf{C} - \mathbf{E})^* \mathbf{D}^{lH} \right] \otimes \mathbf{R}_l, \quad (9)$$

where $\mathbf{D} = \text{diag}\{e^{-j\frac{2\pi}{N}k}\}_{k=0}^{N-1}$, $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of the matrices \mathbf{A} and \mathbf{B} and the superscript $*$ stands for elementwise conjugation. Denoting the nonzero eigenvalues of \mathbf{C}_Y as⁵ $\lambda_i(\mathbf{C}_Y)$ ($i = 0, 1, \dots, r(\mathbf{C}_Y) - 1$) the following result can be established [23]

$$P(\mathbf{C} \rightarrow \mathbf{E}) \leq \prod_{i=0}^{r(\mathbf{C}_Y)-1} \left(1 + \lambda_i(\mathbf{C}_Y) \frac{E_s}{4\sigma_n^2} \right)^{-1}, \quad (10)$$

where⁶ $P(\mathbf{C} \rightarrow \mathbf{E}) = \mathcal{E}_H \left\{ P(\mathbf{C} \rightarrow \mathbf{E} | \mathbf{H}(e^{j\frac{2\pi}{N}k})) \right\}$ is the pairwise error probability averaged over all channel realizations. Next, using the following property of Kronecker products

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{F} \otimes \mathbf{G}) = (\mathbf{A}\mathbf{F}) \otimes (\mathbf{B}\mathbf{G})$$

and the factorizations $\mathbf{R}_l = \mathbf{R}_l^{1/2} \mathbf{R}_l^{H/2}$ ($l = 0, 1, \dots, L-1$), it can be shown [23] that

$$\mathbf{C}_Y = \mathbf{G}(\mathbf{C}, \mathbf{E}) \mathbf{G}^H(\mathbf{C}, \mathbf{E}) \quad (11)$$

⁵ $r(\mathbf{A})$ denotes the rank of the matrix \mathbf{A} .

⁶Here, \mathcal{E} stands for the expectation operator.

with the $NM_R \times M_T M_R L$ matrix

$$\mathbf{G}(\mathbf{C}, \mathbf{E}) = \begin{bmatrix} (\mathbf{C} - \mathbf{E})^T \otimes \mathbf{R}_0^{1/2} & [\mathbf{D}(\mathbf{C} - \mathbf{E})^T] \otimes \mathbf{R}_1^{1/2} \\ \dots & [\mathbf{D}^{L-1}(\mathbf{C} - \mathbf{E})^T] \otimes \mathbf{R}_{L-1}^{1/2} \end{bmatrix}. \quad (12)$$

Based on (10), (11) and (12) we are now able to derive the design criteria for space-frequency codes.

3.2. Design Criteria and Diversity Order

In the following, we assume that $N > M_T L$. The design criteria for space-frequency codes follow from (10) as the well-known *rank and determinant criteria* derived in [11, 9] for the single-carrier narrowband case with the matrix $\mathbf{B}(\mathbf{c}, \mathbf{e})$ defined in Eq. 6 of [11] replaced by $\mathbf{G}^T(\mathbf{C}, \mathbf{E})$. Note that for $L = 1$ and $\mathbf{R}_0 = \mathbf{I}$ the matrix $\mathbf{G}^T(\mathbf{C}, \mathbf{E})$ reduces to $(\mathbf{C} - \mathbf{E}) \otimes \mathbf{I}_{M_R}$. Now, using the fact that every eigenvalue of the $M_T \times M_T$ matrix $(\mathbf{C} - \mathbf{E})(\mathbf{C} - \mathbf{E})^T$ is an eigenvalue of the $M_T M_R \times M_T M_R$ matrix $[(\mathbf{C} - \mathbf{E})(\mathbf{C} - \mathbf{E})^T] \otimes \mathbf{I}_{M_R}$ with multiplicity M_R , it follows that the design criteria in an OFDM system with no delay spread are equivalent to those in a single-carrier-based narrowband system. This is intuitively clear since for $L = 1$ there is no frequency diversity.

Let us next use (12) to establish some results on the maximum achievable diversity order in broadband space-frequency coded OFDM systems. The maximum rank of the $NM_R \times M_T M_R L$ matrix $\mathbf{G}(\mathbf{C}, \mathbf{E})$ is $M_T M_R L$. Since $r(\mathbf{R}^{1/2}) = r(\mathbf{R})$ we can establish the following [23]:

Theorem 1. The maximum achievable diversity order in a space-frequency coded broadband OFDM system is given by

$$d_{max} = s \sum_{l=0}^{L-1} r(\mathbf{R}_l) \leq M_T M_R L,$$

where s is the minimum rank of $(\mathbf{C} - \mathbf{E})$ over all pairs of codewords \mathbf{C} and \mathbf{E} .

