

1. The authors are grateful to Weigutian Ou for pointing out an issue affecting Lemma II.6 and Lemma A.8. Specifically, in Lemma II.6 and Lemma A.8,  $\mathcal{B}(\Psi) = \max_i \{|a_i| \mathcal{B}(\Phi_i)\}$  should be replaced by

$$\mathcal{B}(\Psi) \leq \max \left\{ \sum_{i=1}^n |a_i| \mathcal{B}(\Phi_i), \max_{i \in \{1, \dots, n\}} \mathcal{B}(\Phi_i), \max_{i \in \{1, \dots, n\}} |a_i| \mathcal{B}(\Phi_i) \right\}. \quad (1)$$

In Lemma II.6 this bound is obtained as follows. Given networks  $\Phi_i \in \mathcal{N}_{d_i, d'}$  with associated matrix-vector pairs  $(A_\ell^i, b_\ell^i)$ ,  $i \in \{1, \dots, n\}$ ,  $\ell \in \{1, \dots, L\}$ , the first  $L-1$  matrix-vector pairs of the network  $\Psi \in \mathcal{N}_{\sum_{i=1}^n d_i, d'}$  are obtained as

$$A_\ell = \text{diag}(A_\ell^1, A_\ell^2, \dots, A_\ell^n), \quad b_\ell = (b_\ell^1, b_\ell^2, \dots, b_\ell^n).$$

Hence, we have, for all  $\ell \in \{1, \dots, L-1\}$ , that  $\|A_\ell\|_\infty \leq \max_i \mathcal{B}(\Phi_i)$  and  $\|b_\ell\|_\infty \leq \max_i \mathcal{B}(\Phi_i)$ . In the last layer, i.e., for  $\ell = L$ , we have

$$A_L = (a_1 A_L^1, a_2 A_L^2, \dots, a_n A_L^n), \quad b_L = \sum_{i=1}^n a_i b_L^i,$$

and therefore  $\|A_L\|_\infty \leq \max_i \{|a_i| \mathcal{B}(\Phi_i)\}$  and  $\|b_L\|_\infty \leq \sum_{i=1}^n |a_i| \mathcal{B}(\Phi_i)$ . In the case of Lemma A.8, (1) follows upon noting that the network  $\Psi \in \mathcal{N}_{d, d'}$  is specified by the same matrix-vector pairs as in the modified Lemma II.6, except for the matrix  $A_1$ , which is instead given by

$$A_1 = \begin{pmatrix} A_1^1 \\ \vdots \\ A_1^n \end{pmatrix}.$$

Note that in the special case  $a_i \geq 1$ ,  $b_L^i = 0$ , both for all  $i \in \{1, \dots, n\}$ , the original claim  $\mathcal{B}(\Psi) = \max_i \{|a_i| \mathcal{B}(\Phi_i)\}$  remains correct.

These corrections do not affect the instances where Lemma II.6 and Lemma A.8 are applied. Specifically, in the proof of Theorem VII.2,

$$\mathcal{B}(\Psi_{f, M}) \leq \pi_3(M), \quad \text{for all } f \in \mathcal{C}, M \in \mathbb{N},$$

still holds for some (other) polynomial  $\pi_3$  due to  $I_{f, M} \subseteq \{1, \dots, \pi_1(M)\}$  and  $\max_{i \in I_{f, M}} |c_i| \leq \pi_1(M)$ .

In the proof of Theorem VIII.10, the modification resulting from the correction in Lemma II.6 yields an upper bound on the weight magnitude of the networks approximating  $\psi_m$  which continues to be independent of  $\varepsilon$  and, as such, does not lead to changes in the subsequent arguments.

Finally, in the proof of Lemma IX.5, we consider a linear combination (with  $a_i = 1$ , for all  $i \in \{1, \dots, d\}$ ) of squaring networks in order to

obtain an approximation of  $f(x) = \sum_{i=1}^d x_i^2$ . As  $b_{m+1} = 0$  in the proof of Proposition III.2, the bound  $\mathcal{B}(\Psi_{d,D,\varepsilon}) \leq 1$  in (84) remains valid thanks to the comment at the end of the first paragraph in this document.

2. In the third paragraph on p.32, the formulation “let  $\mathcal{D} = \{\varphi_i\}_{i \in \mathbb{N}}$  be an ordered orthonormal basis for  $\mathcal{C}$ ” is meant to say that  $\mathcal{D} = \{\varphi_i\}_{i \in \mathbb{N}}$  is an ordered orthonormal basis for a space that contains  $\mathcal{C}$ .
3. In the paragraph following Definition IV.3, “for a given  $x \in \mathcal{X}$ ” should read “for a given  $x \in \mathcal{C}$ ” and “the resulting error satisfies  $\|D(E(x)) - x\| \leq \varepsilon$ ” should read “the resulting error satisfies  $\rho(D(E(x)), x) \leq \varepsilon$ ”.
4. In Definition IV.4, “ $M(\varepsilon; \mathcal{X}, \rho)$ ” should read “ $M(\varepsilon; \mathcal{C}, \rho)$ ”.
5. In the caption of Table 1 it should say “bounded Lipschitz domain” instead of “Lipschitz domain”.
6. On p.29, the second and third display should read as follows:

Now, note that

$$\begin{aligned} \left\| \sum_{i \in \tilde{I}_{f, \bar{M}}} \tilde{c}_i \tilde{\varphi}_i \right\|_{L^2(\Omega)} &= \left\| f - \left( f - \sum_{i \in \tilde{I}_{f, \bar{M}}} \tilde{c}_i \tilde{\varphi}_i \right) \right\|_{L^2(\Omega)} \\ &\leq \|f\|_{L^2(\Omega)} + \left\| f - \sum_{i \in I_{f, M}} c_i \varphi_i \right\|_{L^2(\Omega)}. \end{aligned}$$

Making use of the orthonormality of the  $\tilde{\varphi}_i$ , we can conclude that

$$\sum_{i \in \tilde{I}_{f, \bar{M}}} |\tilde{c}_i|^2 \leq \left( \sup_{f \in \mathcal{C}} \|f\|_{L^2(\Omega)} + CM^{-\gamma} \right)^2.$$

7. The authors are grateful to Erwin Riegler for pointing out the following issues in Appendix B. The first sentence should read “We consider, for  $m \in \mathbb{R}_+$  and  $p, q \in [1, \infty]$ , the Besov space ...” and the first paragraph on p.75 should be replaced by the following:

In order to find an upper bound on  $\|(\|A_n(f)\|_{\ell^2})_{n=N+1}^\infty\|_{\ell^2}$ , we first note that

$$\|\cdot\|_{\ell^\alpha} \leq \|\cdot\|_{\ell^\beta}, \quad \leq \beta \leq \alpha \leq \infty, \quad (2)$$

holds for sequences as well as vectors. For  $x \in \mathbb{R}^d$  and  $\alpha \in [2, \infty]$ , we have, in addition, that

$$\|x\|_{\ell^2} \leq d^{\frac{1}{2} - \frac{1}{\alpha}} \|x\|_{\ell^\alpha}, \quad (3)$$

which is a direct consequence of Hölder's inequality applied to the vectors  $(x_1^2, \dots, x_d^2)$  and  $(1, \dots, 1)$  with corresponding Hölder exponents  $\frac{\alpha}{2}$  and  $\frac{\alpha}{\alpha-2}$ , respectively. Upon noting that the  $A_n(f)$  are vectors of length  $|\mathcal{D}_n| = 2^n$ , application of (3) combined with (2) yields, for  $p \in [1, \infty]$ , that

$$\|A_n(f)\|_{\ell^2} \leq 2^{n(\frac{1}{2} - \frac{1}{p})_+} \|A_n(f)\|_{\ell^p}, \quad (4)$$

where  $x_+ := \max\{0, x\}$ . Consequently, we get, for all  $f \in \mathcal{U}(B_{p,q}^m([0, 1]))$ ,  $p \in [1, \infty]$ ,  $q \in [1, 2]$ , that

$$\begin{aligned} & \|(\|A_n(f)\|_{\ell^2})_{n=N+1}^\infty\|_{\ell^2} \\ & \stackrel{(a)}{\leq} \| (2^{n(\frac{1}{2} - \frac{1}{p})_+} \|A_n(f)\|_{\ell^p})_{n=N+1}^\infty \|_{\ell^q} \\ & \stackrel{(b)}{\leq} 2^{-(N+1)(m - (\frac{1}{p} - \frac{1}{2})_+)} \| (2^{n(m + \frac{1}{2} - \frac{1}{p})} \|A_n(f)\|_{\ell^p})_{n=N+1}^\infty \|_{\ell^q} \\ & \stackrel{(c)}{\leq} 2^{-(N+1)(m - (\frac{1}{p} - \frac{1}{2})_+)} \|f\|_{m,p,q} \\ & \stackrel{(d)}{\leq} (2^{N+1})^{-(m - (\frac{1}{p} - \frac{1}{2})_+)}, \end{aligned}$$

which establishes (94) with  $C = 1$  and  $\beta = m - (\frac{1}{p} - \frac{1}{2})_+$ . Note that for (a) we used (4) for the inner norm and (2) for the outer norm. For (b) we employed the fact that the sequence starts with index  $n = N + 1$  and we used that  $(a - b)_+ = (b - a)_+ + a - b$ , for  $a, b \in \mathbb{R}$ , which yields

$$\begin{aligned} 2^{n(\frac{1}{2} - \frac{1}{p})_+} &= 2^{n((\frac{1}{p} - \frac{1}{2})_+ + \frac{1}{2} - \frac{1}{p})} \\ &= 2^{-n(m - (\frac{1}{p} - \frac{1}{2})_+)} 2^{n(m + \frac{1}{2} - \frac{1}{p})}, \end{aligned}$$

for  $n, m \in \mathbb{R}$  and  $p \in [1, \infty]$ , with the convention  $\frac{1}{\infty} = 0$ . Finally, (c) and (d) follow by application of the definitions in (92) and (93), respectively.