From (12) it follows that the l -th multipath can potentially add a diversity order of $s r(\mathbf{R}_l)$. Since the rank of \mathbf{R}_l will be governed by the angle spread of the l -th scatterer cluster and the antenna spacing at the receiver, we conclude that the achievable diversity order critically depends on the propagation environment and the antenna spacing. In [23] it is shown that in order to achieve $M_T M_R L$ -fold diversity it is necessary to have $r(\mathbf{D}^i(\mathbf{C} - \mathbf{E})^T) = M_T$ for $i = 0, 1, \dots, L - 1$ and for all pairs of codewords \mathbf{C} and \mathbf{E} . Furthermore, it is required that $r(\mathbf{R}_l) = M_R$ for $l = 0, 1, \dots, L - 1$. These conditions guarantee that the individual $NM_R \times M_T M_R$ matrices $[\mathbf{D}^i(\mathbf{C} - \mathbf{E})^T] \otimes \mathbf{R}_i$ ($i = 0, 1, \dots, L - 1$) have full rank. Finally, the space-frequency code has to be designed such that the stacked matrix $\mathbf{G}(\mathbf{C}, \mathbf{E})$ has full rank as well.

In the following, in order to simplify the discussion we restrict our attention to the case of no spatial fading correlation and a uniform power delay profile, i.e., $\mathbf{R}_l = \mathbf{I}_{M_R}$ for

$l = 0, 1, \dots, L - 1$. In this case we obtain from (9)

$$\mathbf{C}_Y = \underbrace{\sum_{l=0}^{L-1} [\mathbf{D}^l(\mathbf{C} - \mathbf{E})^T (\mathbf{C} - \mathbf{E})^* \mathbf{D}^{lH}]}_{\mathbf{R}_Y} \otimes \mathbf{I}_{M_R}$$

which implies that the rank of \mathbf{C}_Y will be M_R times the rank of the $N \times N$ matrix

$$\mathbf{R}_Y = \mathbf{F}(\mathbf{C}, \mathbf{E}) \mathbf{F}^H(\mathbf{C}, \mathbf{E})$$

with the $N \times M_T L$ matrix

$$\mathbf{F}(\mathbf{C}, \mathbf{E}) = [(\mathbf{C} - \mathbf{E})^T \mathbf{D}(\mathbf{C} - \mathbf{E})^T \dots \mathbf{D}^{L-1}(\mathbf{C} - \mathbf{E})^T]. \quad (13)$$

The coding gain in this case can be obtained by making use of the fact that every eigenvalue of \mathbf{R}_Y is an eigenvalue of \mathbf{C}_Y with multiplicity M_R . The question of what diversity order a given space-frequency code achieves now reduces to the question of finding the minimum rank of the $N \times M_T L$ matrix $\mathbf{F}(\mathbf{C}, \mathbf{E})$ over the set of all possible codewords \mathbf{C} and \mathbf{E} . In order to achieve full (i.e. $M_T M_R L$ -fold) diversity the matrix $\mathbf{F}(\mathbf{C}, \mathbf{E})$ has to be full rank for every pair of codewords \mathbf{C} and \mathbf{E} . This is the case when $(\mathbf{C} - \mathbf{E})$ has rank M_T for all codewords \mathbf{C} and \mathbf{E} and each of the blocks $\mathbf{B}_i = \mathbf{D}^i(\mathbf{C} - \mathbf{E})^T$ is linearly independent of the other \mathbf{B}_l with $l \neq i$ for every pair \mathbf{C} and \mathbf{E} . While a space-time code designed to achieve full diversity in the narrowband case will have full rank $(\mathbf{C} - \mathbf{E})$ and hence full rank \mathbf{B}_i ($i = 0, 1, \dots, L - 1$) for all codewords \mathbf{C} and \mathbf{E} , the linear independence of the blocks \mathbf{B}_i will not be guaranteed. Note that ensuring linear independence of the blocks \mathbf{B}_i amounts to ensuring that the space-frequency code exploits the available frequency diversity as well. We can, however, make the following statement:

Theorem 2. A space-time code designed to provide a diversity order of s in a single-carrier-based narrowband system provides at least the same diversity order in a broadband OFDM system.

The proof of Theorem 2 follows immediately by noting that if $(\mathbf{C} - \mathbf{E})$ has minimum rank s over the set of all possible codewords \mathbf{C} and \mathbf{E} , then the minimum rank of $\mathbf{F}(\mathbf{C}, \mathbf{E})$ will be at least s . A result similar to Theorem 2 was reported in [24] for single-carrier systems operating in delay-spread environments. We emphasize, however, that a space-time code achieving full spatial diversity in a narrowband environment will in general not achieve full space-frequency diversity in the OFDM broadband case. The next subsection provides an example corroborating this statement. In [23] it is furthermore shown that space-time codes designed for the narrowband *fast fading* case will in general not be guaranteed to achieve full space-frequency diversity in the OFDM broadband case. In fact they may not even achieve full spatial diversity [23]. We note, however, that even though space-time codes designed for the fast fading case and smart greedy space-time codes [11] will be suboptimum in a space-frequency setting, they can be expected to exploit at least some of the available frequency diversity. Summarizing, we conclude that using existing space-time codes in a space-frequency setting will in general be suboptimum. New designs are needed taking into account the criteria derived in this paper. Some first

designs of space-frequency codes are provided in [23]. The systematic design of good space-frequency codes remains an important open research problem.

3.3. Space-Frequency Alamouti Scheme

In the previous subsection we found that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not achieve full space-frequency diversity. We shall next provide an example illustrating this effect. Specifically, we consider the case of two transmit antennas (M_R is arbitrary) and employ the Alamouti scheme [10] across OFDM tones, i.e., the matrices $(\mathbf{C} - \mathbf{E})$ have the following structure

$$(\mathbf{C} - \mathbf{E}) = \begin{bmatrix} d_0 & -d_1^* & d_2 & -d_3^* & \dots & d_{N-2} & -d_{N-1}^* \\ d_1 & d_0^* & d_3 & d_2^* & \dots & d_{N-1} & d_{N-2}^* \end{bmatrix}$$

with $d_i = c_i - e_i$ ($i = 0, 1, \dots, N-1$). From

$$(\mathbf{C} - \mathbf{E})(\mathbf{C} - \mathbf{E})^H = \sum_{i=0}^{N-1} |d_i|^2 \mathbf{I}_2$$

it follows that all possible code difference matrices \mathbf{C} and \mathbf{E} will be of full rank and hence the code achieves full spatial diversity in the narrowband case. This is a well-known fact and has been proved in [10]. Next, take the minimum distance error event where only one out of the N d_i is nonzero say d_0 . In that case the matrix $(\mathbf{C} - \mathbf{E})^T$ will have nonzero entries only in the two top rows. Therefore, all the matrices $\mathbf{D}^i(\mathbf{C} - \mathbf{E})^T$ ($i = 0, 1, \dots, L-1$) will have nonzero entries in the two top rows only and hence it follows from (13) that the matrix $\mathbf{F}(\mathbf{C}, \mathbf{E})$ will have rank 2 irrespectively of the amount of delay spread (and hence frequency diversity) in the channel. We conclude that while the Alamouti scheme continues to achieve second-order diversity in the broadband OFDM case, it fails to exploit the additionally available frequency diversity. This once again shows that the broadband OFDM case calls for new code designs.

4. SIMULATION RESULTS

In this section, we provide simulation results demonstrating the performance of known space-time codes employed as space-frequency codes and studying the influence of physical propagation parameters on the performance of space-frequency codes. We simulated an OFDM system with $M_T = 2$, $N = 64$ tones and CP of length 16 using the two transmit antenna 16-state 4-PSK code proposed in [11]. The signal-to-noise-ratio (SNR) was defined as $\text{SNR} = 10 \log \left(\frac{E_s}{\sigma_n^2} \right)$. All results were obtained by averaging over 10,000 independent Monte Carlo trials, where each realization consisted of one burst (i.e. one OFDM symbol).

Simulation Example 1. In the first simulation example we study the impact of delay spread on the performance of space-frequency codes. For $M_R = 2$ and uncorrelated spatial fading Fig. 2 shows the symbol error rate as a function of SNR for the low delay spread case (i.e. one path with $|\beta_0|^2 = 1$) and for the high delay spread case (i.e. six paths with $|\beta_i|^2 = 1$), respectively. We can see that the presence of delay spread drastically improves the performance of the

space-frequency code. From the slopes of the two curves we can conclude that the space-frequency code does indeed seem to be able to exploit at least some of the available frequency diversity, but certainly not all of it. Recall that since $M_T = M_R = 2$ and six paths with path gain 1 are present the channel provides 24-th order diversity.

Simulation Example 2. In Sec. 3.2 we have demonstrated that the rank of the correlation matrices \mathbf{R}_l has critical impact on the available diversity. Furthermore, we showed in Sec. 2.1 that the rank of the correlation matrices is driven by the angle spread of the individual scatterer clusters and by the antenna spacing at the receiver. In this simulation example, we study the influence of spatial fading correlation on the performance of space-frequency codes. We assumed the following power delay profile [1 0.77 0.56]. Fig. 3 shows the symbol error rate for $M_R = 2$ and $M_R = 3$ and low and high spatial fading correlation, respectively. The solid curves correspond to $M_R = 2$ whereas the dashed-dotted curves correspond to $M_R = 3$. The upper two curves show the symbol error rate in the case of high spatial fading correlation which was achieved by making the cluster angle spreads small and choosing the relative antenna spacing to be $\Delta = 1/4$. Low spatial fading correlation was achieved by setting $\Delta = 1/2$ and choosing large cluster angle spreads. We can clearly see that in the case of low spatial fading correlation the performance of the code is significantly better than in the case of high spatial fading correlation. This is consistent with our findings (and intuition) that high spatial fading correlation reduces the diversity order and hence yields degraded performance in terms of symbol error rate. We can furthermore see that adding a receive antenna has much less impact in the case of high spatial fading correlation. This is so, since due to the small cluster angle spreads and hence large spatial fading correlation only a small increase in spatial diversity can be expected from additional receive antennas.

5. CONCLUSION

We studied space-frequency coded broadband OFDM systems where both spatial and frequency diversity are available. Considering a strategy which consists of coding across OFDM tones and employing a spatial broadband channel model taking into account physical propagation parameters and antenna spacing, we *derived the design criteria for space-frequency codes*. We furthermore showed that space-time codes designed to achieve full spatial diversity in the narrowband case will in general not achieve full space-frequency diversity in the broadband case. Space-time codes designed for the fast fading case and smart greedy space-time codes [11] can be expected to be able to exploit at least some of the available frequency diversity. Nevertheless, these codes will generally be suboptimum when employed as space-frequency codes. Hence, new designs taking into account the design criteria derived in this paper are needed. We furthermore *established the maximum achievable diversity order* in space-frequency coded OFDM systems and we studied the impact of spatial fading correlation on the performance of space-frequency codes.

Our simulation results showed that space-time Trellis codes designed for the narrowband case do seem to exploit at least some of the available frequency diversity (but certainly not all of it) when employed as space-frequency codes.

Acknowledgment

The authors would like to thank R. W. Heath Jr. for providing part of the MATLAB code used to obtain the simulation results in Sec. 4.

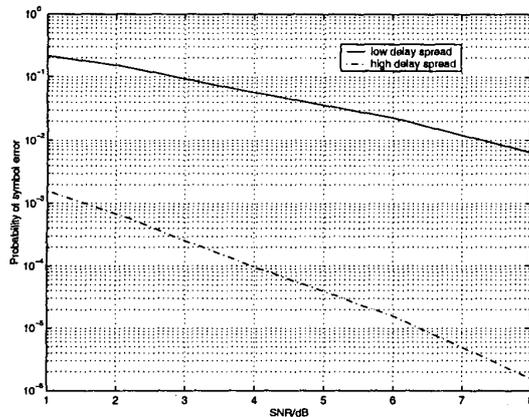


Fig. 2. Impact of delay spread on the performance of space-frequency codes.

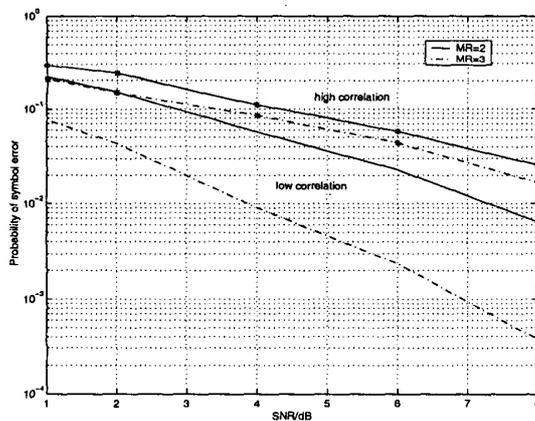


Fig. 3. Impact of spatial fading correlation and M_R on the performance of space-frequency codes.

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